Teknologi A
Mekanik
Lösningar

I detta dokument finns lösningsförslag till samtliga uppgifter i boken ”Teknologi A – Mekanik”.

På YouTube kanalen, ”Lösningar Teknologi A Mekanik”, finns inspelade lösningsförslag till många av uppgifterna. Det finns även en spellista med föreläsningar som är anpassad till boken (dock ej kombinerade lastfall och knäckning).

Jag använder mig att Karl Björks ”Formler och Tabeller för Mekanisk konstruktion” när jag löser uppgifterna.

Om du inte har en formelsamling i maskinkonstruktion så rekommenderar jag starkt Karl Björks formelsamling. Denna finns att beställa på bjorksforlag.se

En komplett översikt av kanalen och materialet finns på edume.nu

/Madeleine

YouTube SE https://www.youtube.com/channel/UCZWty6uAUlkab9XyHQIAu9Q

Madeleine Hermann

EduME – Education and Mechanical Engineering

2.1 $\rho = 2{,}65 \cdot 10^3 \ kg/m^3$

$V = 1{,}21 \cdot 2{,}03 \cdot 0{,}545 = 1{,}339 \ m^3$

$m = \rho \cdot V = 2{,}65 \cdot 10^3 \cdot 1{,}339 = 3550 \ kg$

2.2 $L = 547 \ mm$

$d = 40{,}0 \ mm$

$\rho = 7{,}80 \cdot 10^3 \ kg/m^3$

$G = mg = V \cdot \rho \cdot g = 0{,}547 \cdot \frac{0{,}04^2 \cdot \pi}{4} \cdot 7{,}8 \cdot 10^3 \cdot 9{,}81 = 52{,}6 \ N$

2,3 $d_y = 40 mm$ $d_i = 35 mm$

a) $m = V \cdot \rho = 0{,}547 \cdot \left(\frac{0{,}04^2 \cdot \pi}{4} - \frac{0{,}035^2 \cdot \pi}{4} \right) \cdot 7{,}8 \cdot 10^3 =$

$= 0{,}547 \ \frac{(0{,}04^2 - 0{,}035^2)\pi}{4} \cdot 7{,}8 \cdot 10^3 = 1{,}257 \ kg$

b) $\% \text{ viktförändring} = \frac{m_{rör}}{m_{stång}} = \frac{A_{rör} \cdot L \cdot \rho}{A_{stång} \cdot L \cdot \rho} = \frac{A_{rör}}{A_{stång}}$

$\% \text{ viktförändring} = \frac{(40^2 - 35^2) \cdot \pi \cdot 4}{40^2 \cdot \pi \cdot 4} = 0{,}2344$

röret har 23,4% av stångens massa

Minskningen blir $\underline{\underline{76{,}6\%}}$

2,4 IPE 180 =

$L = 5{,}50\,m \qquad m = 18{,}8\ kg/m$

$G = m \cdot L \cdot g = 18{,}8 \cdot 5{,}50 \cdot 9{,}81 = 1\,kN$

2.5 a) $1{,}51 \cdot 10^5\ mm = 1{,}51 \cdot 10^2\ m$

b) $2{,}00 \cdot 10^7\ mm^2 = 2{,}00 \cdot 10^7 \cdot (10^{-3})^2 = 20\ m^2$

c) $3{,}10\ m^3 = 3{,}10 \cdot 100^3 = 3{,}10 \cdot 10^6\ cm^3$

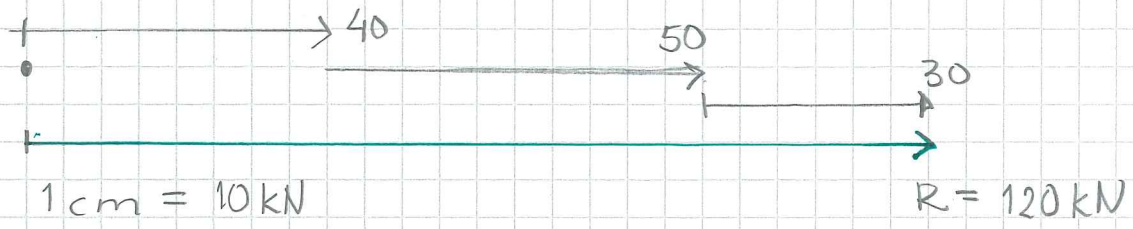
d) $1{,}00\ N/mm^2 = 0{,}001\ kN/mm^2 = 0{,}001 \frac{1}{0{,}001^2} = 1000\ kN/m^2$

2.6 a) $1{,}00 \cdot 10^6\ mm = 1{,}00\ km$

b) $2{,}05 \cdot 10^{11}\ W = 205\ GW$

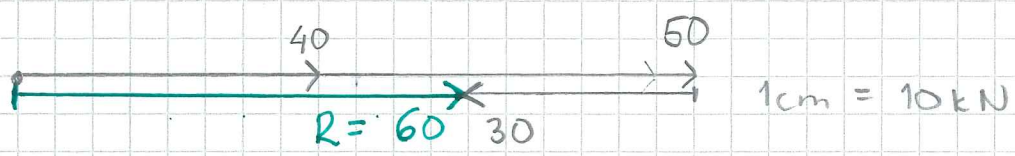
c) $2{,}05 \cdot 10^{-4}\ s = 205\ \mu s = 0{,}205\ ms$

3.1 a)



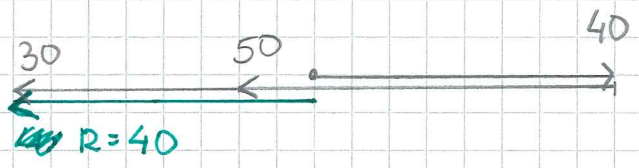
$\rightarrow R_x = 40 + 50 + 30 = 120$ kN.

b



$\rightarrow R_x = 40 + 50 - 30 = 60$ kN

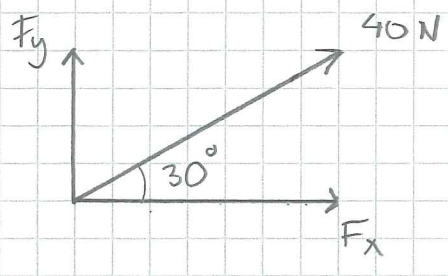
c



$\rightarrow R_x = 40 - 50 - 30 = -40$ kN

3.2

a)



1cm = 10 N

Mätning => $F_x = 35$ N

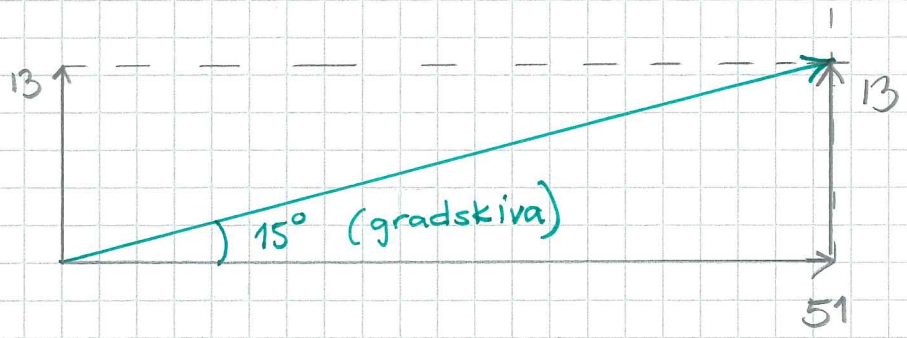
$F_y = 20$ N

b) $F_x = 40 \cos 30° = 34{,}6$ N

$F_y = 40 \sin 30° = 20$ N

3.3

a) 2 cm = 10 N



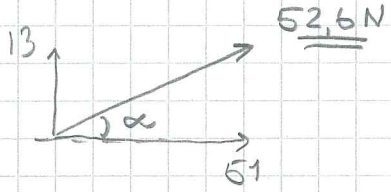
R = 52,5 N (10,5 cm)

b) $\rightarrow R_x = 51\ N$

$\uparrow R_y = 13\ N$

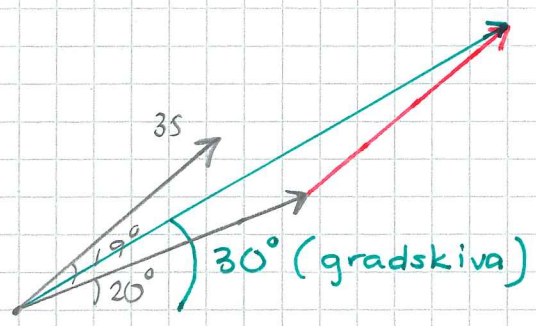
$R = \sqrt{51^2 + 13^2} = 52{,}6\ N$

Skiss



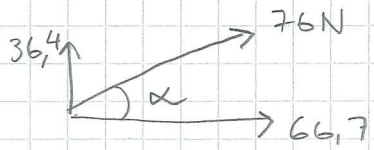
$\tan\alpha = \frac{13}{51} \Rightarrow \alpha = \arctan\frac{13}{51} = \underline{\underline{14{,}3°}}$

3.4 a)



R = 75 kN (7,5 cm)

Skiss



b $\rightarrow R_x = 42\cos 20° + 35\cos(20° + 19°) = 66{,}7\ kN$

$\uparrow R_y = 42\sin 20° + 35\sin(20° + 19°) = 36{,}4\ kN$

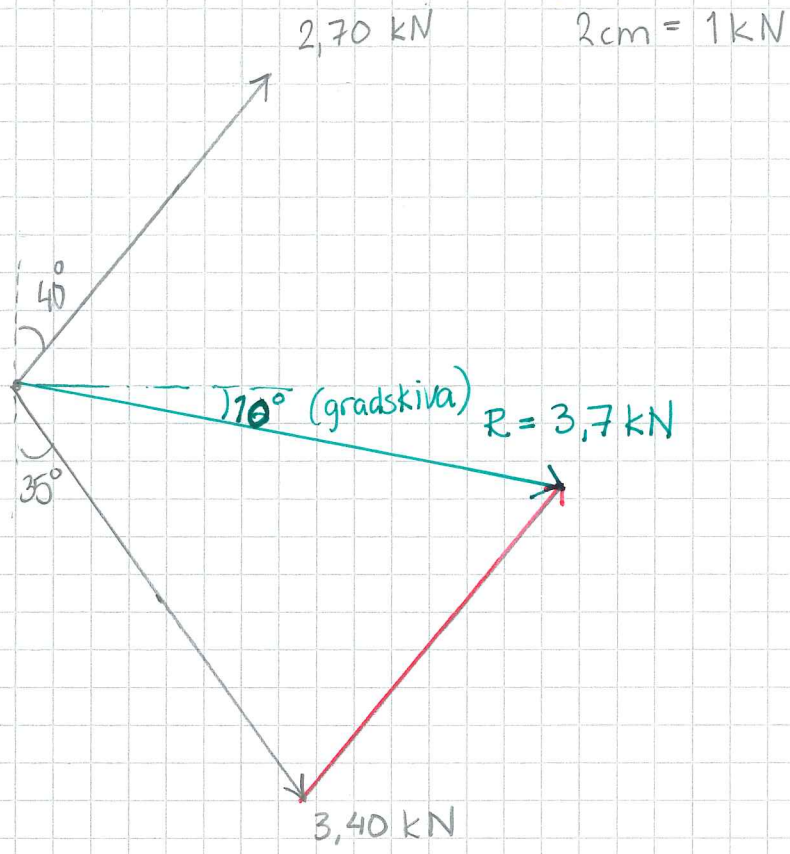
$R = \sqrt{66{,}7^2 + 36{,}4^2} = 76\ kN$

$\alpha = \arctan\left(\frac{36{,}4}{66{,}7}\right) = \underline{\underline{28{,}6°}}$

3.5

• Flytta längs verkningslinje för gemensam angreppspunk.

a)



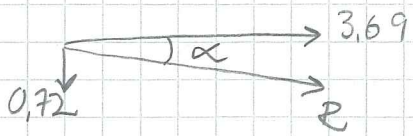
b) $\vec{R}_x = 2{,}7 \sin 40^\circ + 3{,}4 \sin 35^\circ = 3{,}69 \text{ kN}$

$\uparrow R_y = 2{,}7 \cos 40^\circ - 3{,}4 \cos 35^\circ = -0{,}72 \text{ kN}$

$R = \sqrt{3{,}69^2 + 0{,}72^2} = \underline{\underline{3{,}76 \text{ kN}}}$

$\alpha = \arctan\left(\frac{0{,}72}{3{,}69}\right) = \underline{\underline{11^\circ}}$

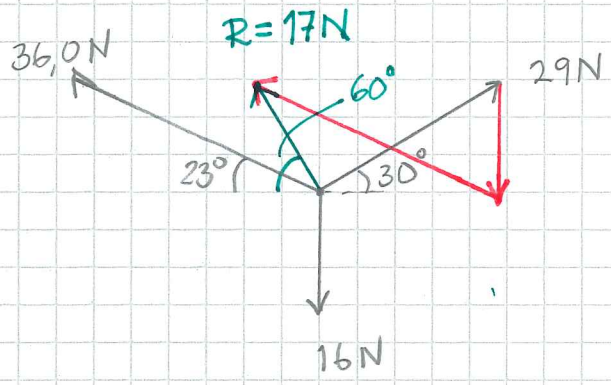
Skiss



3.6

1cm = 10N

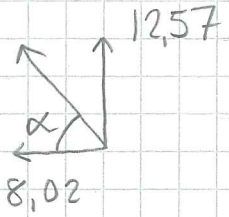
Huvudsvans (eller kraftpolygon)



$\vec{R}_x = 29\cos 30° - 36\cos 23° = -8{,}02\ N$

$\uparrow R_y = 29\sin 30° + 36\sin 23° - 16 = 12{,}57\ N$

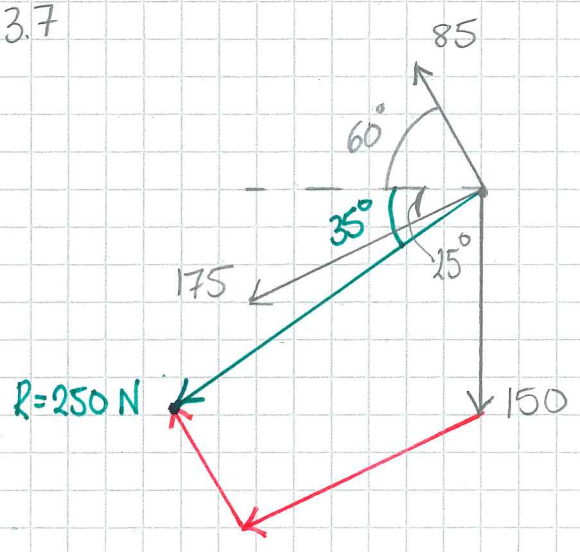
Skiss



$R = \sqrt{8{,}02^2 + 12{,}57^2} = \underline{\underline{14{,}9\ N}}$

$\alpha = \arctan \frac{12{,}57}{8{,}02} = \underline{\underline{57{,}5°}}$

3.7



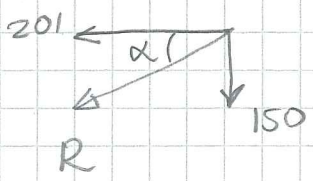
1cm = 50N

$\uparrow R_y = 85 \sin 60^\circ - 175 \sin 25^\circ - 150 = -150 \text{ N}$

$\rightarrow R_x = -85 \cos 60^\circ - 175 \cos 25^\circ = -201$

Skiss

$R = \sqrt{150^2 + 201^2} = \underline{\underline{251 \text{ N}}}$



$\alpha = \arctan\left(\frac{150}{201}\right) = \underline{\underline{36{,}7^\circ}}$

3,8 a Huvud-svansmetoden

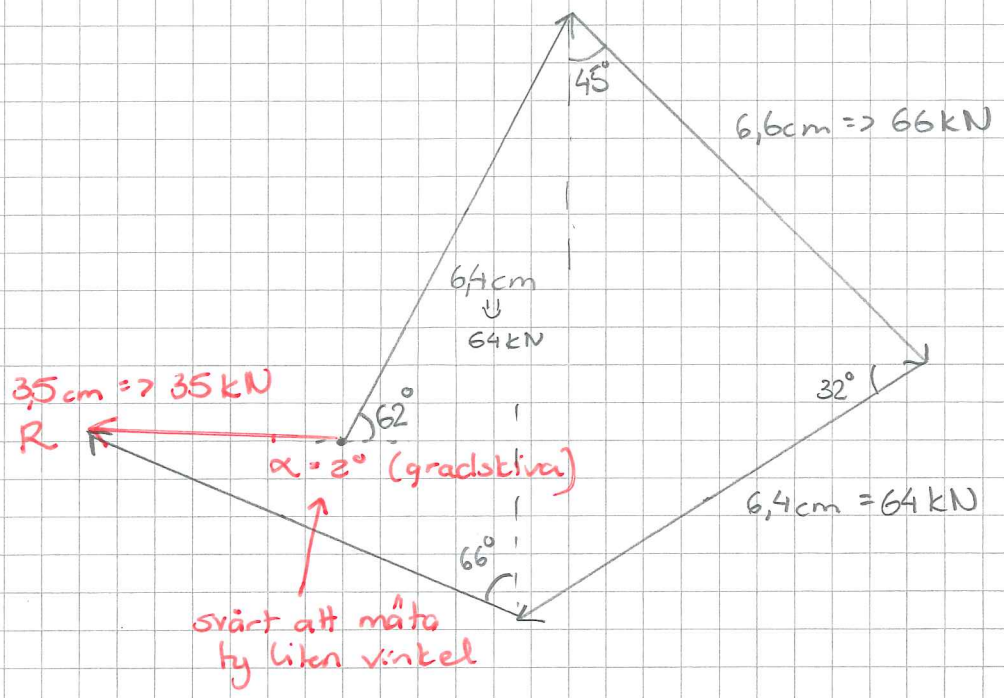
Börja med en av krafterna (valfri kraft)

Jag väljer 64 kN (62°)

Skalan jag väljer 1 cm = 10 kN

Rita med rätt riktning (gradskiva)

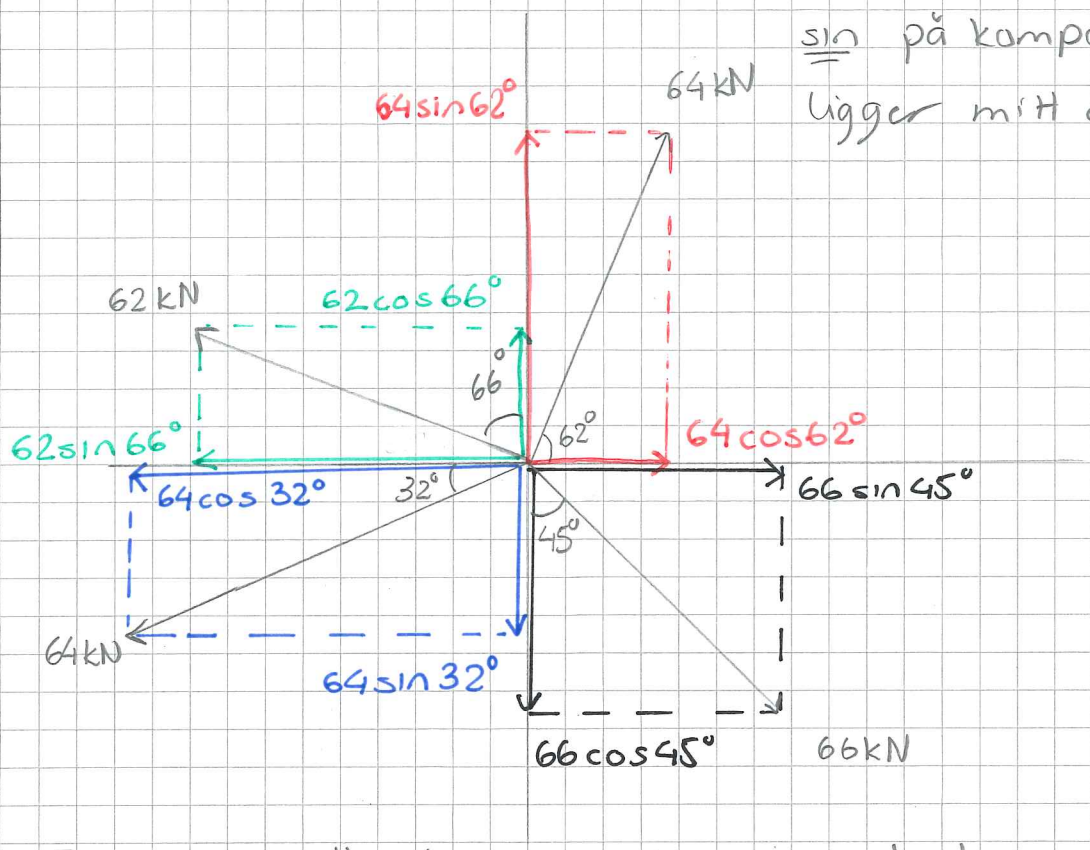
Fortsätt med 66 kN, 64 kN (66°+56°-90° = 32°), 62 kN (66°)



3:8 b

cos på komposant som ligger nära vinkeln.

sin på komposant som ligger mitt emot.



Summera alla komposanter i x-led

pilen anger min valda positiva riktning

$\rightarrow R_x = 64\cos 62° + 66\sin 45° - 62\sin 66° - 64\cos 32°$

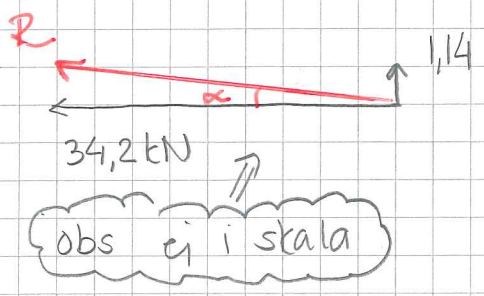
$R_x = -34{,}2$ kN.

positivt uppåt

$\uparrow R_y = 64\sin 62° + 62\cos 66° - 64\sin 32° - 66\cos 45°$

$R_y = 1{,}14$ kN

Rita R_x & R_y



$R = \sqrt{34{,}2^2 + 1{,}14^2} = 34{,}2$ kN

ty liten R_y

$\tan\alpha = \frac{1{,}14}{34{,}2}$

$\alpha = \arctan\left(\frac{1{,}14}{34{,}2}\right) = 1{,}9°$

3.8 a) Lös grafiskt med huvud/svans-metoden (kraftpolygon)

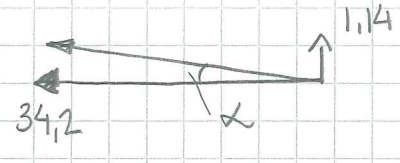
b) $\overrightarrow{R_x} = 64\cos 62^\circ - 62\sin 66^\circ - 64\cos 32^\circ + 66\sin 45^\circ$

$R_x = -34{,}2\ kN$

$\uparrow R_y = 64\sin 62^\circ + 62\cos 66^\circ - 64\sin 32^\circ - 66\cos 45^\circ$

$R_y = 1{,}14\ kN$

Skiss

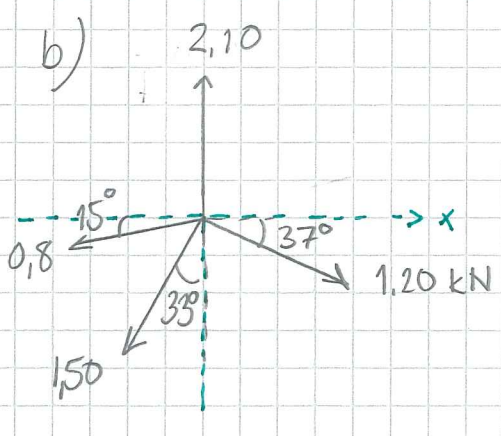


$R = \sqrt{34{,}2^2 + 1{,}14^2} = 34{,}2\ kN$

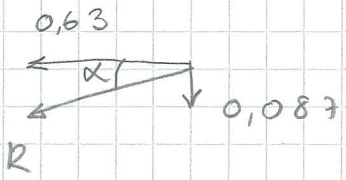
$\alpha = \arctan\left(\frac{1{,}14}{34{,}2}\right) = 1{,}9^\circ$

3.9 a) se ovan

b)



Skiss



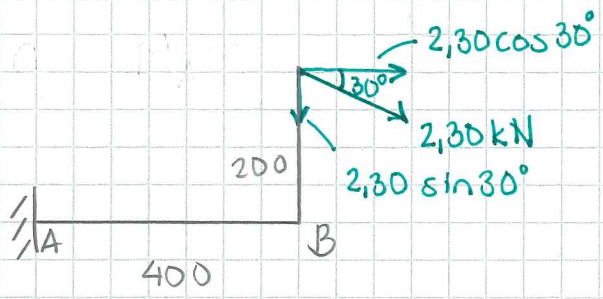
$\alpha = \arctan\left(\frac{0{,}087}{0{,}63}\right) = \underline{\underline{7{,}9^\circ}}$

$\uparrow R_y = 2{,}10 - 0{,}8\sin 15^\circ - 1{,}5\cos 33^\circ - 1{,}20\sin 37^\circ = -0{,}087$

$\rightarrow R_x = 1{,}20\cos 37^\circ - 0{,}8\cos 15^\circ - 1{,}50\sin 33^\circ = -0{,}63$

$R = \sqrt{0{,}087^2 + 0{,}63^2} = \underline{\underline{0{,}63\ kN}}$

3.10



Anta: mm

a) $\overset{\curvearrowright}{M_A} = 2{,}30 \sin 30^\circ \cdot 0{,}4 + 2{,}30 \cos 30^\circ \cdot 0{,}2$

$\overset{\curvearrowright}{M_A} = 0{,}858 \text{ kNm} = \underline{\underline{858 \text{ Nm}}}$ medurs

b) $\overset{\curvearrowright}{M_B} = 0{,}2 \cdot 2{,}30 \cos 30^\circ = 0{,}398 \text{ kNm} = \underline{\underline{398 \text{ Nm}}}$ medurs

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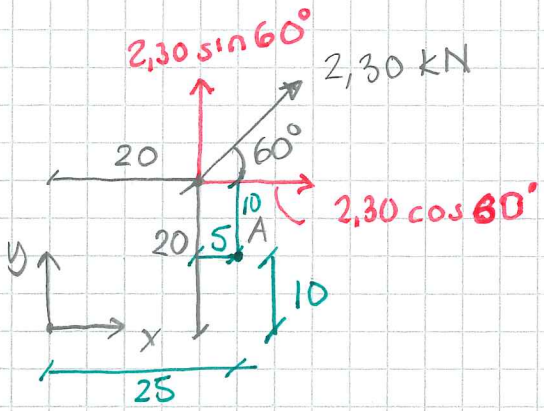
3.11

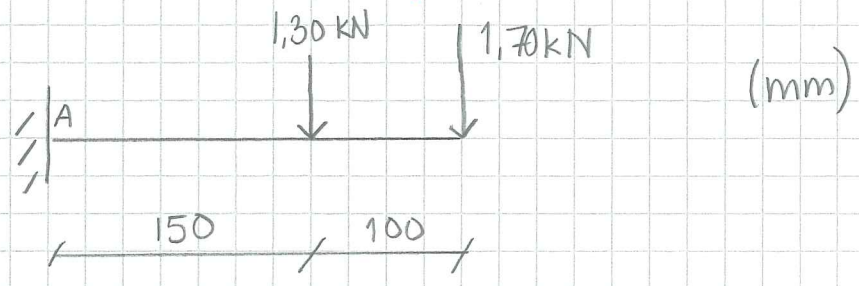


Anta: mm

a) $\overset{\curvearrowleft}{M_O} = 0{,}02 \cdot 2{,}30 \sin 60^\circ - 0{,}02 \cdot 2{,}30 \cos 60^\circ = 16{,}8 \text{ Nm}$ moturs

b) $\overset{\curvearrowright}{M_A} = 0{,}01 \cdot 2{,}30 \cos 60^\circ + 0{,}005 \cdot 2{,}30 \sin 60^\circ = 21{,}5 \text{ Nm}$ medurs

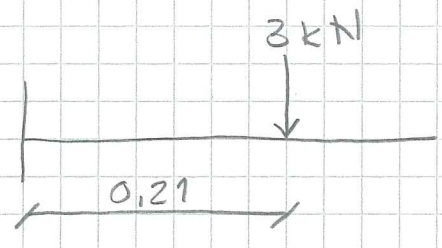
3.12



a) $\overset{\curvearrowright}{M_A} = 1{,}30 \cdot 0{,}150 + 1{,}70 \cdot 0{,}250 = 0{,}62 \text{ kNm}$

$\overset{\curvearrowright}{M_A} = 620 \text{ Nm}$

b) $R = 1{,}30 + 1{,}70 = 3 \text{ kN}$



$R \cdot r = M_A$

$r = \frac{M_A}{R} = \frac{620}{3000} = 0{,}21 \text{ m}$

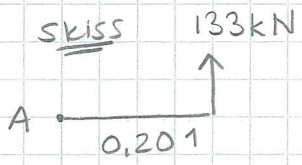
3.13

$\uparrow R_y = 39 + 44 + 50 = 133 \text{ kN}$

$\overset{\curvearrowleft}{M_A} = 44 \cdot 0{,}203 + 50(0{,}154 + 0{,}203) = 26{,}8 \text{ kNm}$

(39 kN ger inget moment m.a.p A då hävarmen är noll)

$M_A = r \cdot R_y \qquad r = \frac{26{,}8}{133} = 0{,}201$



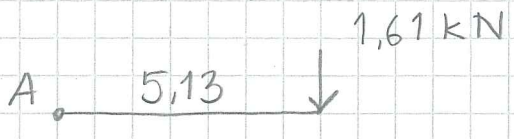
3.14

$\downarrow R_y = R = 2{,}54 + 1{,}17 - 2{,}10 = 1{,}61\ kN$

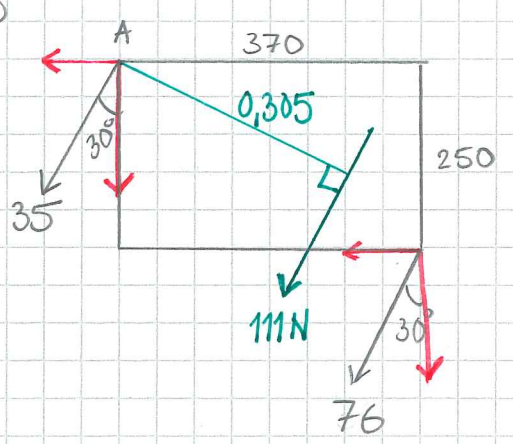
$\curvearrowright M_A = 1{,}17 \cdot 1{,}7 - 2{,}1(1{,}7 + 2{,}03) + 2{,}54(1{,}7 + 2{,}03 + 1{,}82)$

$\curvearrowright M_A = 8{,}253\ kNm$

$M_A = r \cdot R \qquad r = \frac{8{,}253}{1{,}61} = 5{,}13\ m$



3,15



Båda krafterna har samma riktning =>
att resultanten kommer ha lika riktning (30°)

$R = 35 + 76 = 111\ N$

Om inte detta inses kan man räkna R_x & R_y

$\leftarrow R_x = 35 \sin 30° + 76 \sin 30° = 55{,}5\ N$

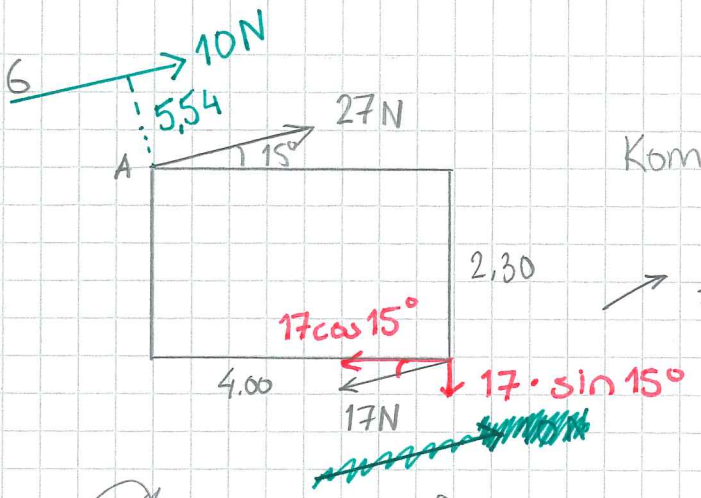
$\downarrow R_y = 35 \cos 30° + 76 \cos 30° = 96{,}13\ N$

$R = \sqrt{55{,}5^2 + 96{,}13^2} = 111\ N$ som ovan ☺

$\curvearrowright M_A = 76 \sin 30° \cdot 0{,}250 + 76 \cos 30° \cdot 0{,}370 = 33{,}85\ Nm$

$r = \frac{33{,}85}{111} = 0{,}305\ m$

3,16



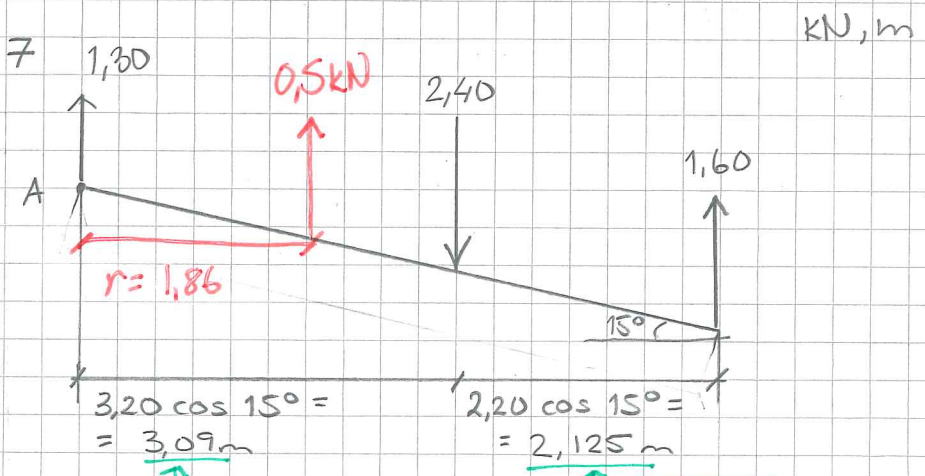
Kommentar se uppg. 3.15

$\nearrow \quad R = 27 - 17 = 10\,N$

$$\curvearrowright M_A = 17 \sin 15^\circ \cdot 4{,}0 + 17 \cos 15^\circ \cdot 2{,}3 = 55{,}37\,Nm$$

$$r = \frac{55{,}37}{10} = 5{,}54\,m$$

3,17



$\uparrow \quad R_y = R = 1{,}3 - 2{,}4 + 1{,}6 = 0{,}5\,kN$

· Bestäm hävarmar till A, vinkelräta avstånd

$$\curvearrowright M_A = 2{,}40 \cdot 3{,}09 - 1{,}6(3{,}09 + 2{,}125) = -0{,}93\,kNm$$

$$\curvearrowleft M_A = 0{,}93\,kNm \quad \text{(moturs)}$$

$$r = \frac{M_A}{R} = \frac{0{,}93}{0{,}5} = \underline{\underline{1{,}86\,m}}$$

3.18

a) $M_A = 1{,}30 \cdot 7{,}35 = 9{,}56 \text{ kNm}$

b) $M_B = 1{,}30 \cdot 7{,}35 = 9{,}56 \text{ kNm}$

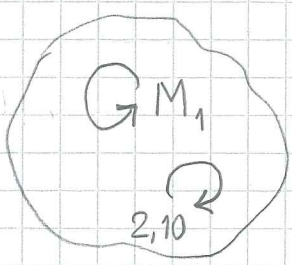
c) $M_C = 1{,}30 \cdot 4{,}50 + 1{,}3\,(7{,}35 - 4{,}50)$

$M_C = \cancel{1{,}30 \cdot 4{,}50} + 1{,}3 \cdot 7{,}35 - \cancel{1{,}3 \cdot 4{,}50} = 1{,}3 \cdot 7{,}35$

$M_C = 9{,}56 \text{ kNm}$

3.19

Ett kraftpar är ett rent moment och in platsberoende på stela kroppar



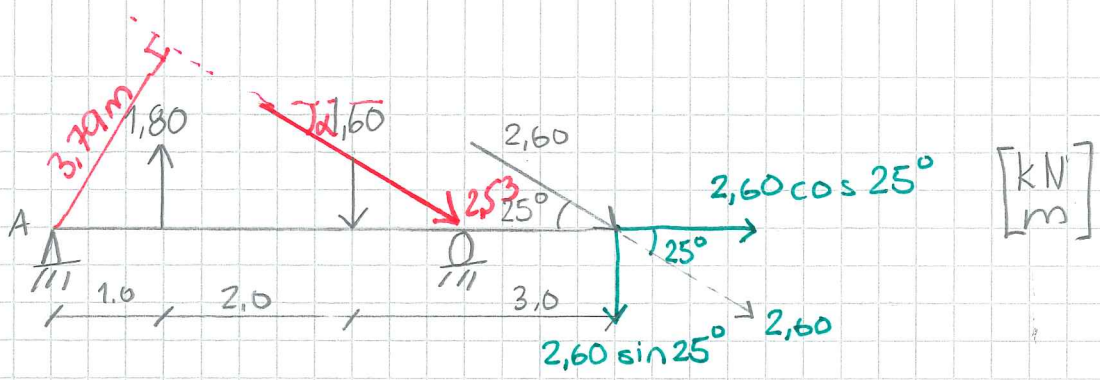
$M_1 = 10 \cdot 0{,}5 = 5 \text{ kNm}$

$M_{tot} = 5 - 2{,}10 = 2{,}90 \text{ kNm}$

3.20

$M_A = 3{,}50 \cdot 2{,}70 - 9{,}30 = 0{,}15 \text{ kNm}$

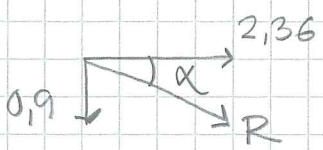
3.21



$\rightarrow R_x = 2{,}60 \cos 25^\circ = 2{,}36 \text{ kN}$

$\uparrow \; R_y = 1{,}8 - 1{,}6 - 2{,}6 \sin 25^\circ = -0{,}90 \text{ kN}$

Skiss



$R = \sqrt{0{,}9^2 + 2{,}36^2} = 2{,}53 \text{ kN}$

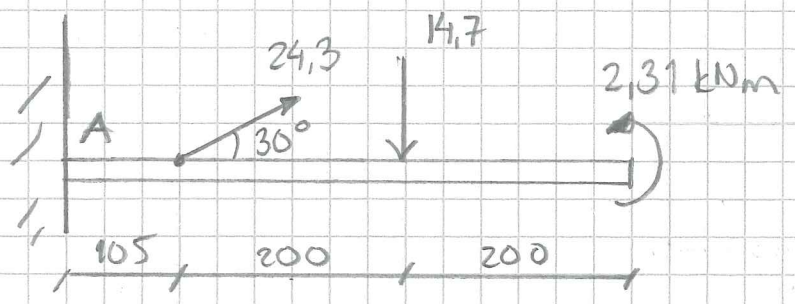
$\alpha = \arctan\left(\frac{0{,}9}{2{,}36}\right) = 20{,}9^\circ$

$\curvearrowright M_A = 1{,}6 \cdot 3{,}0 - 1{,}8 \cdot 1{,}0 + 6{,}0 \cdot 2{,}6 \sin 25^\circ = 9{,}59 \text{ kNm}$

$r = \frac{M_A}{R} = \frac{9{,}59}{2{,}53} = 3{,}79 \text{ m}$

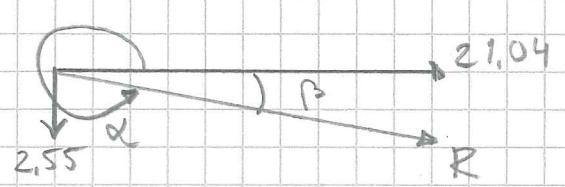
3.22 Sökt: Resultanten.

kN, cm



$\rightarrow F_{Rx} = 24{,}3 \cos 30^\circ = 21{,}04 \text{ kN}$

$\uparrow F_{Ry} = 24{,}3 \sin 30^\circ - 14{,}7 = -2{,}55 \text{ kN}$



$R = \sqrt{21{,}04^2 + 2{,}55^2} = \underline{\underline{21{,}19 \text{ kN}}}$

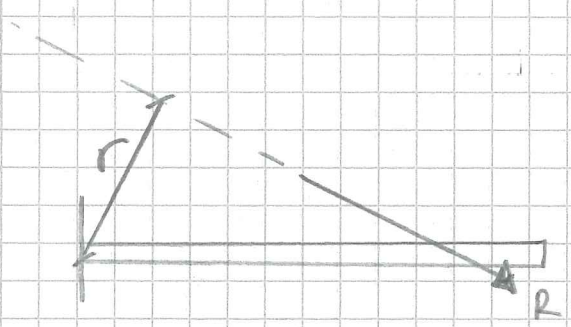
$\beta = \arctan\left(\frac{2{,}55}{21{,}04}\right) = \underline{\underline{6{,}91^\circ}}$

"boken" $\alpha = 360 - 6{,}91 = 353^\circ$

R ska ge samma vridande förmåga som de två krafterna + momentet

$\overset{\curvearrowright}{M}_A = 2{,}31 - 14{,}7 \cdot 3{,}05 + 1{,}05 \cdot 24{,}3 \sin 30^\circ =$

$= -29{,}8 \text{ kNm} \Rightarrow \overset{\curvearrowright}{M}_A = 29{,}8 \text{ kNm}$

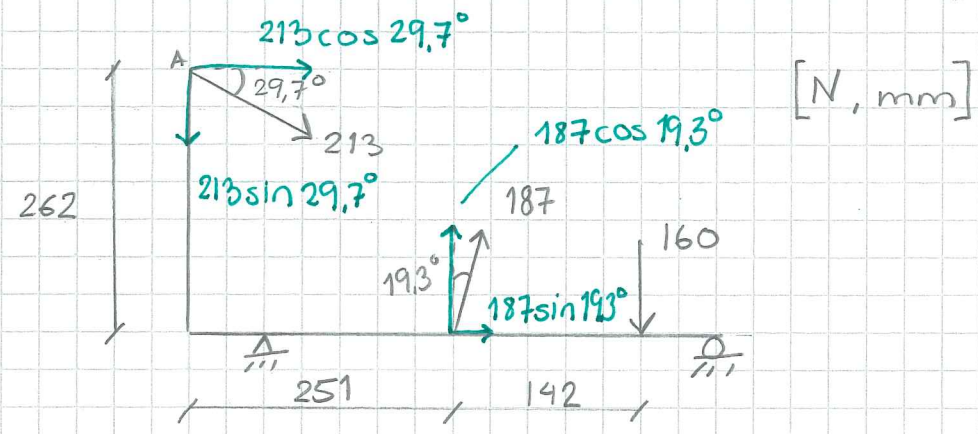


$\overset{\curvearrowright}{M}_A = r \cdot R$

$r = \frac{29{,}8}{21{,}2} = \underline{\underline{1{,}4 \text{ m}}}$

$R_y x = M_A \Rightarrow x = \frac{M_A}{R_y} = \frac{29{,}8}{2{,}55}$

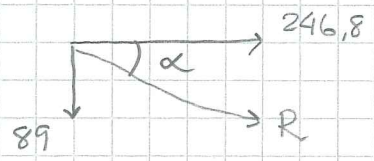
3.23



$$\downarrow R_y = 213\sin 29{,}7^\circ - 187\cos 19{,}3^\circ + 160 = 89{,}04\ \text{N}$$

$$\rightarrow R_x = 213\cos 29{,}7^\circ + 187\sin 19{,}3^\circ = 246{,}8\ \text{N}$$

Stiss

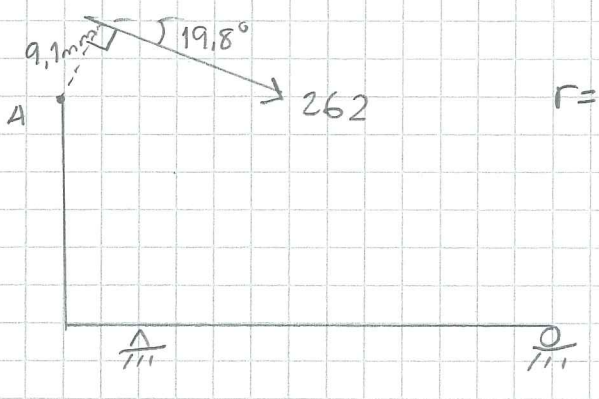


$$R = \sqrt{246{,}8^2 + 89^2} = 262\ \text{N}$$

$$\alpha = \arctan\left(\frac{89}{246{,}8}\right) = 19{,}8^\circ$$

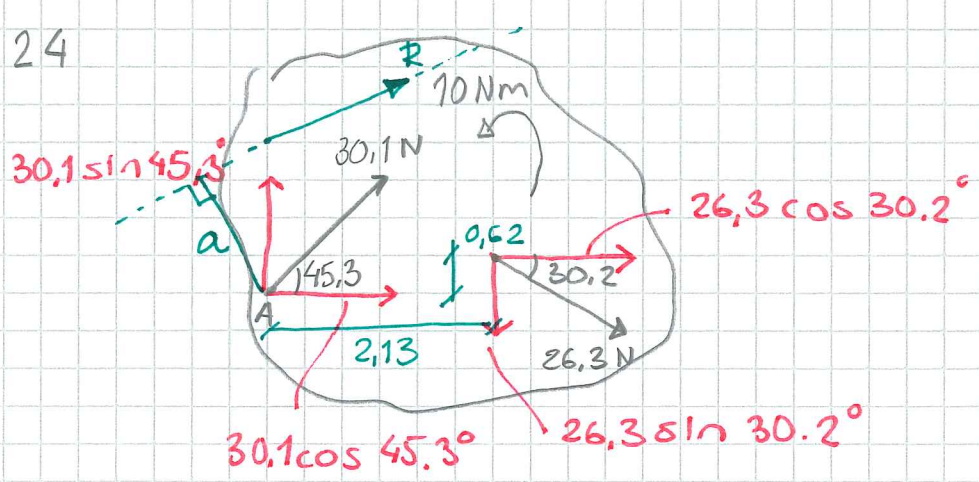
$$\circlearrowright M_A = 160(0{,}251 + 0{,}142) - 187\cos 19{,}3^\circ \cdot 0{,}251 - \ldots$$

$$\ldots\ 187\sin 19{,}3^\circ \cdot 0{,}262 = 2{,}39\ \text{Nm}$$



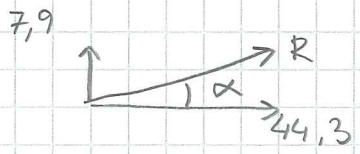
$$r = \frac{M_A}{R} = \frac{2{,}39}{262} = 9{,}1\ \text{mm}$$

3.24



$\uparrow \quad R_y = 30{,}1 \sin 45{,}3^\circ - 26{,}3 \sin 30{,}2^\circ = 7{,}9\,N$

$\rightarrow \quad R_x = 26{,}3 \cos 30{,}2^\circ + 30{,}1 \cos 45{,}3^\circ = 44{,}3\,N$



$R = \sqrt{7{,}9^2 + 44{,}3^2} = 45\,N$

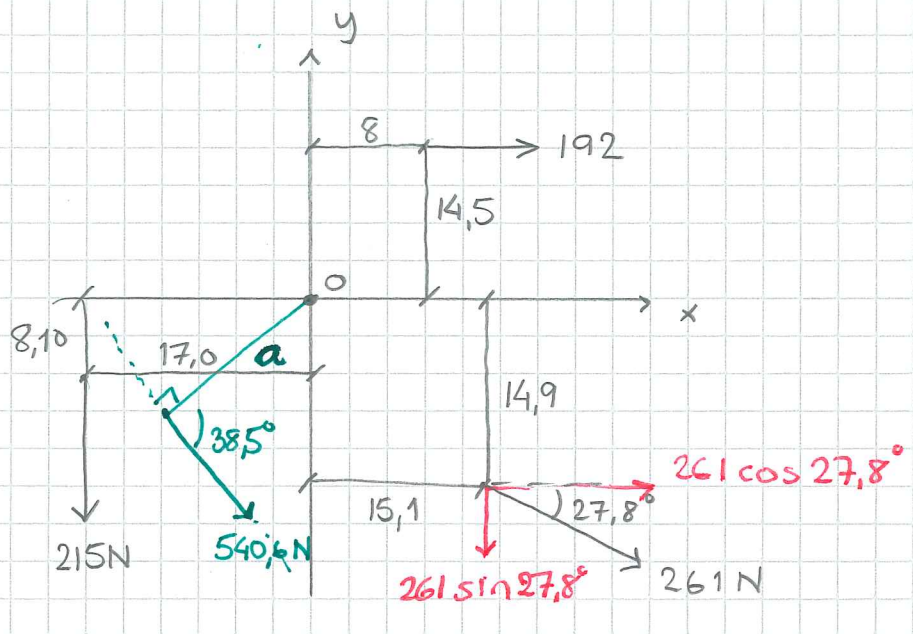
$\alpha = \arctan\left(\frac{7{,}9}{44{,}3}\right) = 10{,}1^\circ$

$\curvearrowright M_A = 0{,}62 \cdot 26{,}3 \cos 30{,}2^\circ + 2{,}13 \overset{26{,}3}{\sin 30{,}2^\circ} - 10 = 33\,Nm$

$R \cdot a = M_A \qquad a = \frac{33}{45} = 0{,}733\,m$

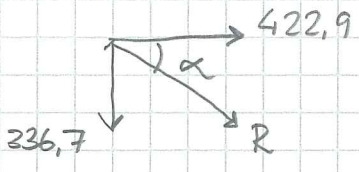
a enligt figur (vinkelrät avstånd från A till kraftens verkningslinje, placera R så att det blir medurs i förhållande till A.

3.25



$\rightarrow \quad R_x = 192 + 261 \cos 27{,}8^\circ = 422{,}9\,N$

$\downarrow \quad R_y = 215 + 261 \sin 27{,}8^\circ = 336{,}7\,N$



$R = \sqrt{422{,}9^2 + 336{,}7^2} = 540{,}6\,N$

$\alpha = \arctan\left(\frac{336{,}7}{422{,}9}\right) = 38{,}5^\circ$

$\curvearrowright M_O = 192 \cdot 14{,}5 - 215 \cdot 17 + 15{,}1 \cdot 261 \sin 27{,}8^\circ \ldots$

$\ldots - 14{,}9 \cdot 261 \cos 27{,}8^\circ = -2470\ Nmm$

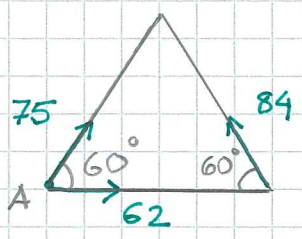
$\curvearrowleft M_O = 2470\ Nmm$

$a = \frac{2470}{540{,}6} = 4{,}57\,mm$

3.26

3×65 cm

(N, cm)

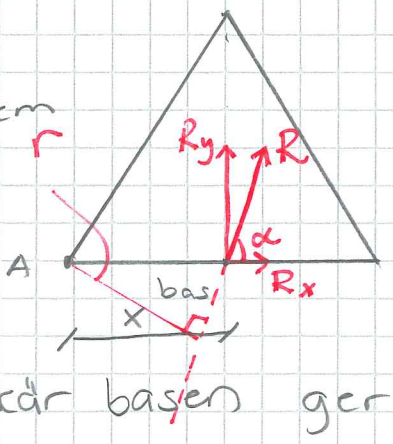


$\uparrow \quad R_y = 75 \sin 60^\circ + 84 \sin 60^\circ = 138{,}7\,N$

$\rightarrow \quad R_x = 75 \cos 60^\circ - 84 \cos 60^\circ + 62 = 57{,}5\,N$

$\curvearrowleft A \quad M_A = 84 \sin 60^\circ \cdot 65 = 4728\,Ncm$

$r = \frac{4728}{150} = 31{,}5\,cm$



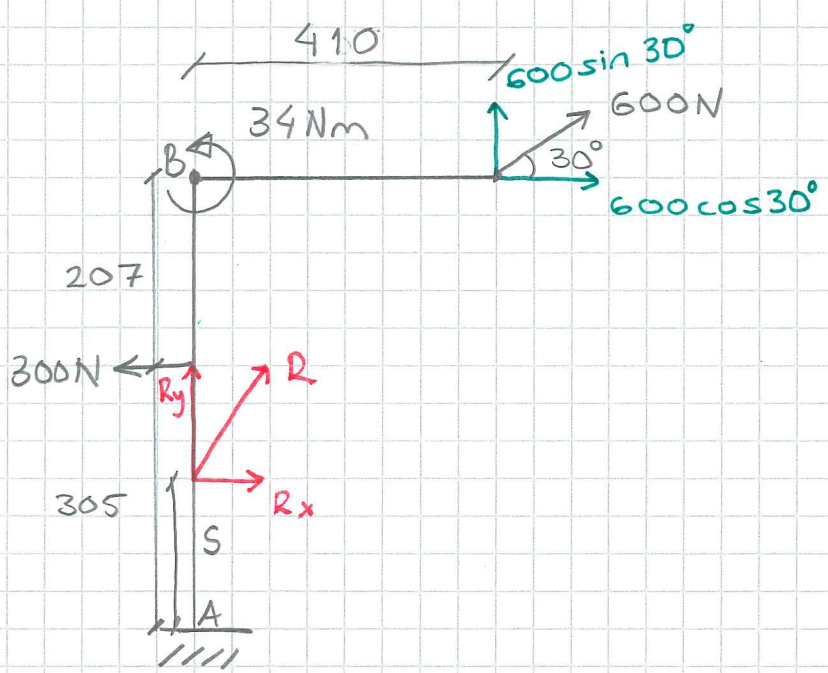
$\alpha = \arctan \frac{138{,}7}{57{,}5} = 67{,}5^\circ$

$R = \sqrt{138{,}7^2 + 57{,}5^2} = 150\,N$

b) Då R skär basen ger endast Ry moment i A

$R_y \cdot x = M_A \Rightarrow x = \frac{M_A}{R_y} = \frac{4728}{138{,}7} = 34\,cm$

3,27



$$\uparrow \quad R_y = 600 \sin 30^\circ = 300\,N$$

$$\rightarrow \quad R_x = 600 \cos 30^\circ - 300 = 220\,N$$

$$R = \sqrt{300^2 + 220^2} = 372\,N$$

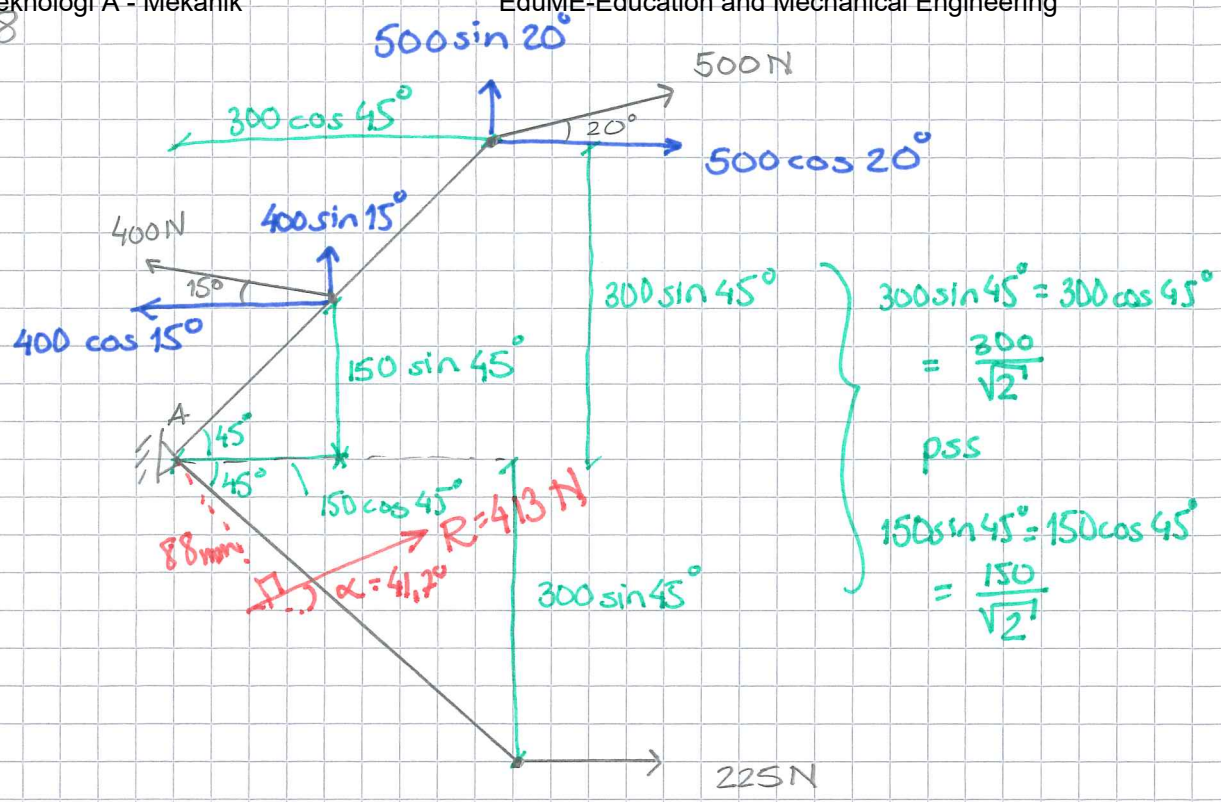
$$\alpha = \arctan\left(\frac{300}{220}\right) = 53{,}7^\circ$$

$$\circlearrowright M_A = 0{,}512 \cdot 600 \cos 30^\circ - 0{,}41 \cdot 600 \sin 30^\circ - 34 - 300 \cdot 0{,}305$$

$$\circlearrowright M_A = 17{,}5\,Nm$$

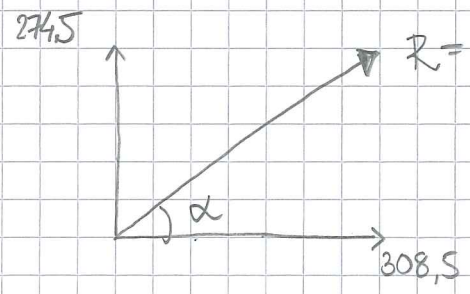
$$s = \frac{M_A}{R_x} = \frac{17{,}5}{220} = \underline{\underline{0{,}08\,m}}$$

3.28



$\uparrow \quad R_y = 500 \sin 20^\circ + 400 \sin 15^\circ = 274{,}5 \text{ N}$

$\rightarrow \quad R_x = 225 + 500 \cos 20^\circ - 400 \cos 15^\circ = 308{,}5 \text{ N}$



$R = \sqrt{274{,}5^2 + 308{,}5^2} = \underline{\underline{413 \text{ N}}}$

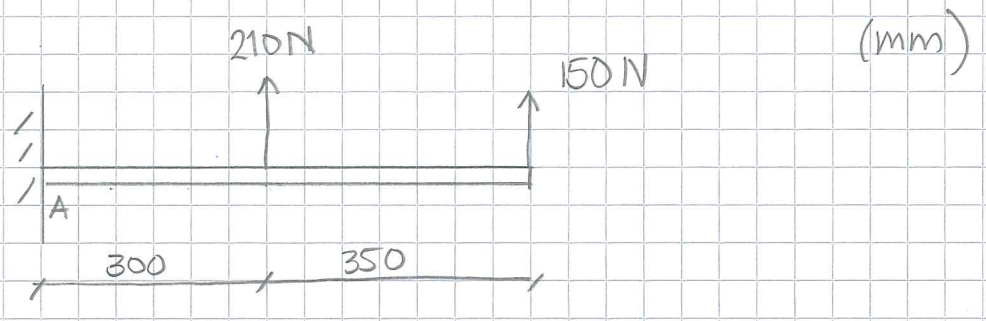
$\alpha = \arctan \frac{274{,}5}{308{,}5} = \underline{\underline{41{,}7^\circ}}$

$\overset{\curvearrowleft}{A} \quad 225 \cdot \frac{300}{\sqrt{2}} + 400 \cos 15^\circ \cdot \frac{150}{\sqrt{2}} + 400 \sin 15^\circ \cdot \frac{150}{\sqrt{2}} + 500 \sin 20^\circ \cdot \frac{300}{\sqrt{2}} \ldots$

$\ldots - 500 \cos 20^\circ \cdot \frac{300}{\sqrt{2}} = M_A \quad \Rightarrow \quad \overset{\curvearrowleft}{M_A} = 36298 \text{ Nmm}$

$a = \frac{36298}{413} = \underline{\underline{88 \text{ mm}}}$

3.29

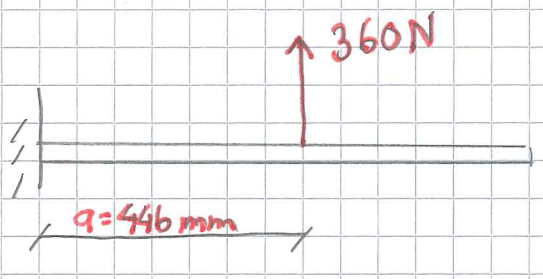


$\uparrow \quad R = 210 + 150 = 360\,N$

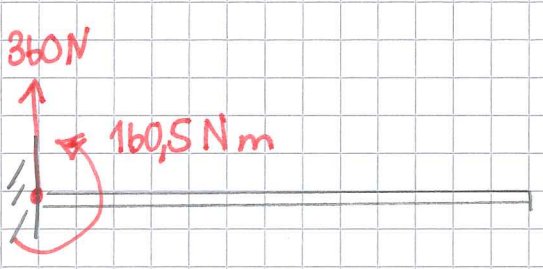
$\curvearrowleft \quad M_A = 210 \cdot 300 + 150 \cdot 650 = 160500\,Nmm$

$$a = \frac{M_A}{R} = \frac{160500}{360} = 446\,mm$$

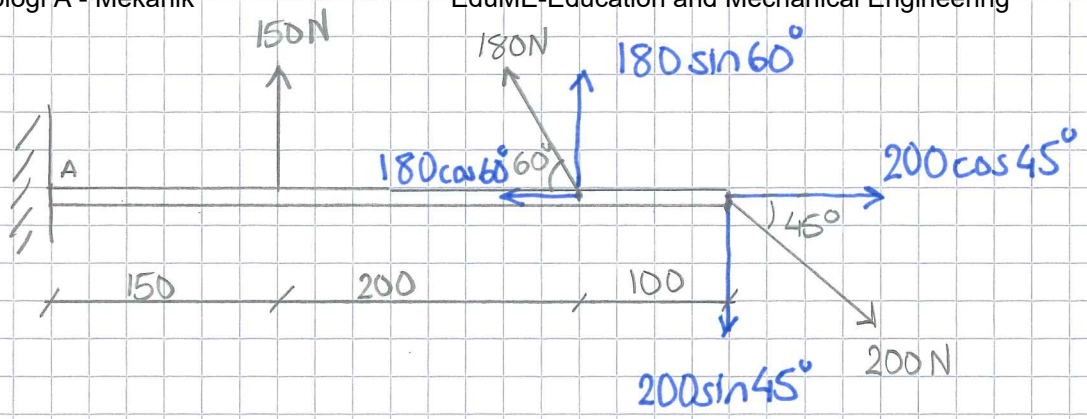
a)



b)



3:30

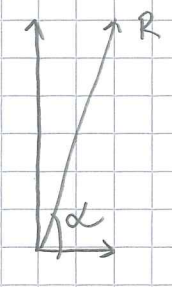


$$\uparrow \quad R_y = 150 + 180\sin 60^\circ - 200\sin 45^\circ = 164{,}5\ N$$

$$\rightarrow \quad R_x = 200\cos 45^\circ - 180\cos 60^\circ = 51{,}4\ N$$

$$R = \sqrt{51{,}4^2 + 164{,}5^2} = 172\ N$$

$$\alpha = \arctan\left(\frac{164{,}5}{51{,}4}\right) = 72{,}6^\circ$$

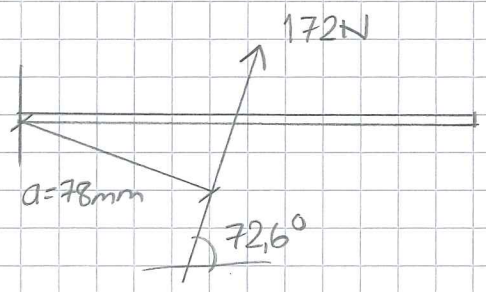


$$\curvearrowleft M_A = 150 \cdot 150 + 180\sin 60^\circ \cdot (150+200) - 200\sin 45^\circ (150+200+100)$$

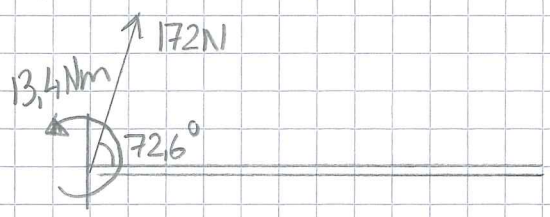
$$\curvearrowleft M_A = 13420\ Nmm$$

$$a = \frac{M_A}{R} = \frac{13420}{172} = 78\ mm$$

a)

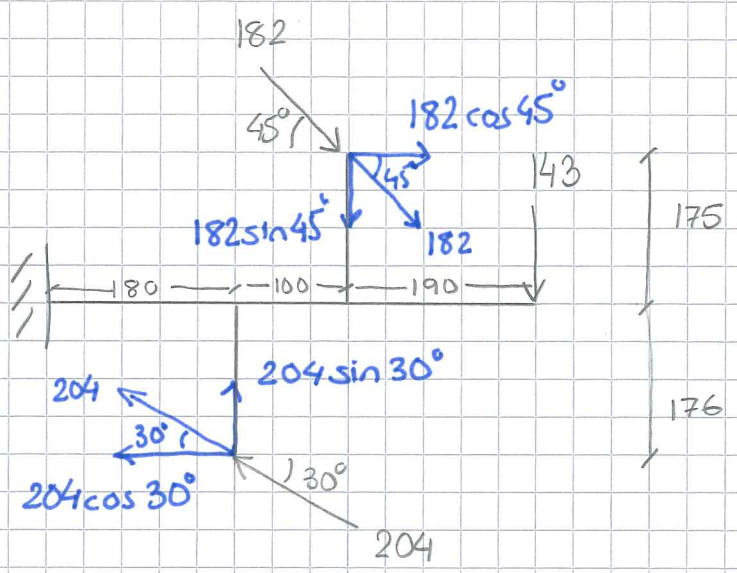


b



3.31

(N, mm)

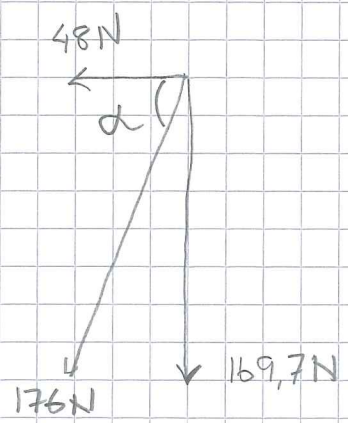


$$\uparrow \quad R_y = 204\sin 30^\circ - 143 - 182\sin 45^\circ = -169{,}7\ \text{N}$$

$$\rightarrow \quad R_x = 182\cos 45^\circ - 204\cos 30^\circ = -48{,}0\ \text{N}$$

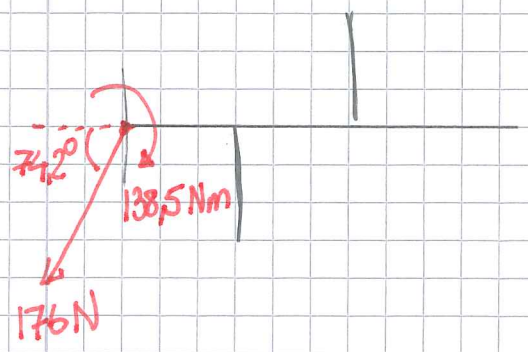
$$R = \sqrt{169{,}7^2 + 48^2} = 176\ \text{N}$$

$$\alpha = \arctan\left(\frac{169{,}7}{48}\right) = 74{,}2^\circ$$

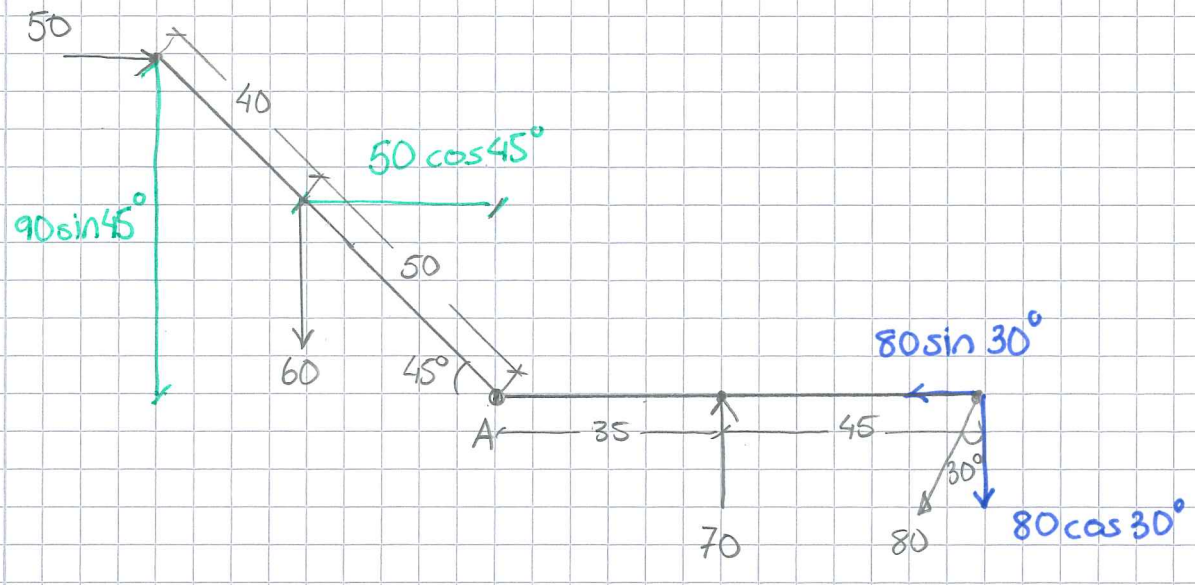


$$M_A = 143(180+100+190) + 182\sin 45^\circ(180+100) + 182\cos 45 \cdot 175 \ldots$$

$$+ 204\cos 30^\circ \cdot 176 - 204\sin 30^\circ \cdot 180 = 138499\ \text{Nmm}$$



3.32

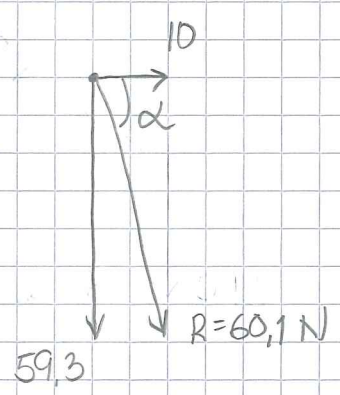


$\uparrow \; R_y = 70 - 60 - 80\cos 30^\circ = -59{,}3 \text{ N}$

$\rightarrow \; R_x = 50 - 80\sin 30^\circ = 10$

$R = \sqrt{10^2 + 59{,}3^2} = 60{,}1 \text{ N}$

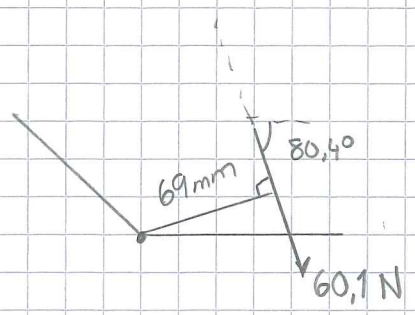
$\alpha = \arctan\left(\frac{59{,}3}{10}\right) = 80{,}4^\circ$



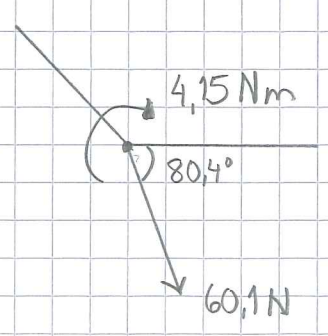
$\overset{\curvearrowright}{M_A} = 50 \cdot 90\sin 45^\circ - 60 \cdot 50\cos 45^\circ - 70 \cdot 35 + 80\cos 30^\circ \cdot 80$

$\overset{\curvearrowright}{M_A} = 4153 \text{ Nmm}$ $\qquad a = \frac{M_A}{R} = \frac{4153}{60{,}1} = 69 \text{ mm}$

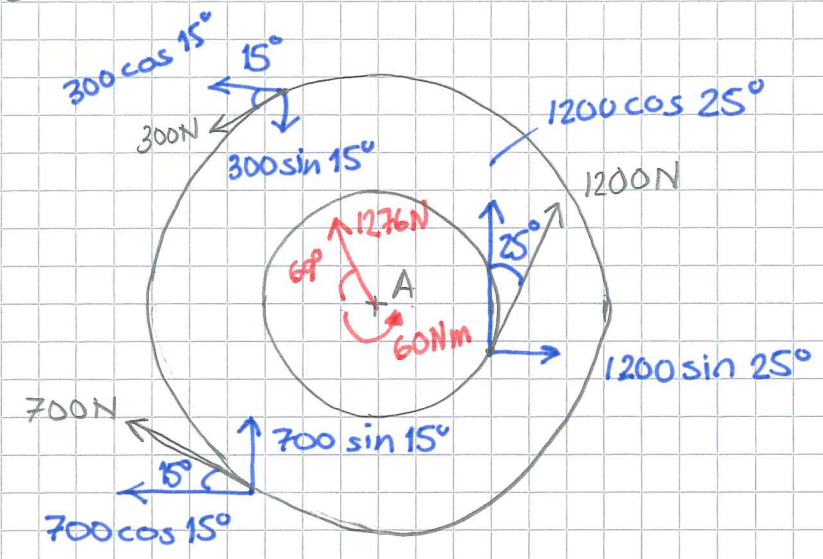
a)



b)



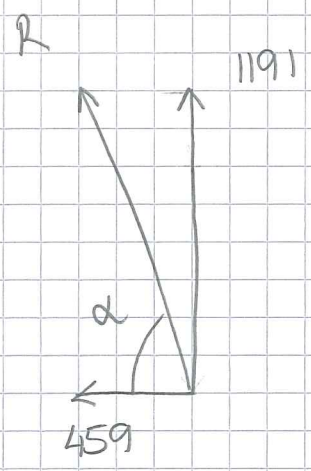
3.33



$$\rightarrow R_x = 1200\sin 25^\circ - 700\cos 15^\circ - 300\cos 15^\circ = -459\ \text{N}$$

$$\uparrow R_y = 1200\cos 25^\circ + 700\sin 15^\circ - 300\sin 15^\circ = 1191\ \text{N}$$

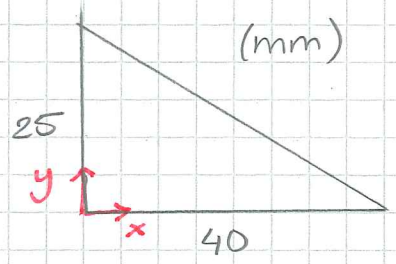
$$\circlearrowleft M_A = 300 \cdot 300 + 1200 \cdot 150 - 700 \cdot 300 = \underline{\underline{60000}}\ \text{Nmm}$$



$$R = \sqrt{1191^2 + 459^2} = \underline{\underline{1276\ \text{N}}}$$

$$\alpha = \arctan\left(\frac{1191}{459}\right) = \underline{\underline{69^\circ}}$$

4.1



Formelsamling => $t_p = \frac{h}{3}$

Mitt råd: Använd alltid $\frac{h}{3}$ för att beräkna t_p inte

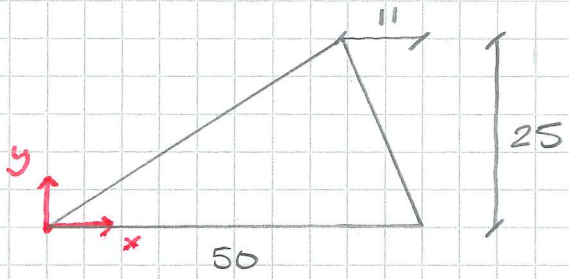
$x_{tp} = \frac{A+C+D}{3}$ (Malmendahl)

$\frac{h}{3}$ gäller från basen.

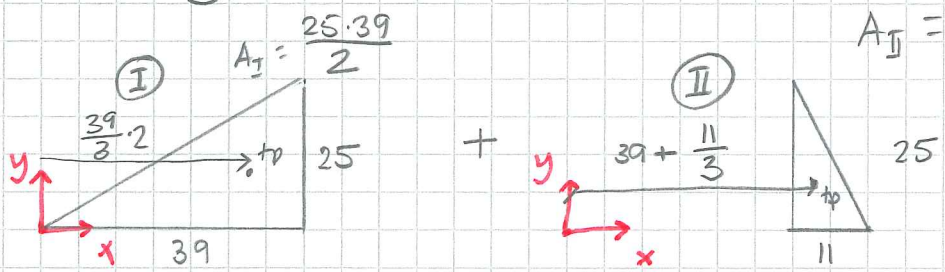
$$y_{tp} = \frac{25}{3} = 8{,}33\,mm$$

$$x_{tp} = \frac{40}{3} = 13{,}3\,mm$$

4.2



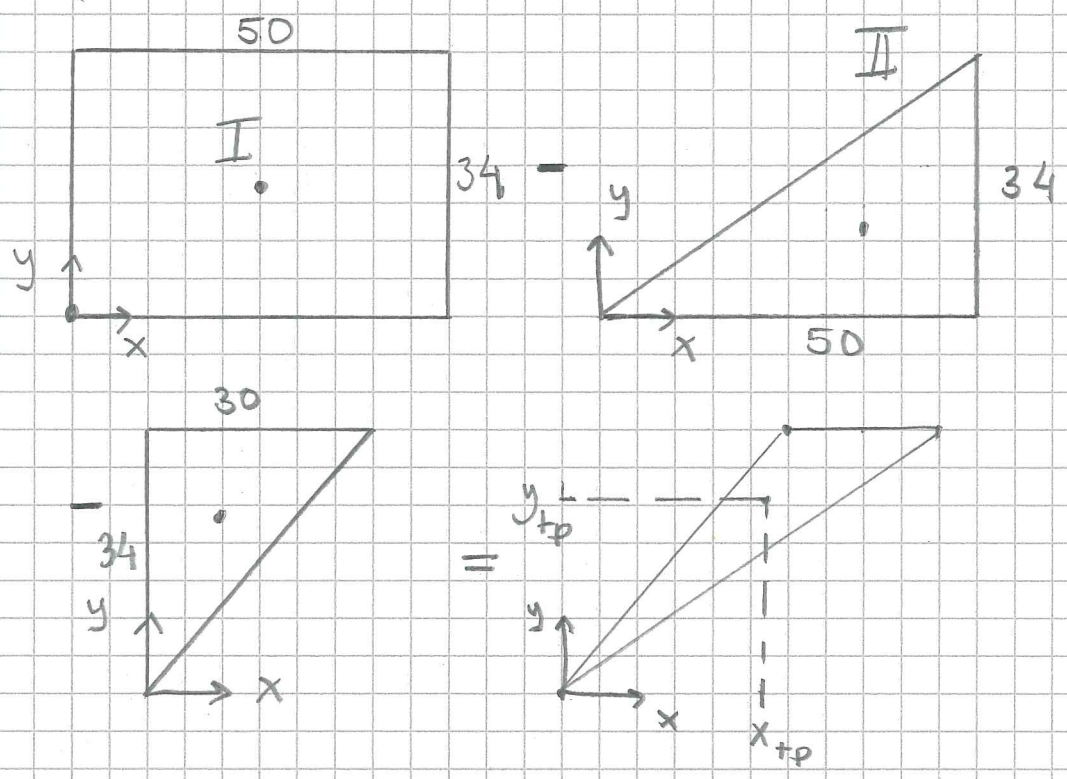
$$y_{tp} = \frac{25}{3} = 8{,}33\,mm$$



	A	x_{tp}	$A\,x_{tp}$
I	487,5	26	12675
II	137,5	42,7	5867
	625	\	18542

$$x_{tp} = \frac{18542}{625} = \underline{\underline{29{,}7\,mm}}$$

4.3 Sökt: x_{tp}, y_{tp}



n	A_n	x_{tp}^n	y_{tp}^n	$A_n \cdot x_{tp}^n$	$A_n \cdot y_{tp}^n$
I	1700	25	17	42500	28 900
II	-850	$\frac{2 \cdot 50}{3} = 33{,}3$	$\frac{34}{3} = 11{,}3$	-28 333	-9633
III	-510	$\frac{30}{3} = 10$	$\frac{2 \cdot 34}{3} = 22{,}7$	-5100	-11560
Σ	340			9067	7707

$$x_{tp} = \frac{\Sigma A_n x_{tp}^n}{A_{tot}} = \frac{9067}{340} = \underline{\underline{26{,}7 \text{ mm}}}$$

$$y_{tp} = \frac{7707}{340} = \underline{\underline{22{,}7 \text{ mm}}}$$

4.4

$$x_{tp} = \frac{2R \sin\alpha}{3\alpha}$$

Formelbladet säger α i rad

α i nämnaren måste <u>alltid</u> vara i radianer.

sin α => om α i rad måste miniräknare stå på radianer.

om α i grader kan miniräknaren vara på grader (enklast)

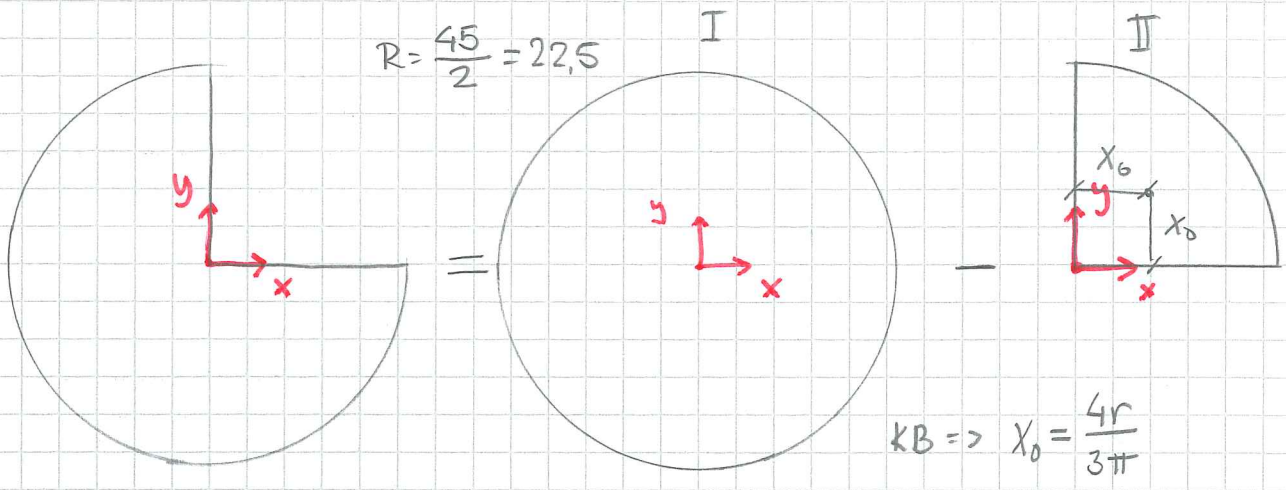
Mitt råd: radianer i nämnaren
grader för sinα för miniräknare
står på grader.

$\alpha = 13°$ $\qquad \alpha_{rad} = \frac{13}{360} \cdot 2\pi = 13 \cdot \frac{\pi}{180}$

$$x_{tp} = \frac{2 \cdot 30 \cdot \sin 13°}{3 \cdot 13 \cdot \frac{\pi}{180}} = 19{,}8 \text{ mm}$$

4,5

$R = \frac{45}{2} = 22{,}5$



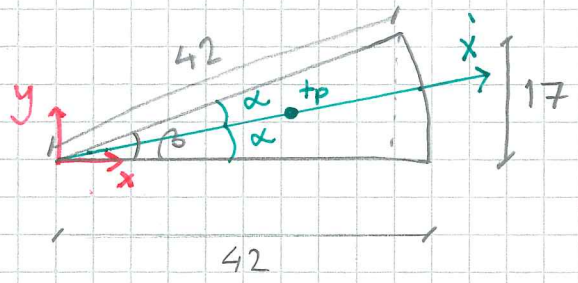
KB => $X_0 = \frac{4r}{3\pi}$

$X_0 = \frac{4 \cdot 22{,}5}{3\pi} = 9{,}55$

	A	X_{tp}	y_{tp}	Ax_{tp}	Ay_{tp}
I	$22{,}5^2\pi = 1590$	0	0	0	0
II	$-\frac{1590}{4} = -397{,}5$	9,55	9,55	−3796	−3796
	1193			−3796	−3796

$$X_{tp} = \frac{-3796}{1193} = \underline{\underline{-3{,}18}} = y_{tp}$$

4,6



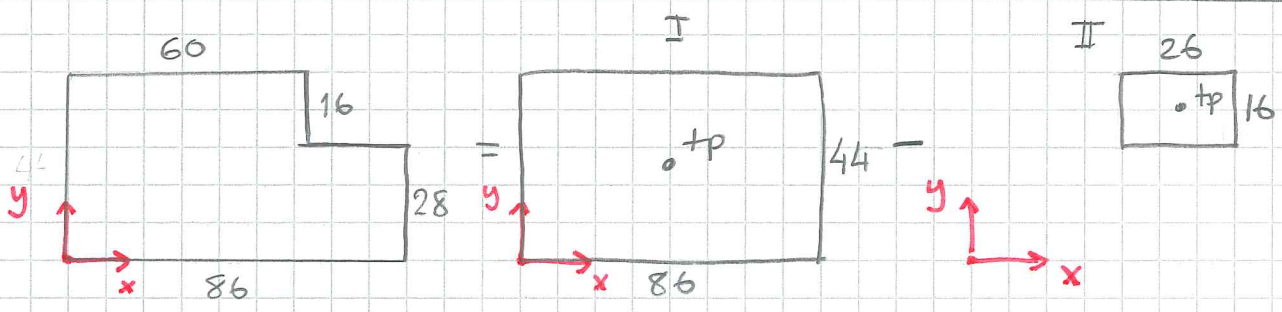
$$\beta = \arcsin\left(\frac{17}{42}\right) = 23{,}88^\circ$$

$$x'_{tp} = \frac{2 \cdot 42 \cdot \sin 11{,}9^\circ}{3 \cdot 11{,}9 \cdot \frac{\pi}{180}} = 27{,}8 \text{ mm}$$

$$x_{tp} = 27{,}8 \cdot \cos 11{,}9^\circ = \underline{\underline{27{,}2 \text{ mm}}}$$

$$y_{tp} = 27{,}8 \sin 11{,}9^\circ = \underline{\underline{5{,}73 \text{ mm}}}$$

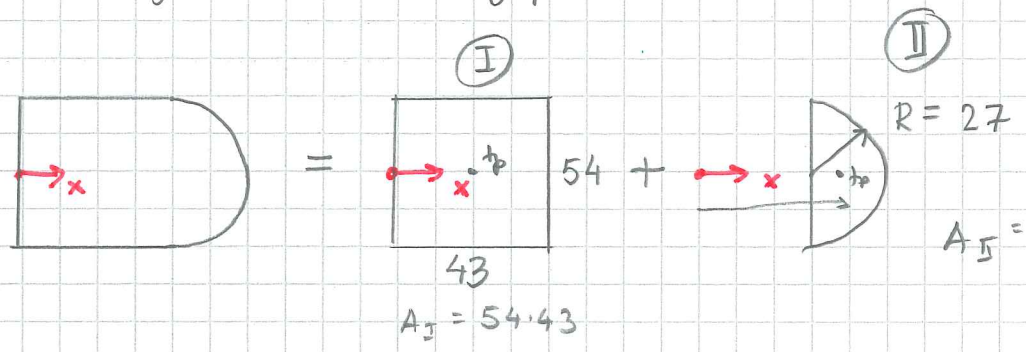
4,7



	A	x_{tp}	y_{tp}	$A x_{tp}$	$A y_{tp}$
I	$44 \cdot 86 = 3784$	$\frac{86}{2} = 43$	$\frac{44}{2} = 22$	162712	83248
II	$-26 \cdot 16 = -416$	$60 + \frac{26}{2} = 73$	$28 + \frac{16}{2} = 36$	−30368	−14976
	3368			132 344	68272

$$x_{tp} = \frac{132\,344}{3368} = \underline{\underline{39{,}3}} \qquad y_{tp} = \frac{68272}{3368} = \underline{\underline{20{,}3}}$$

4.8 Symmetri => $y_{tp} = 0$



$A_I = 54 \cdot 43$

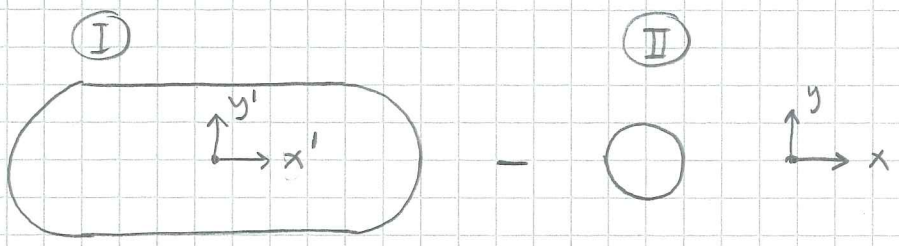
$A_{II} = \frac{27^2 \cdot \pi}{2}$

	A	x_{tp}	$A\,x_{tp}$
I	2322	21,5	49923
II	1145	$43 + \frac{4 \cdot 27}{3\pi} = 54{,}5$	62402
Σ	3467	—	112325

$$x_{tp} = \frac{112325}{3467} = \underline{\underline{32.4}} \text{ mm}$$

4.9 Tänk till INNAN! Leta symmetrier & kända tyngdpunkter

Här: flytta koordinatsystemet



$A_I = 19 \cdot 26 + 9{,}5^2 \pi = 777$

$x'_{tp} = 0$

$A_{II} = 5^2 \pi = 78{,}5$

$x'_{tp} = -\left(\frac{45}{2} - 9{,}5\right) = -13$

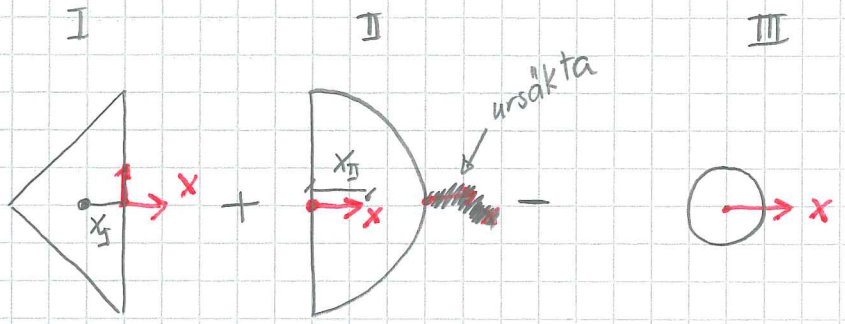
	A	x'_{tp}	$A x'_{tp}$
I	777	0	0
II	−78,5	−13	1020,5
	698,5		1020,5

$$x'_{tp} = \frac{1020{,}5}{698{,}5} = 1{,}46$$

Koordinatsystem enl. uppg. Δ 13 mm

$$x_{tp} = 13 + 1{,}46 = \underline{\underline{14{,}5\,mm}}$$

4.10 Symmetri => $y_{tp} = 0$



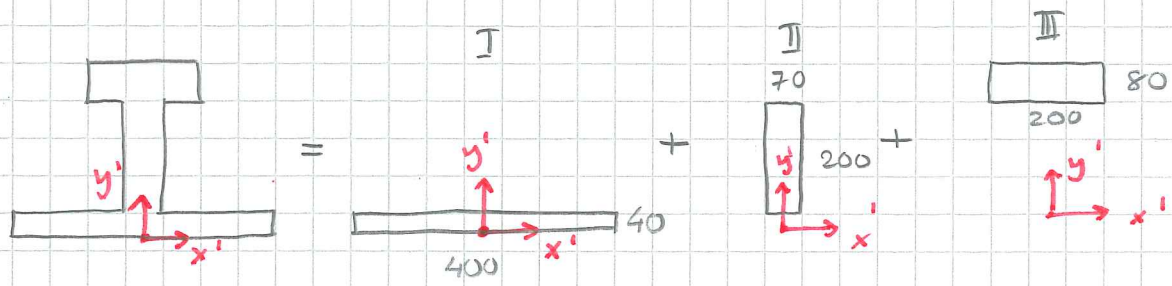
$A_I = \frac{17 \cdot 2 \cdot 18}{2} = 306$ $\quad A_{II} = \frac{18^2 \cdot \pi}{2} = 509$ $\quad A_{III} = \frac{11^2 \cdot \pi}{4} = 95$

$x_I = -\frac{17}{3} = 5{,}67$ $\quad x_{II} = \frac{4 \cdot 18}{3\pi} = 7{,}64$ $\quad x_{III} = 0$

	A	x_{tp}	$A x_{tp}$
I	306	-5,67	-1735
II	509	7,64	3888
III	-95	0	0
Σ	720	\	2153

$x_{tp} = \frac{2153}{720} = \underline{\underline{2{,}99}}$

4.11 Flytta koordinatsystemet



Symmetri => $x'_{tp} = 0$ eller $x_{tp} = 200\,mm$

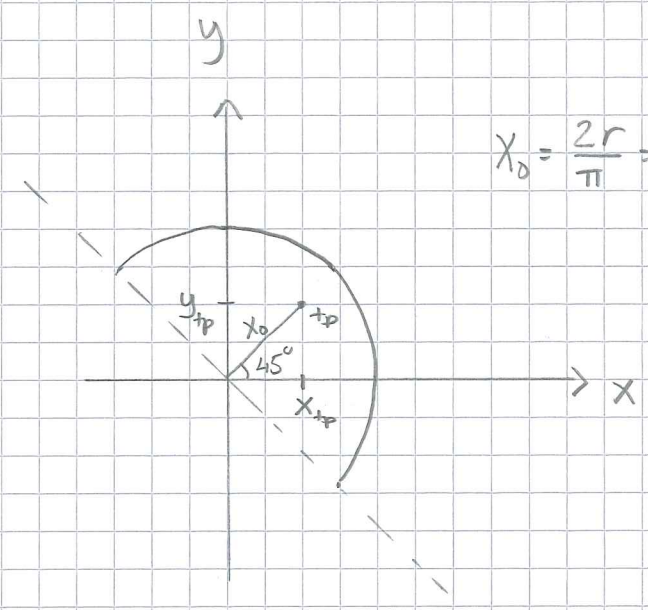
$A_I = 400 \cdot 40$ $\quad A_{II} = 200 \cdot 70$ $\quad A_{III} = 200 \cdot 80$

$y_I = 20$ $\quad y_{II} = 100 + 40$ $\quad y_{III} = 240 + 40$

	A	y_{tp}	Ay_{tp}
I	16000	20	320 000
II	14000	140	1960 000
III	16000	280	4480 000
	46000	/	6760000

$$y_{tp} = \frac{6760\,000}{46\,000} = \underline{\underline{147\,mm}}$$

4.12

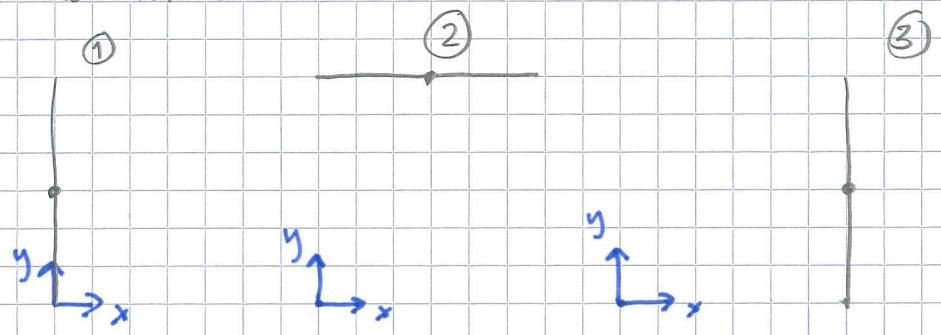


$$x_0 = \frac{2r}{\pi} = \frac{2 \cdot 15}{\pi} = 9{,}55$$

$$x_{tp} = 9{,}55 \cos 45^\circ = \underline{\underline{6{,}75}}$$

$$y_{tp} = x_{tp} = \underline{\underline{6{,}75}}$$

4.13 Area ersätts med längd då tvärsnitt är konstant.

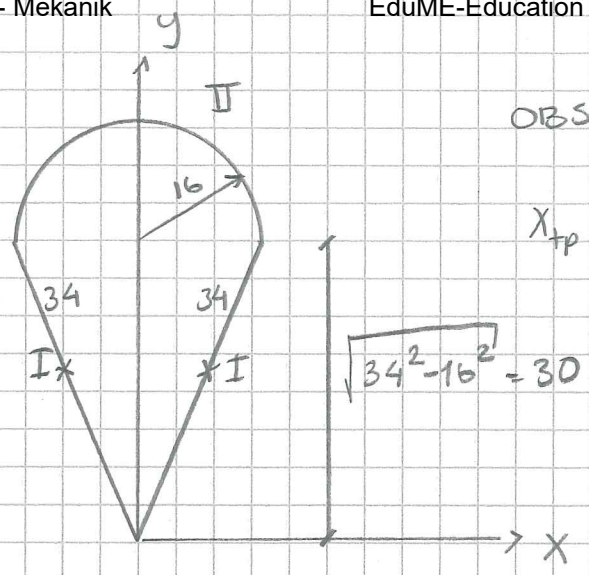


$L = 30$	$L = 30$	$L = 30$
$x_{tp}^1 = 0$	$x_{tp}^2 = 15$	$x_{tp}^3 = 30$
$y_{tp}^1 = 15$	$y_{tp}^2 = 30$	$y_{tp}^3 = 15$

$$x_{tp} = \frac{\Sigma L \cdot x_{tp}}{L_{tot}} = \frac{30 \cdot 0 + 30 \cdot 15 + 30 \cdot 30}{3 \cdot 30} = \underline{\underline{15}} \quad \text{"inses"}$$

$$y_{tp} = \frac{30 \cdot 15 + 30 \cdot 30 + 30 \cdot 15}{3 \cdot 30} = \underline{\underline{20}}$$

4.14



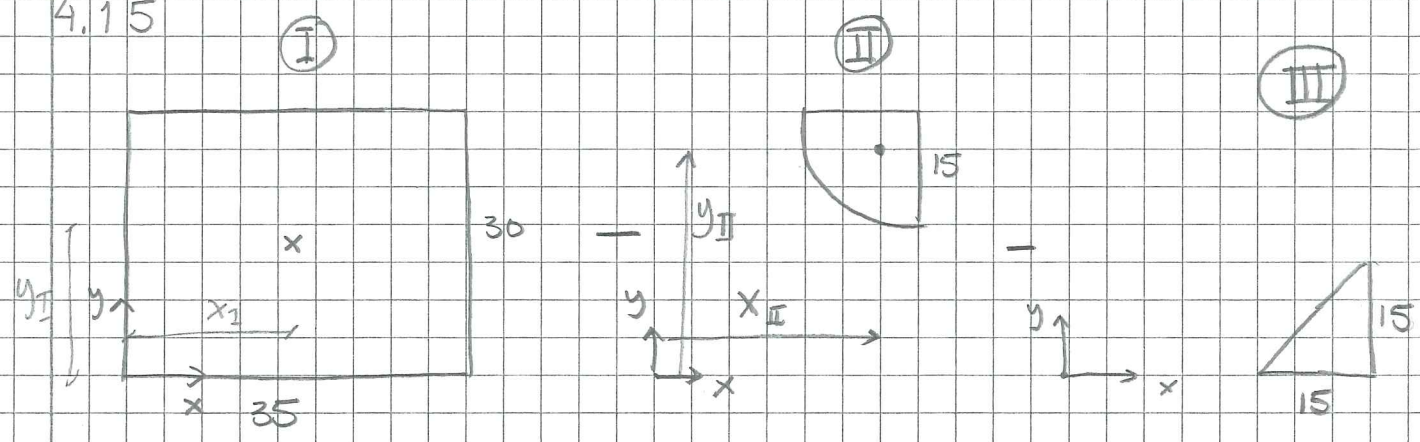
OBS! tråd

$x_{tp}=0$ ty symmetri

	L	y_{tp}	$L\cdot y_{tp}$
I	$2\cdot 34$	$\frac{\sqrt{34^2-16^2}}{2}=15$	1020
II	$\frac{32\cdot\pi}{2}=$ $=16\pi$	$30+\frac{2\cdot 16}{\pi}=40{,}18$	2020
Σ	118,27		3039,7

$$y_{tp}=\frac{3039{,}7}{118{,}27}=\underline{\underline{25{,}7}} \qquad \underline{\underline{x_{tp}=0}}$$

4.15



I $A_I = 35 \cdot 30 = 1050$

$x_I = \frac{35}{2} = 17{,}5$

$y_I = \frac{30}{2} = 15$

II $A_{II} = \frac{15^2 \cdot \pi}{4} = 176{,}7$

$x_{II} = 35 - x_0 = 35 - \frac{4 \cdot 15}{3\pi} = 28{,}6$ (6,37)

$y_{II} = 30 - x_0 = 30 - 6{,}37 = 23{,}6$

III $A_{III} = \frac{15 \cdot 15}{2} = 112{,}5$

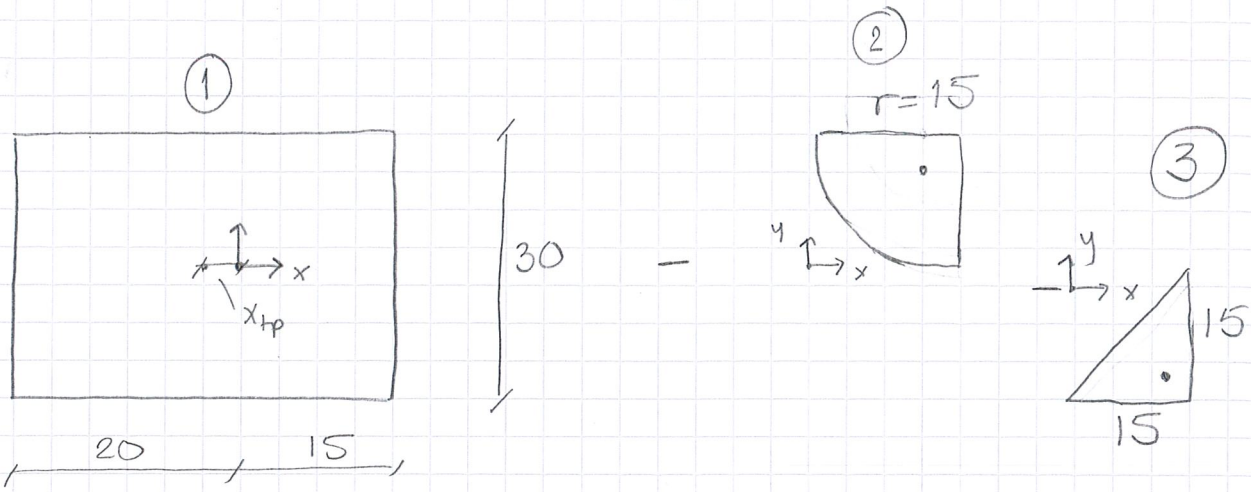
$x_{III} = 35 - \frac{h}{3} = 35 - 5 = 30$

$y_{III} = \frac{15}{3} = 5$

$$x_{tp} = \frac{1050 \cdot 17{,}5 - 177 \cdot 28{,}6 - 112{,}5 \cdot 30}{1050 - 177 - 112{,}5} = 13{,}07 \text{ mm}$$

$$y_{tp} = \frac{1050 \cdot 15 - 177 \cdot 23{,}6 - 112{,}5 \cdot 5}{1050 - 177 - 112{,}5} = 14{,}48 \text{ mm}$$

4.15 Bokens koordinatsystem

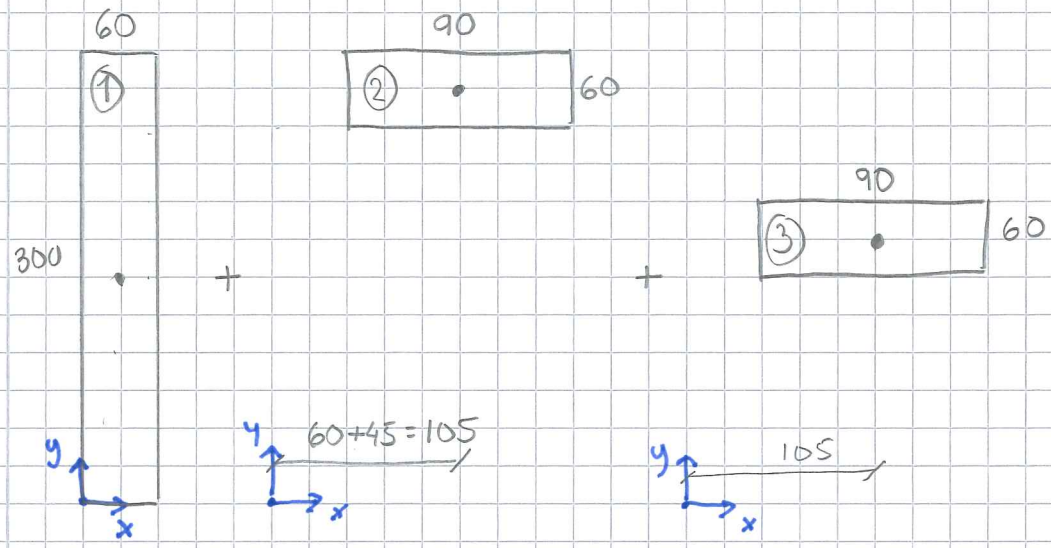


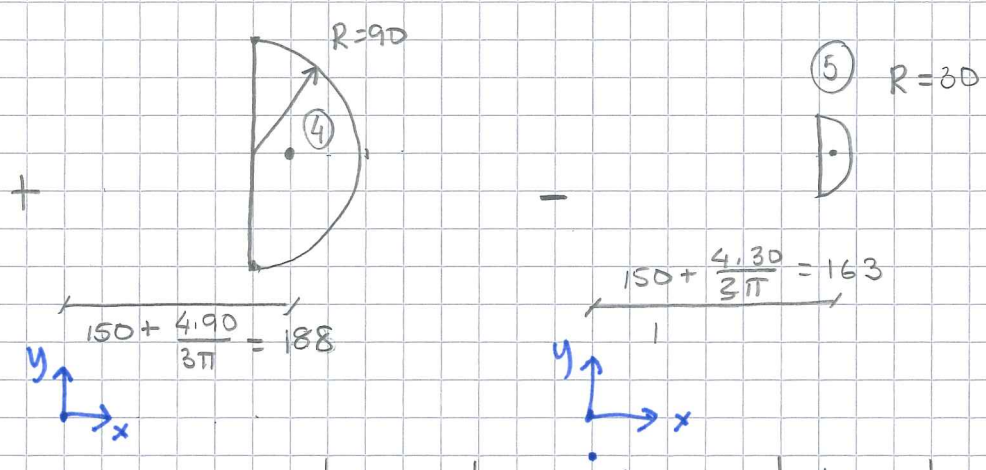
	A	x_{tp}	y_{tp}	$A x_{tp}$	$A y_{tp}$
1	$30 \cdot 35 = 1050$	$-2{,}5$	0	-2625	0
2	$-15^2\pi/4 = -177$	$15 - \frac{4 \cdot 15}{3\pi} = 8{,}63$	8,63	-1527	-1527
3	$-15^2/2 = -112{,}5$	10	-10	-1125	1125
	760,5			-5277	-402

$$x_{tp} = \frac{-5277}{760{,}5} = -6{,}9$$

$$y_{tp} = \frac{-402}{760{,}5} = 0{,}53$$

4.16 Många lösningsstrategier är möjliga.
Försök att minska antal delytor





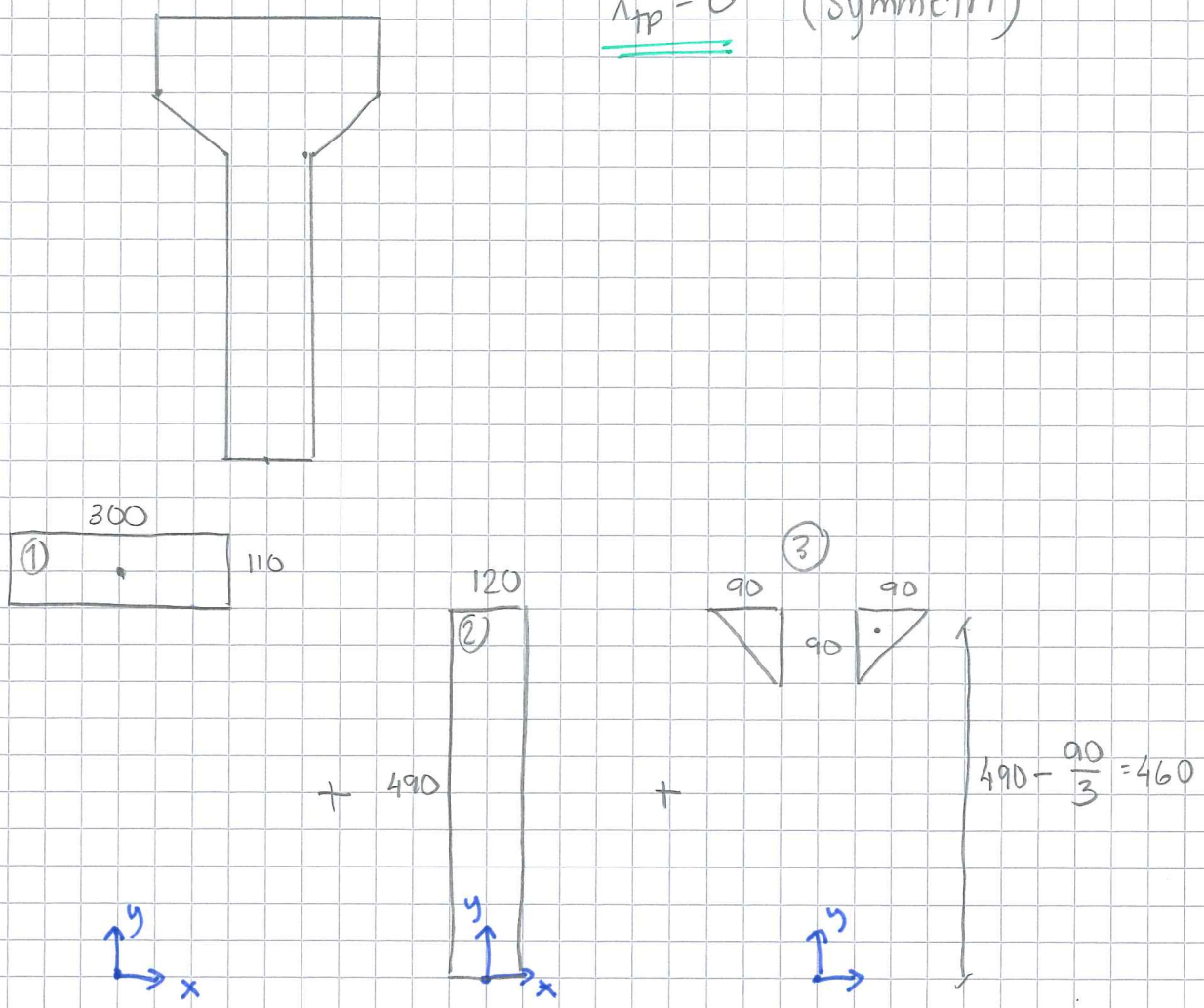
	A_n	x_{tp}	y_{tp}	$A_n x_{tp}$	$A_n y_{tp}$
1	$60 \cdot 300 = 18000$	30	150	540000	2700000
2	$90 \cdot 60 = 5400$	105	270	567000	1458000
3	$90 \cdot 60 = 5400$	105	150	567000	810000
4	$\frac{90^2 \cdot \pi}{2} = 12723$	188	210	2391924	2671830
5	$\frac{-30^2 \cdot \pi}{2} = -1414$	163	210	−230482	−296940
Σ	40109	\	\	3835442	7342890

$$x_{tp} = \frac{3835442}{40109} = \underline{\underline{95{,}6}}$$

$$y_{tp} = \frac{7342890}{40109} = \underline{\underline{183}}$$

4.17

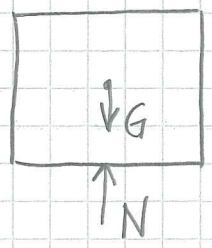
$x_{tp} = 0$ (symmetri)



	A_n	y_{tp}	$A_n y_{tp}$
1	$300 \cdot 110 = 33000$	545	17985 000
2	$490 \cdot 120 = 58800$	245	14406000
3	$\frac{90^2}{2} \cdot 2 = 8100$	460	3726000
	99900		36117 000

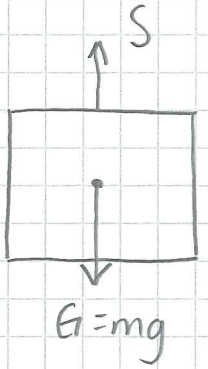
$$y_{tp} = \frac{36117000}{99900} = \underline{\underline{362\ mm}}$$

5,1

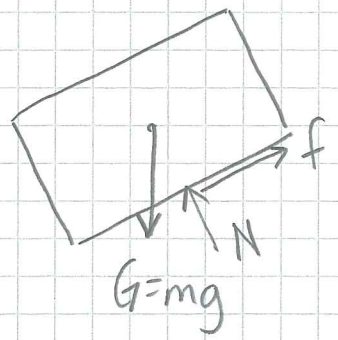


$G = mg$

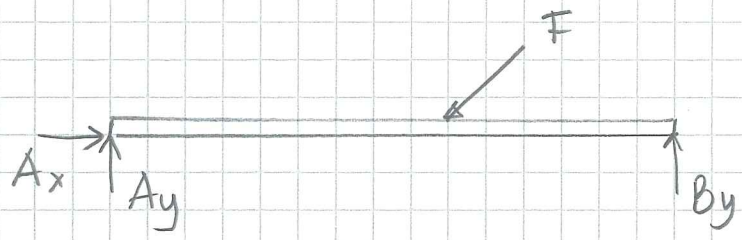
5,2



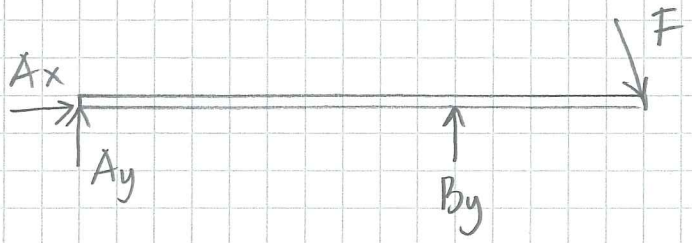
5,3



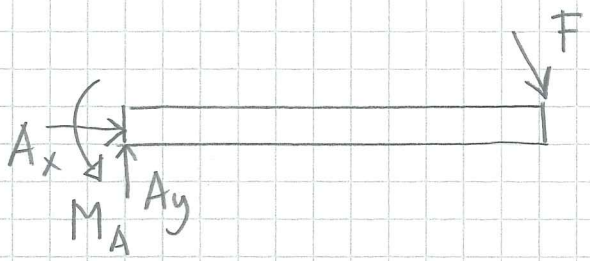
5,4



5,5

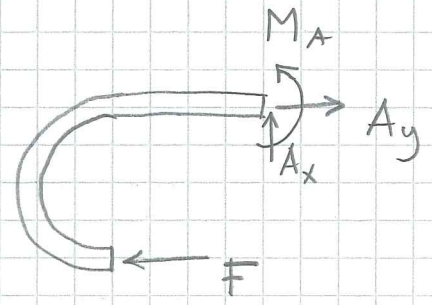


5,6



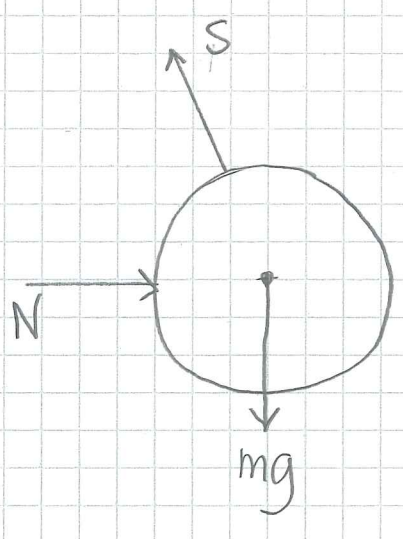
5,7

5,8



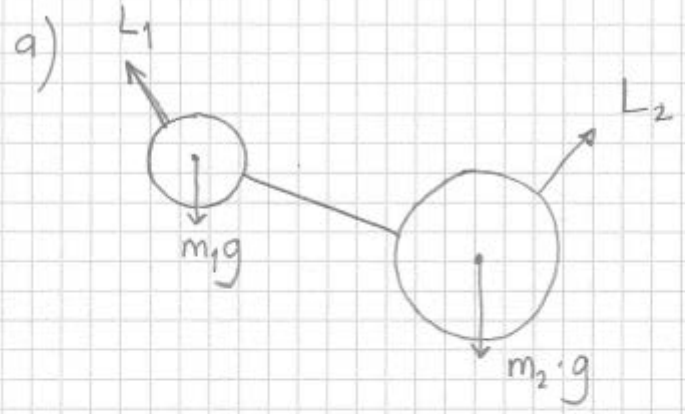
$A_x = 0$

5,9

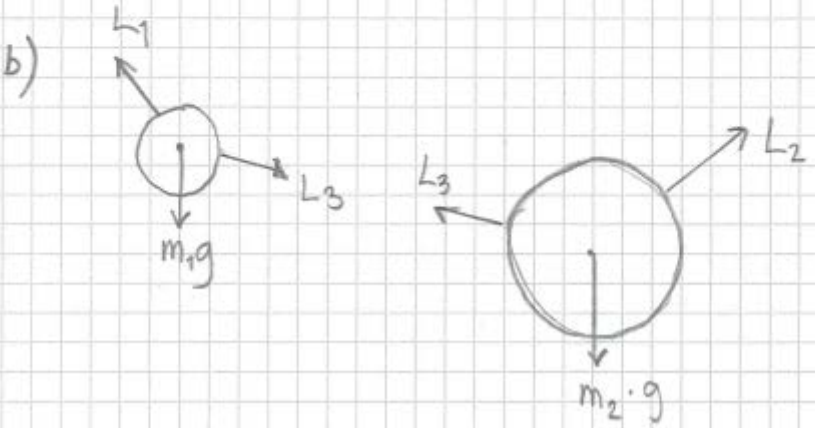


5.10

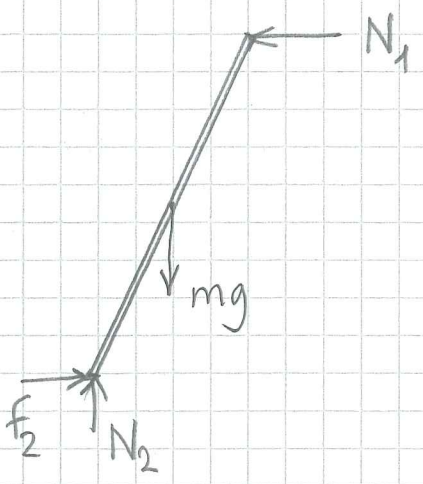
a)



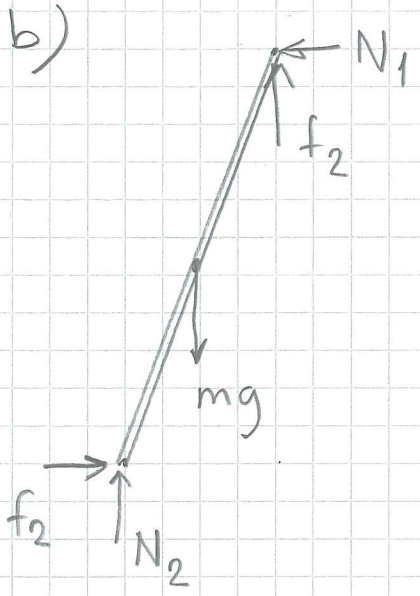
b)



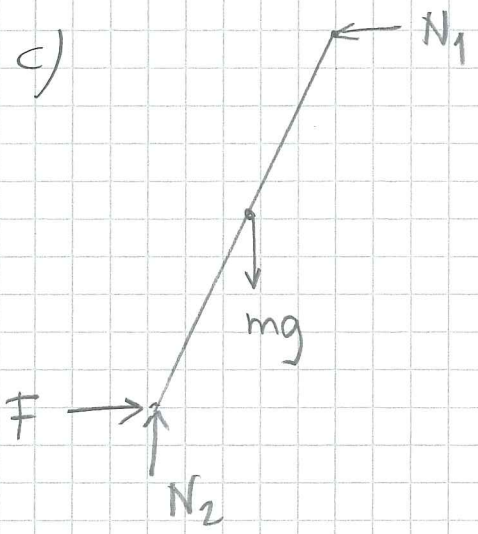
5.11 a



b)

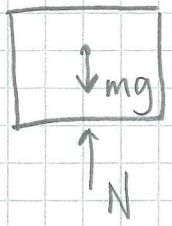


c)



5.12

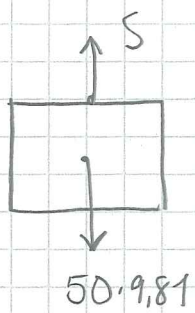
$m = 75\,kg$



$\uparrow \quad N - 75 \cdot 9{,}81 = 0$

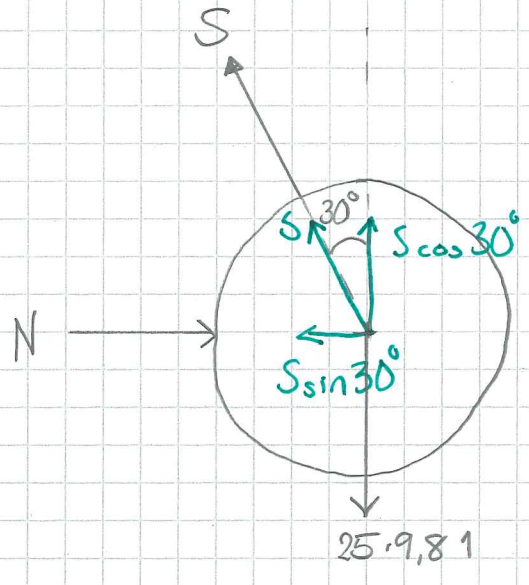
$N = 736\,N$

5.13



$\uparrow \quad S - 50 \cdot 9{,}81 = 490\,N$

5.14



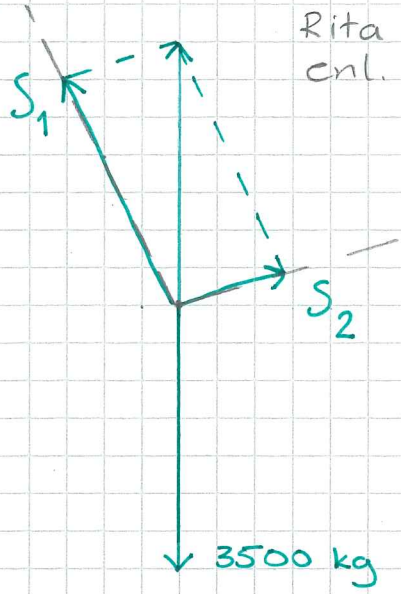
$\uparrow \quad S\cos 30^\circ - 25 \cdot 9{,}81 = 0$

$S = \dfrac{25 \cdot 9{,}81}{\cos 30^\circ} = \underline{\underline{283\,N}}$

$\rightarrow \quad N - S\sin 30^\circ = 0$

$N = 283 \sin 30^\circ = \underline{\underline{142\,N}}$

5.15a



Rita med rätt proportioner enl. mått i bild.

1 cm = 1000 kg

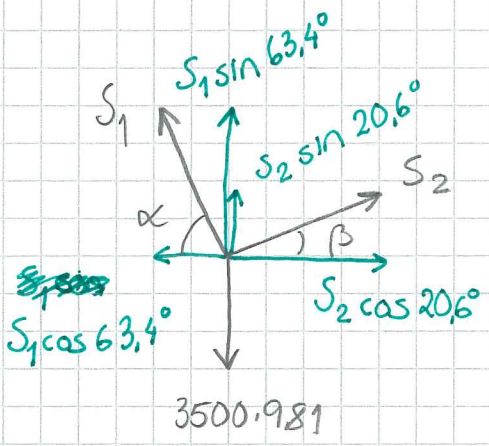
$S_2 = 1{,}5\,cm = 1500\,kg$

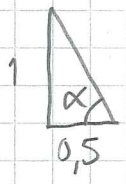
$S_2 = 9{,}81 \cdot 1500 = 14{,}7\,kN$

$S_1 = 3{,}3\,cm = 3300\,kg$

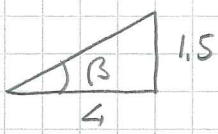
$S_1 = 9{,}81 \cdot 3300 = 32{,}4\,kN$

b)





$\alpha = \arctan\left(\frac{1}{0{,}5}\right) = 63{,}4°$



$\beta = \arctan\left(\frac{1.5}{4}\right) = 20{,}6°$

$\rightarrow \quad S_2 \cos 20{,}6° - S_1 \cos 63{,}4° = 0$

$S_2 = S_1 \dfrac{\cos 63{,}4°}{\cos 20{,}6°} = S_1 \cdot 0{,}478$

$\uparrow \quad S_1 \sin 63{,}4° + S_2 \sin 20{,}6° - 3500 \cdot 9{,}81 = 0$

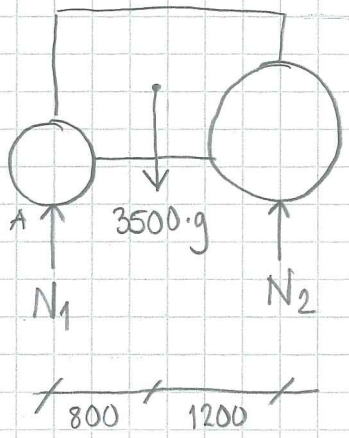
$S_1 \sin 63{,}4° + S_1 \cdot 0{,}478 \sin 20{,}6° = 3500 \cdot 9{,}81$

$S_1 \left(\sin 63{,}4° + 0{,}478 \cdot \sin 20{,}6° \right) = 3500 \cdot 9{,}81$

$S_1 \cdot 1{,}062 = 3500 \cdot 9{,}81 \quad \Rightarrow \quad S_1 = \underline{\underline{32{,}3\,kN}}$

$S_2 = 32{,}3 \cdot 0{,}478 = \underline{\underline{15{,}4\,kN}}$

5.16



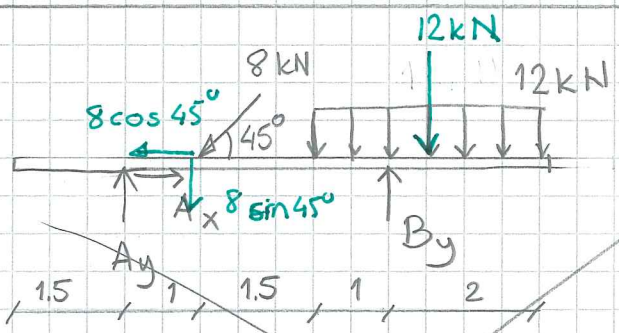
$\overset{\curvearrowright}{A}$ $3500 \cdot 9{,}81 \cdot 0{,}8 - N_2 \cdot (0{,}8 + 1{,}2) = 0$

$$N_2 = \frac{3500 \cdot 9{,}81 \cdot 0{,}8}{0{,}8 + 1{,}2} = 13734\,N = \underline{\underline{13{,}7\,kN}}$$

$\uparrow$ $N_1 + N_2 - 3500 \cdot 9{,}81 = 0$

$$N_1 = 3500 \cdot 9{,}81 - 13734 = 20601\,N = \underline{\underline{20{,}6\,kN}}$$

5.17



$\overset{\curvearrowright}{A}$ $12 \cdot 3 + 1 \cdot 8 \sin 45° - B_y \cdot 3{,}5 = 0$

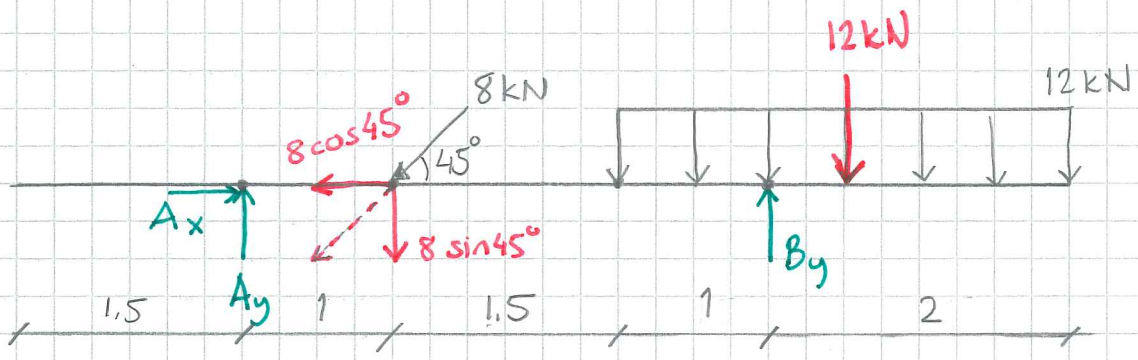
$$B_y = \frac{12 \cdot 3 + 1 \cdot 8 \sin 45°}{3.5} \Rightarrow B_y = 11{,}9\,kN$$

$\uparrow$ $A_y + B_y - 12 - 8 \sin 45° = 0$

$$A_y = 12 + 8 \sin 45° - 11{,}9 = 5{,}76\,kN$$

$\rightarrow$ $A_x - 8 \cos 45° = 5{,}66\,kN$

5,17



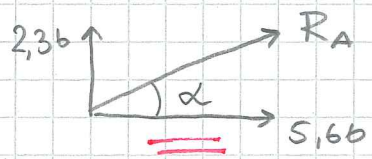
$\overset{\curvearrowright}{A}$ $\quad 1 \cdot 8 \sin 45^\circ + 4 \cdot 12 - 3{,}5 \cdot B_y = 0$

$$B_y = \frac{8 \sin 45^\circ + 4 \cdot 12}{3{,}5} = \underline{\underline{15{,}3 \text{ kN}}}$$

$\uparrow \quad A_y + B_y - 8 \sin 45^\circ - 12 = 0$

$$A_y = 8 \sin 45^\circ + 12 - 15{,}3 = 2{,}36 \text{ kN}.$$

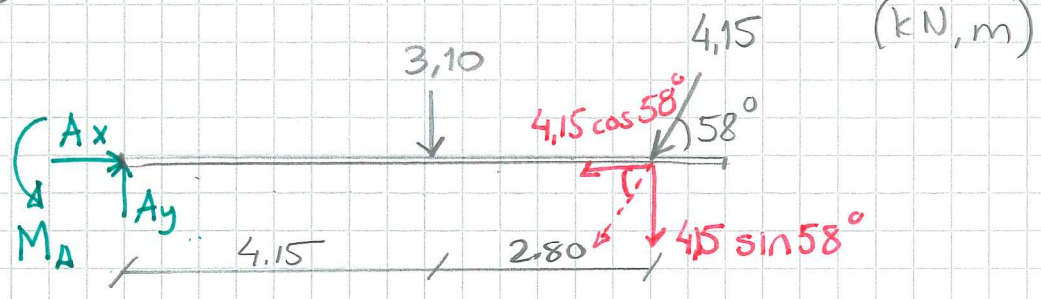
$\rightarrow \quad A_x - 8 \cos 45^\circ = 5{,}66 \text{ kN}$



$$R_A = \sqrt{5{,}66^2 + 2{,}36^2} = \underline{\underline{6{,}1 \text{ kN}}}$$

$$\alpha = \arctan \frac{2{,}36}{5{,}66} = \underline{\underline{22{,}6^\circ}}$$

5,18

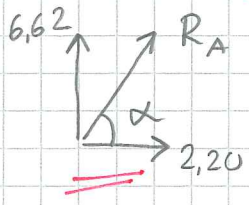


$\uparrow \quad A_y - 3{,}10 - 4{,}15 \sin 58^\circ = 0 \quad \Rightarrow \quad A_y = 6{,}62 \text{ kN}$

$\rightarrow \quad A_x - 4{,}15 \cos 58^\circ = 0 \quad \Rightarrow \quad A_x = 2{,}20 \text{ kN}$

$\circlearrowleft A \quad M_A - 3{,}10 \cdot 4{,}15 - 4{,}15 \sin 58^\circ \cdot (4{,}15 + 2{,}80) = 0$

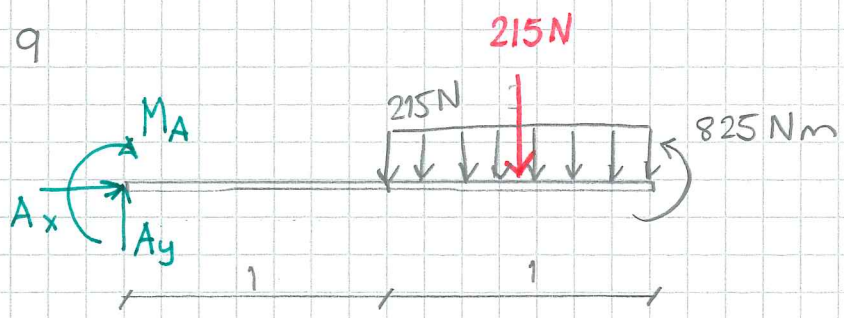
$M_A = \underline{\underline{37{,}3 \text{ kNm}}} \circlearrowleft$



$R_A = \sqrt{6{,}62^2 + 2{,}20^2} = \underline{\underline{7{,}0 \text{ kN}}}$

$\alpha = \arctan\left(\frac{6{,}62}{2{,}20}\right) = \underline{\underline{71{,}6^\circ}}$

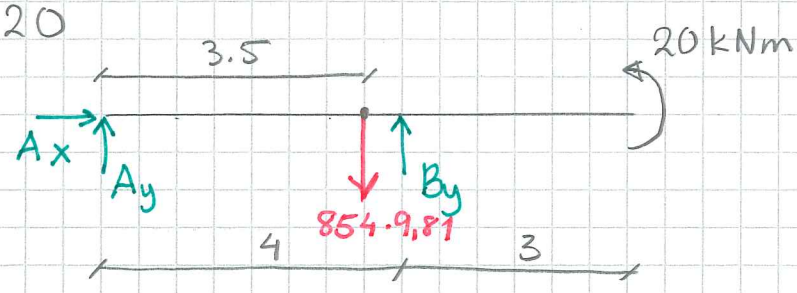
5.19



$\rightarrow$ $A_x = 0$ $\quad\uparrow$ $A_y - 215 = 0$ $\quad A_y = 215\ N$

$\curvearrowright A$ $\quad M_A + 215 \cdot 1{,}5 - 825 = 0 \Rightarrow M_A = 503\ Nm$

5.20



$m = 122 \cdot 7 = 854\ kg$

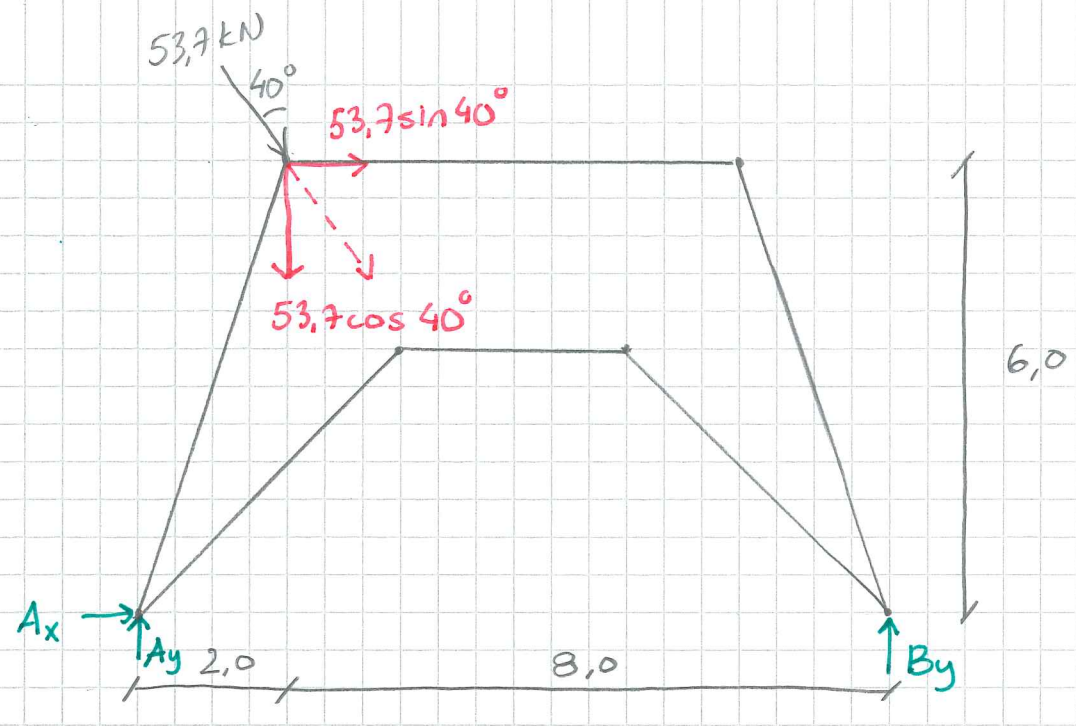
$\curvearrowright A$ $\quad 854 \cdot 9{,}81 \cdot 3{,}5 - 20 \cdot 10^3 - B_y \cdot 4 = 0$

$$B_y = \frac{854 \cdot 9{,}81 \cdot 3{,}5 - 20 \cdot 10^3}{4} = 2331\ N$$

$\rightarrow$ $A_x = 0$ $\quad\uparrow$ $B_y + A_y - 854 \cdot 9{,}81 = 0$

$A_y = 854 \cdot 9{,}81 - 2331 = 6047\ N$

5.21



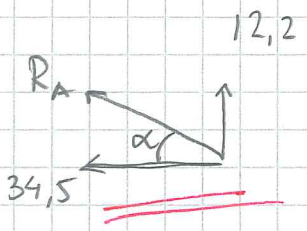
$$\overset{\curvearrowright}{A} \quad 6 \cdot 53{,}7 \sin 40^\circ + 2 \cdot 53{,}7 \cos 40^\circ - 10{,}0 \cdot B_y = 0$$

$$B_y = \frac{6 \cdot 53{,}7 \sin 40^\circ + 2 \cdot 53{,}7 \cos 40^\circ}{10} = \underline{\underline{28{,}9 \text{ kN}}}$$

$$\uparrow \quad A_y + B_y - 53{,}7 \cos 40^\circ = 0$$

$$A_y = 12{,}2 \text{ kN}$$

$$\rightarrow \quad A_x + 53{,}7 \sin 40^\circ = 0 \quad \Rightarrow \quad A_x = -34{,}5 \text{ kN}.$$



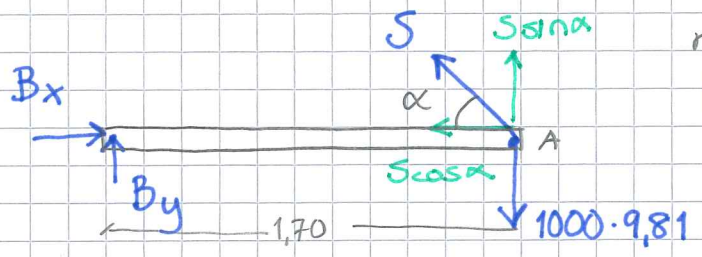
$$R_A = \sqrt{34{,}5^2 + 12{,}2^2} = \underline{\underline{36{,}6 \text{ kN}}}$$

$$\alpha = \arctan\left(\frac{12{,}2}{34{,}5}\right) = \underline{\underline{19{,}5^\circ}}$$

5,22 Sökt: Lin & stångkrafter

Alt 1

Lina ger dragkraft i linans riktning.



$\tan\alpha = \frac{1,25}{1,70}$

$\alpha = 36,33^\circ$

$\overset{\curvearrowright}{A}$ $B_y \cdot 1,70 = 0 \Rightarrow B_y = 0$

$\uparrow$ $S \sin 36,33^\circ - 1000 \cdot 9,81 + \underbrace{\cancel{B_y}}_{=0} = 0$

$S = 16559$ N

$\rightarrow$ $B_x - S\cos 36,33^\circ = 0$

$B_x = 16559 \cdot \cos 36,33^\circ = 13340$ N

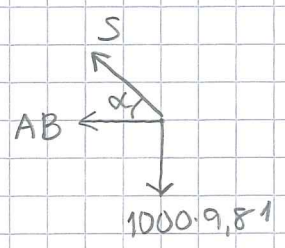
Bx ger tryck enligt frilägning

Alt 2: Inse att systemet består av tvåkraftskroppar

leder: - Momentfritt sammankopplade

- Kraft angriper i knutpunkt

Frilägg punkt A



$\uparrow$ $S \sin 36,33^\circ - 1000 \cdot 9,81 = 0$

$S = 16559$ N

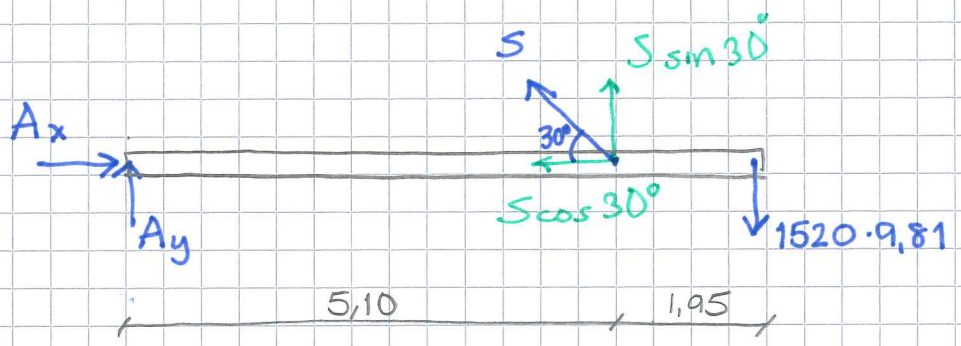
$\leftarrow$ $AB - S\cos 36,33^\circ = 0$

$AB = -13340$ N (tryck)

5.23 Sökt: krafter på balken AB

Frilägg ning

S har riktning 30° ty ledad



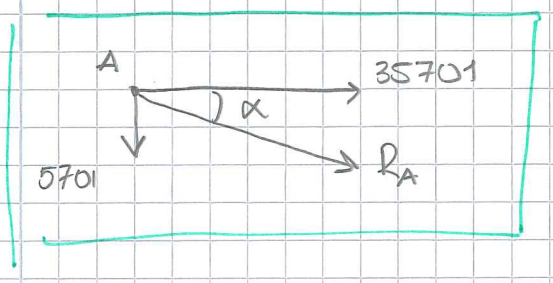
$\overset{\curvearrowleft}{A}$ $5{,}10 \cdot S \cdot \sin 30° - 1520 \cdot 9{,}81 (5{,}10 + 1{,}95) = 0$

$S = \underline{\underline{41225 \text{ N}}}$

$\uparrow$ $A_y + S \sin 30° - 1520 \cdot 9{,}81 = 0$

$A_y = 1520 \cdot 9{,}81 - 41225 \cdot \sin 30° = -5701 \text{ N}$

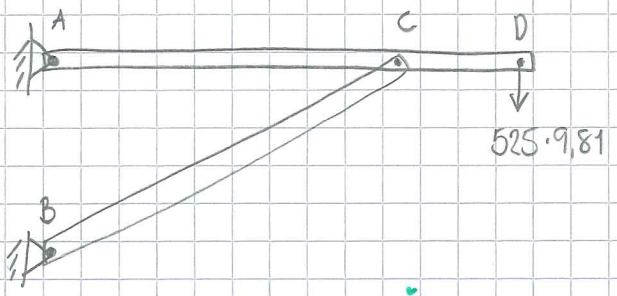
$\rightarrow$ $A_x - S \cos 30° = 0$ $A_x = 41225 \cos 30° = 35701 \text{ N}$



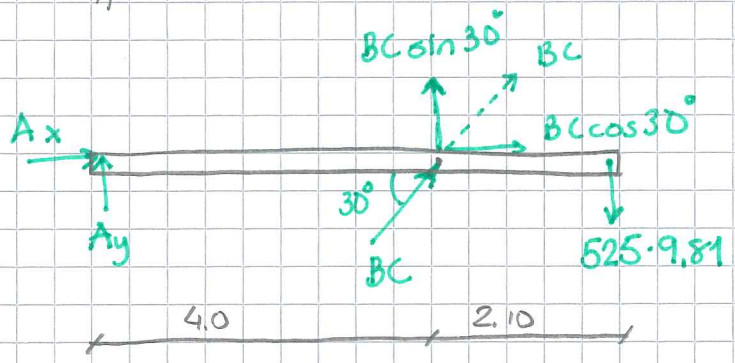
$R_A = \sqrt{35701^2 + 5701^2} = \underline{\underline{36{,}1 \text{ kN}}}$

$\alpha = \arctan\left(\frac{5701}{35701}\right) = \underline{\underline{9{,}1°}}$

5.24 Sökt: Krafterna på ACD & BC



BC => led => endast axiella krafter



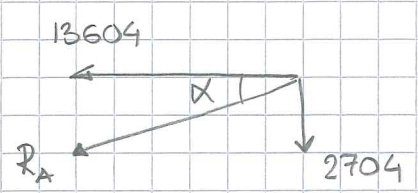
$\overset{\curvearrowright}{A}$ $525 \cdot 9{,}81(4+2{,}1) - BC \sin 30° \cdot 4 = 0$

$BC = 15708\,N$

$\rightarrow$ $A_x + BC \cos 30° = 0 \Rightarrow A_x = -15708 \cos 30° = -13604\,N$

$\uparrow$ $A_y + BC \sin 30° - 525 \cdot 9{,}81 = 0$

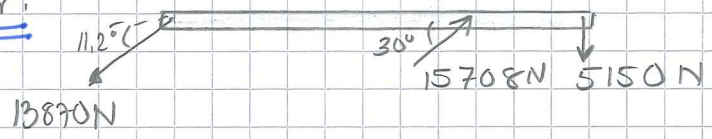
$A_y = 525 \cdot 9{,}81 - 15708 \sin 30° = -2704\,N$

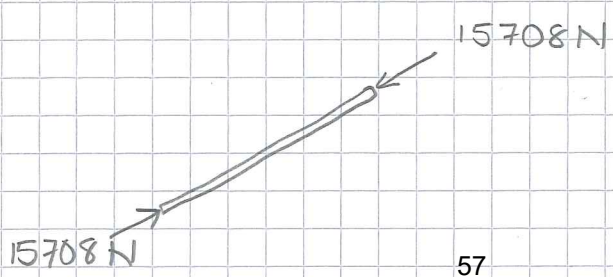


$R_A = \sqrt{2704^2 + 13604^2} = 13870\,N$

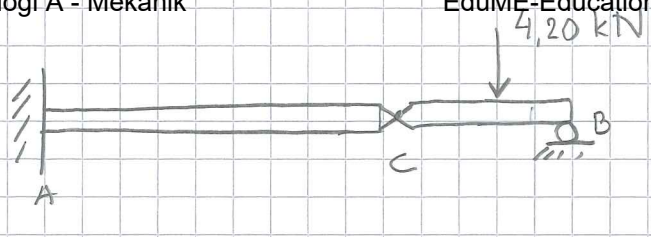
$\alpha = \arctan\left(\frac{2704}{13604}\right) = 11{,}2°$

Svar:





5.25

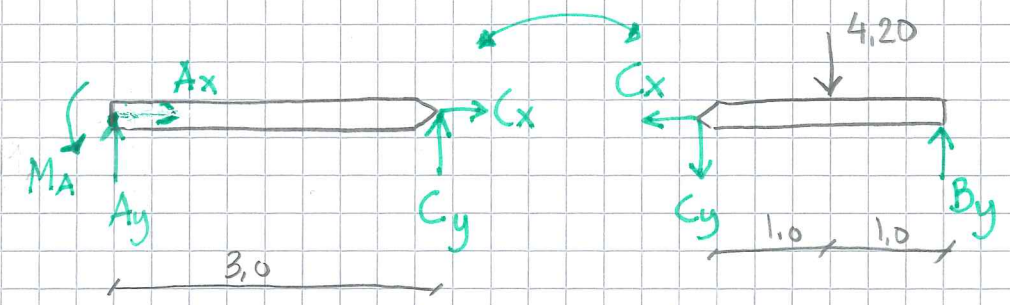


Sökt: Stödreaktioner R_A, R_B, M_A

Börja med jämvikt här 3 obekanta ≟ 3 ekv lösbart.

Frilägg

lika men motriktade (kN)



$C_x = 0$ ⟸ ← $C_x = 0$

$C_y = -2{,}1$ kN

↺ C $B_y \cdot 2 - 4{,}2 \cdot 1 = 0$

$B_y = 2{,}1$ kN

→ $A_x = 0$

↑ $B_y - 4{,}20 - C_y = 0$

$C_y = 2{,}1 - 4{,}2 = -2{,}1$ kN

↺ A $M_A + C_y \cdot 3 = 0$

$M_A = -(-2{,}1) \cdot 3 = 6{,}3$ kNm

Antagit fel riktning på C_y i frilägningen därav negativ.

↑ $A_y + C_y = 0$

$A_y = -C_y = -(-2{,}1) = 2{,}1$ kN

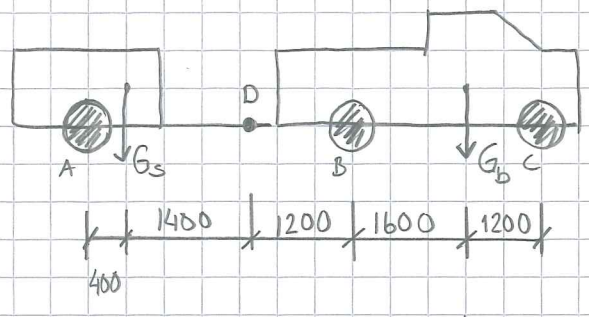
Då lasten är mitt på BC kan man inse att 4,20 kN fördelas lika på C och B samt är riktade uppåt, vilket jämvikten visar.

Svar: $R_A = A_y = 2{,}1$ kN ↑

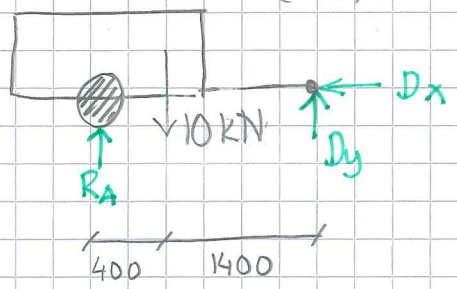
$R_B = B_y = 2{,}1$ kN ↑

$M_A = 6{,}3$ kNm ↺

5.26 Sökt: R_A, R_B, R_C



(kN) alla x-komposanter (ABC) är noll ty endast vertikala krafter.



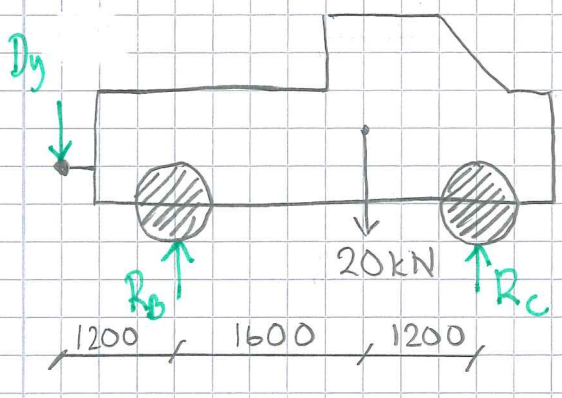
$\leftarrow$ $D_x = 0$ (förstås)

$\overset{\curvearrowright}{D}$ $R_A \cdot 1800 - 10 \cdot 400 = 0$

$R_A = 7{,}78$ kN

$\uparrow$ $R_A + D_y - 10 = 0$

$D_y = 2{,}22$ kN



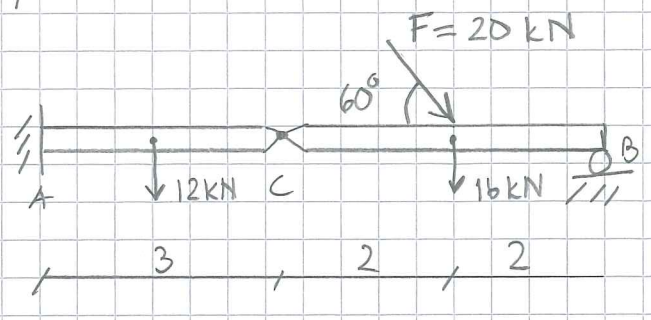
$\overset{\curvearrowleft}{B}$ $1200 \cdot 2{,}22 - 20 \cdot 1600 + R_C(1600 + 1200) = 0$

$R_C = 10{,}48$ kN

$\uparrow$ $R_B + R_C - 2{,}22 - 20 = 0$

$R_B = 20 + 2{,}22 - 10{,}48 = 11{,}74$ kN

5,27

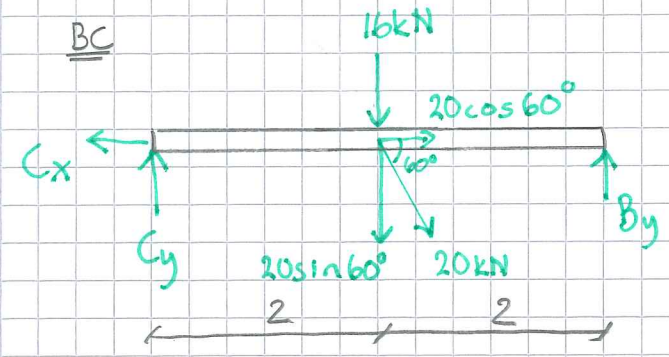


Egentyngd: 4kN/m

$m_{AC} = 3 \cdot 4 = 12\ kN$

$m_{CB} = 4 \cdot 4 = 16\ kN$

BC



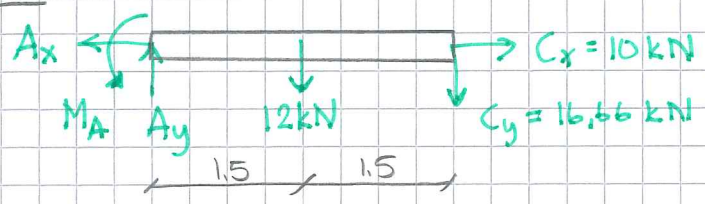
$\leftarrow \quad C_x - 20\cos 60^\circ = 0 \quad \Rightarrow \quad C_x = 10\ kN$

$\circlearrowleft C \quad B_y \cdot 4 - (16 + 20\sin 60^\circ) \cdot 2 = 0 \qquad \underline{\underline{B_y = 16{,}66\ kN}}$

$\uparrow \quad C_y + B_y - 16 - 20\sin 60^\circ = 0$

$C_y = 16 + 20\sin 60^\circ - 16{,}66 = 0 \quad \Rightarrow \quad C_y = 16{,}66\ kN$ (förstås)

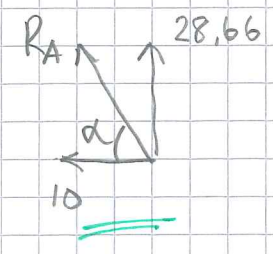
AC



$\leftarrow \quad A_x - C_x = 0 \quad \Rightarrow \quad A_y = C_x = 10\ kN$

$\circlearrowleft A \quad M_A - 12 \cdot 1{,}5 - C_y \cdot 3 = 0 \quad \Rightarrow \quad \underline{\underline{M_A}} = 12 \cdot 1{,}5 + 16{,}66 \cdot 3 = \underline{\underline{68\ kNm}}$

$\uparrow \quad A_y - 12 - C_y = 0 \qquad A_y = 12 + 16{,}66 = 28{,}66\ kN$

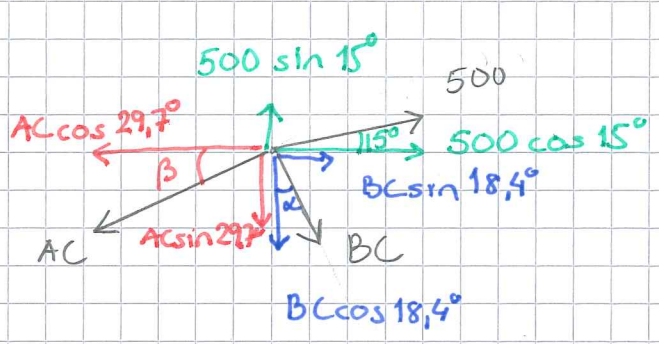


$\underline{\underline{R_A}} = \sqrt{28{,}66^2 + 10^2} = \underline{\underline{30{,}35\ kN}}$

$\underline{\underline{\alpha}} = \arctan\left(\frac{28{,}66}{10}\right) = \underline{\underline{70{,}7^\circ}}$

5.28 Sökt: Krafter på AC & BC

AC & BC är ledat i fästa, lasten (500N) påverkar en ledpunkt (C).
Detta innebär att det blir krafter i stångernas riktning.
Krafterna bestäms enklast med en knutpunktsanalys av C



Antar drag i AC och BC.

$\alpha = \arctan\left(\frac{16}{48}\right) = 18{,}4°$

$\beta = \arctan\left(\frac{48}{84}\right) = 29{,}7°$

$\uparrow \quad 500\sin 15° - AC\sin 29{,}7° - BC\cos 18{,}4° = 0 \qquad (1)$

$\rightarrow \quad 500\cos 15° - AC\cos 29{,}7° + BC\sin 18{,}4° = 0 \qquad (2)$

$(1) \Rightarrow \quad AC = \frac{500\sin 15°}{\sin 29{,}7°} - \frac{BC\cos 18{,}4°}{\sin 29{,}7°} = 261{,}19 - 1{,}915\,BC \qquad (3)$

(3) i (2)

$500\cos 15° - (261{,}19 - 1{,}915\,BC)\cdot\cos 29{,}7° + BC\sin 18{,}4° = 0$

$500\cos 15° - 226{,}88 + 1{,}663\,BC + 0{,}316\,BC = 0$

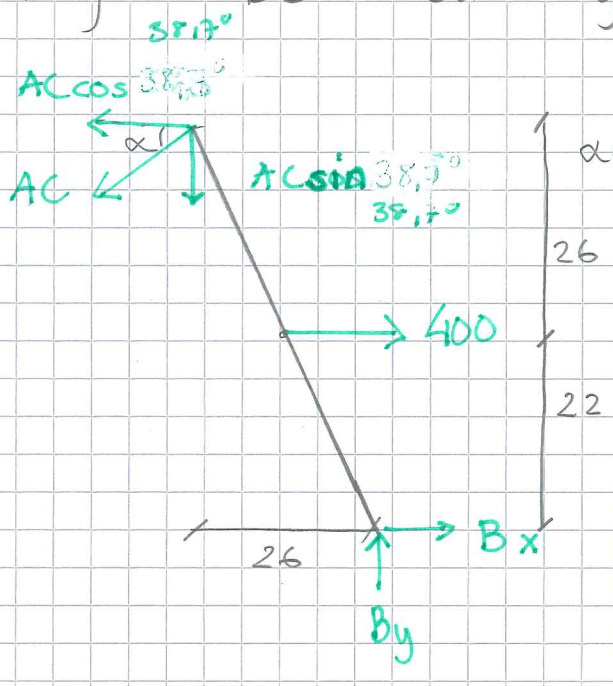
$\underline{BC} = \underline{-129{,}4\,N}$ (tryckkraft, rimligt)

(3) $\underline{AC} = 261{,}19 - 1{,}915\cdot(-129{,}4) = \underline{509\,N}$ (drag, rimligt)

5,29 Sökt: Krafterna på AC & BC

En yttre kraft påverkar mitt på BC-balken
Detta innebär att man får kraft-riktningar
som skiljer från balkens riktning i BC.
AC får däremot en kraft i dess riktning.

Friläggning av BC => samtliga krafter



$\alpha = \arctan\left(\frac{48}{60}\right) = 38{,}7^\circ$

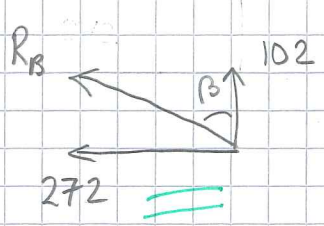
$\overset{\curvearrowright}{B}$ $400 \cdot 22 - AC\sin 38{,}7^\circ \cdot 26 - AC\cos 38{,}7^\circ \cdot 48 = 0$

$$AC = \frac{400 \cdot 22}{26\sin 38{,}7^\circ + 48\cos 38{,}7^\circ} = \underline{\underline{163{,}8\ N}}$$

$\rightarrow$ $Bx + 400 - AC\cos 38{,}7^\circ = 0$

$Bx = 163{,}8\cos 38{,}7^\circ - 400 = -272\ N$

$\uparrow$ $By - AC\sin 38{,}7^\circ = 0 \Rightarrow By = 163{,}8 \cdot \sin 38{,}7^\circ = 102\ N$

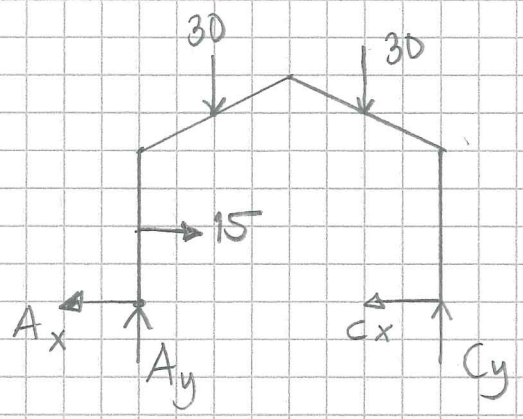


$R_B = \sqrt{102^2 + 272^2} = \underline{\underline{291\ N}}$

$\beta = \arctan\left(\frac{102}{272}\right) = \underline{\underline{21^\circ}}$

5.30 Sökt: R_A, R_C

Global frilägning



(kN

4 obekanta
3 jämviksekv } ej lösbart

=> gör vad du kan
därefter inre frilägning

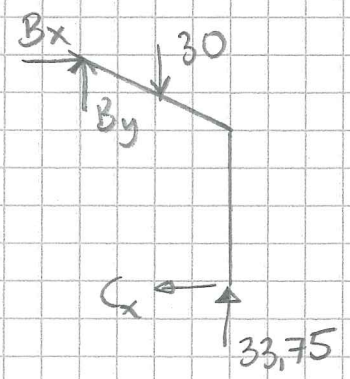
$\overset{\curvearrowleft}{A}$ $C_y \cdot 8 - 15 \cdot 2 - 30 \cdot 2 - 30 \cdot 6 = 0$

$C_y = 33{,}75$ kN

$\uparrow$ $A_y + C_y - 2 \cdot 30 = 0$

$A_y = 60 - 33{,}75 = 26{,}25$ kN

Inre frilägning => C_x

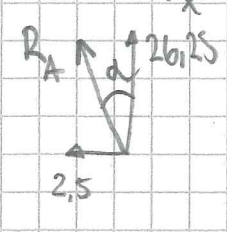


$\overset{\curvearrowright}{B}$ $C_x \cdot 6 + 30 \cdot 2 - 33{,}75 \cdot 4 = 0$

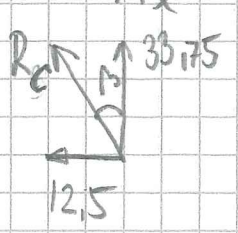
$C_x = 12{,}5$ kN.

Global frilägning

$\leftarrow$ $A_x - 15 + 12{,}5 = 0$ => $A_x = 2{,}5$ kN

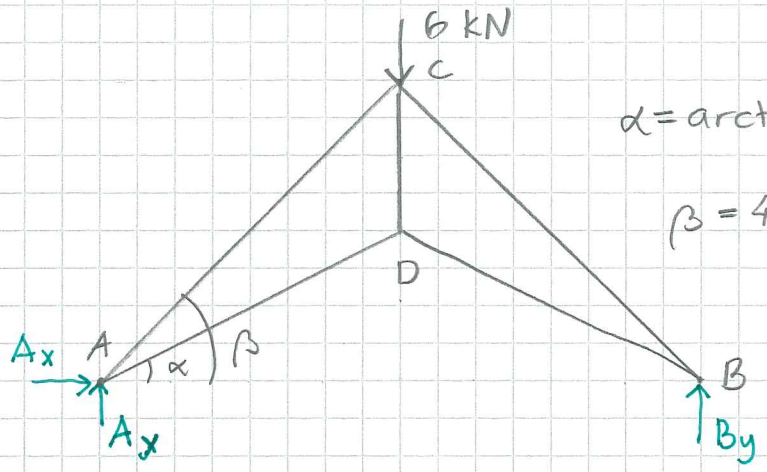


$R_A = 26{,}4$ kN
$\alpha = 5{,}4°$



$R_C = 36$ kN
$\beta = 20{,}3°$

5,31



$\alpha = \arctan\left(\frac{2}{4}\right) = 26{,}6^\circ$

$\beta = 45$

Symmetri => $A_x = 0$ $A_y = B_y = 3$ kN

Symmetri => $AC = CB$

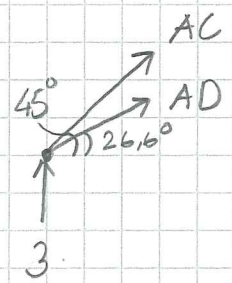
$AD = DB$

Svar: $AC = CB = -8{,}50$ kN

$AD = DB = 6{,}72$ kN

$CD = 6$ kN

Knutpunkt A



$\uparrow \; AC \sin 45^\circ + AD \sin 26{,}6^\circ + 3 = 0 \qquad (1)$

$\rightarrow \; AC \cos 45^\circ + AD \cos 26{,}6^\circ = 0$

$AC = -AD \cdot \frac{\cos 26{,}6^\circ}{\cos 45^\circ} = -AD \cdot 1{,}265$

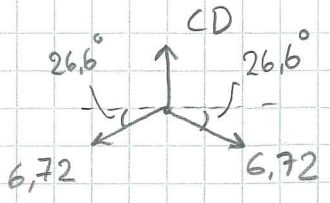
$-AD \cdot 1{,}265 \sin 45^\circ + AD \sin 26{,}6^\circ + 3 = 0$

$AD(1{,}265 \sin 45^\circ - \sin 26{,}6^\circ) = 3$

$AD = 6{,}72$ kN

$AC = -6{,}72 \cdot 1{,}265 = -8{,}50$ kN

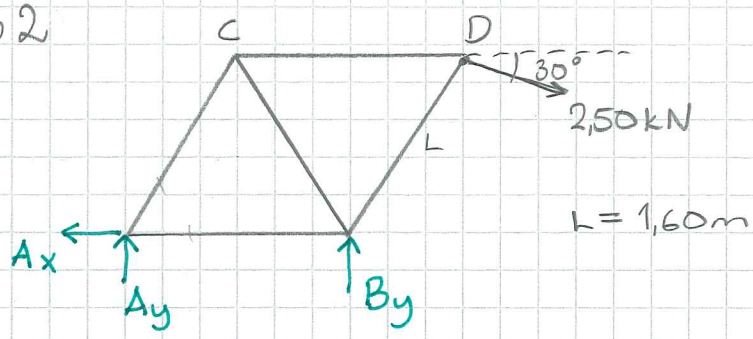
Knutpunkt D



$\uparrow \; CD - 2 \cdot 6{,}72 \cdot \sin 26{,}6^\circ = 0$

$CD = 6$ kN

5.32



b Svar:

AC = 2,89 kN
AB = 0,725 kN
CB = − 2,89 kN
BD = − 1,44 kN
CD = 2,89 kN

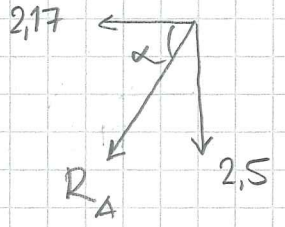
a) $\overset{\curvearrowright}{A}$ $2{,}50 \cdot \cos 30^\circ \cdot 1{,}60 \cdot \sin 60^\circ + 2{,}50 \sin 30^\circ \cdot (1{,}60 + 0{,}8) \ldots$

$- B_y \cdot 1{,}60 = 0 \quad \Rightarrow \quad B_y = \underline{\underline{3{,}75}}$ kN

$\uparrow \quad A_y + B_y - 2{,}50 \sin 30^\circ = 0$

$A_y = 2{,}50 \sin 30^\circ - 3{,}75 = -2{,}5$ kN

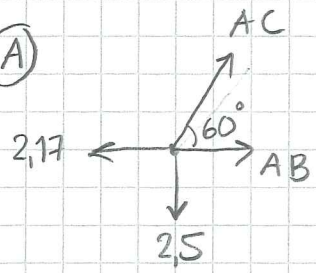
$\leftarrow \quad A_x - 2{,}50 \cos 30^\circ = 0 \quad \Rightarrow \quad A_x = 2{,}17$ kN



$R_A = \sqrt{2{,}17^2 + 2{,}5^2} = \underline{\underline{3{,}31}}$ kN

$\alpha = \arctan\left(\frac{2{,}5}{2{,}17}\right) = \underline{\underline{49^\circ}}$

b) (A)



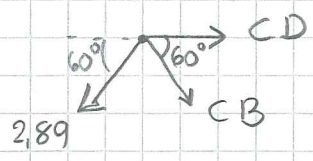
$\uparrow \quad AC \sin 60^\circ - 2{,}5 = 0$

$AC = 2{,}89$ kN

$\rightarrow \quad AB + AC \cos 60^\circ - 2{,}17 = 0$

$AB = 2{,}17 - 2{,}89 \cos 60^\circ = 0{,}725$ kN

(C)



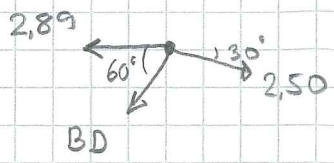
$\downarrow \quad CB \cancel{\sin 60^\circ} + 2{,}89 \cancel{\sin 60^\circ} = 0$

$CB = -2{,}89$ kN

$\rightarrow \quad CD + CB \cos 60^\circ - 2{,}89 \cos 60^\circ = 0$

$CD = \cos 60^\circ (2{,}89 - (-2{,}89)) = 2{,}89$ kN

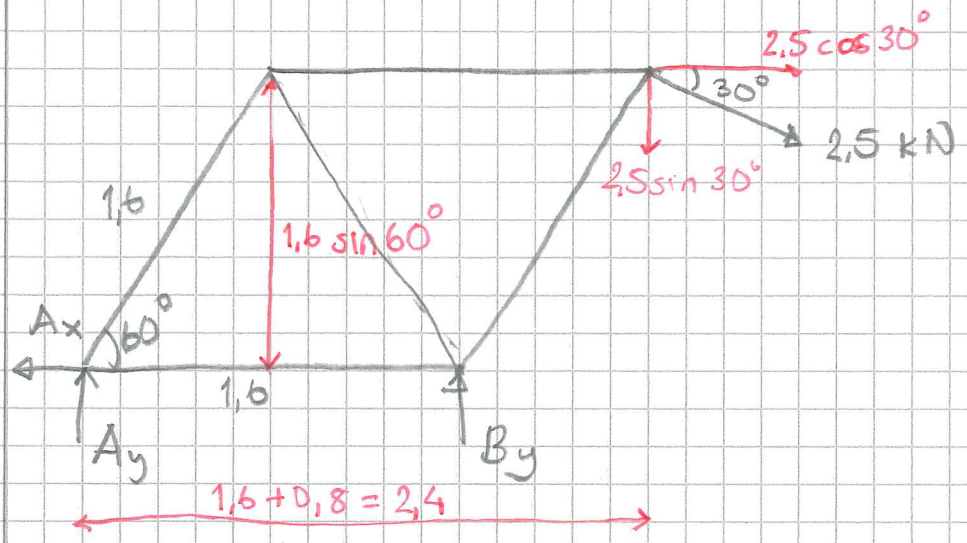
(D)



$\downarrow \quad BD \sin 60^\circ + 2{,}50 \sin 30^\circ = 0$

$BD = -1{,}44$ kN

5.32a Sökt: R_A, R_B



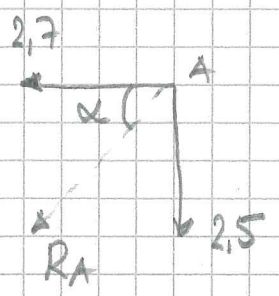
$\overset{\curvearrowleft}{A}$ $B_y \cdot 1{,}6 - 2{,}4 \cdot 2{,}5 \sin 30^\circ - 1{,}6 \sin 60^\circ \cdot 2{,}5 \cos 30^\circ = 0$

$B_y = 3{,}75$ kN

$\rightarrow$ $2{,}5 \cos 30^\circ - A_x = 0$ $\quad A_x = 2{,}17$ kN

$\uparrow$ $A_y + B_y - 2{,}5 \sin 30^\circ = 0$

$A_y = 1{,}25 - 3{,}75 = -2{,}5$ kN



$R_A = \sqrt{2{,}7^2 + 2{,}5^2} = \underline{\underline{3{,}68 \text{ kN}}}$

$\alpha = \arctan \frac{2{,}5}{2{,}17} = \underline{\underline{49^\circ}}$

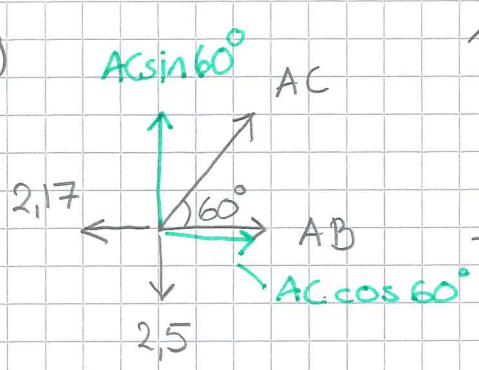
eller $180 + 49 = 229$ från x-axeln

$R_B = 3{,}75$ kN $\uparrow$

5.32 b Knutpunktsanalys

Börja med punkt A eller D då det finns två okända stångkrafter.

(A)



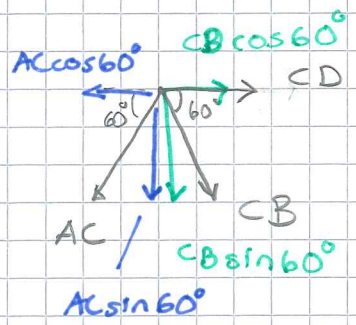
$\uparrow \; AC\sin 60^\circ - 2{,}5 = 0$

$AC = \frac{2{,}5}{\sin 60^\circ} = 2{,}89 \text{ kN (drag)}$

$\rightarrow \; AC\cos 60^\circ + AB - 2{,}17 = 0$

$AB = 2{,}17 - 2{,}89\cos 60^\circ = 0{,}73 \text{ kN (drag)}$

(C)



$AC = 2{,}89 \text{ kN}$ Sökta: CD & CB

$\downarrow \; CB\sin 60^\circ + AC\sin 60^\circ = 0$

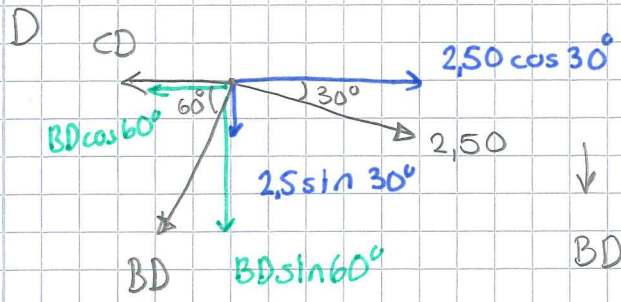
$CB = -AC = -2{,}89 \text{ kN}$

$\rightarrow \; CD + CB\cos 60^\circ - AC\cos 60^\circ = 0$

$CD = \cos 60^\circ (AC - CB) = \cos 60^\circ (2{,}89 - (-2{,}89))$

$CD = 2{,}89 \text{ kN}$

D



$\downarrow \; BD\sin 60^\circ + 2{,}5\sin 30^\circ = 0$

$BD = -2{,}5\frac{\sin 30^\circ}{\sin 60^\circ} = -1{,}44 \text{ kN}$

Svar: AC = 2,89 kN (drag)

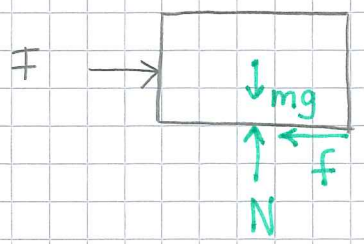
AB = 0,73 kN (drag)

CB = −2,89 kN (tryck)

CD = 2,89 kN (drag)

BD = −1,44 kN (tryck)

5.33



$\uparrow \; N - mg = 0 \quad N = 35 \cdot 9{,}81 = 343\,N$

$\rightarrow \; F - f = 0 \quad F = f \quad (1)$

a) vid glidning $f = f_{max} = N \cdot \mu$

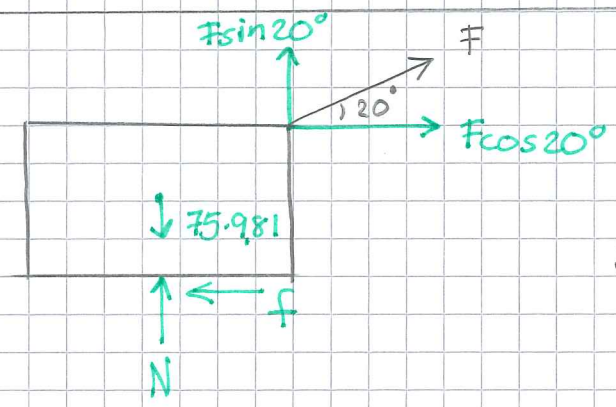
$\mu = 0{,}350 \Rightarrow f = 343 \cdot 0{,}35 = 120\,N$

$(1) \Rightarrow \underline{\underline{F = 120\,N}}$

b) Om $F = 100\,N$ utnyttjat friktionstal blir

$(1) \Rightarrow F = f = \mu_b N \Rightarrow \mu_b = \frac{100}{343} = \underline{\underline{0{,}292}}$

5.34



$\mu = 0{,}220$

Sökt: F vid glidning

Vid glidning $f = f_{max} = N \cdot \mu$

$\uparrow \; N - 75 \cdot 9{,}81 + F\sin 20^\circ = 0 \quad N = 735{,}75 - F\sin 20^\circ \quad (1)$

$\rightarrow \; F\cos 20^\circ - N\mu = 0$

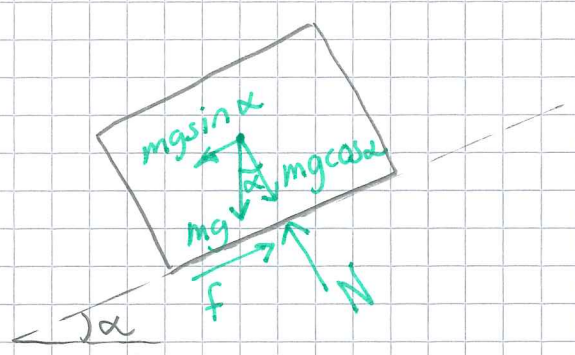
$(1) \Rightarrow F\cos 20^\circ - \mu(735{,}75 - F\sin 20^\circ) = 0$

$F\cos 20^\circ + \mu F \sin 20^\circ = \mu \cdot 735{,}75$

$$F = \frac{0{,}220 \cdot 735{,}75}{\cos 20^\circ + 0{,}22 \sin 20^\circ} = \underline{\underline{160\,N}}$$

5,35

Sökt: α vid glidning



$m = 30\,kg$

$\mu = 0{,}3$

Vid glidning $f = f_{max} = \mu N$

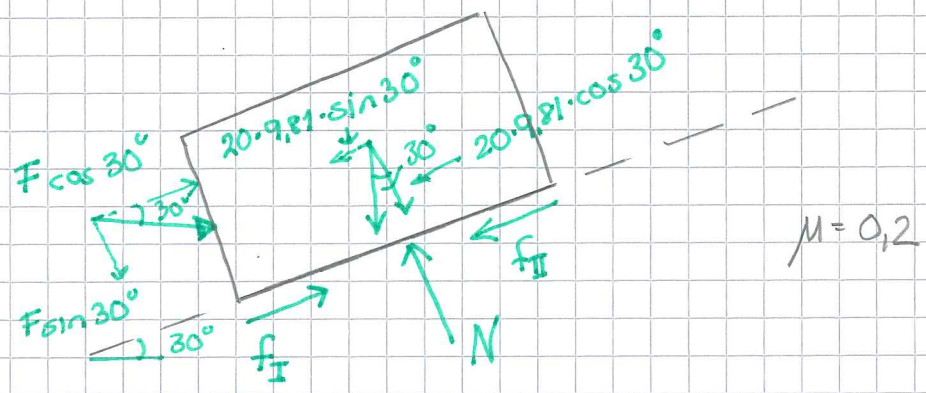
$\swarrow$ $\mu N - mg\sin\alpha = 0 \quad N = \frac{mg}{\mu}\sin\alpha \quad (1)$

$\nwarrow$ $N - mg\cos\alpha = 0 \quad N = mg\cos\alpha \quad (2)$

$(1) = (2) \quad \frac{\cancel{mg}}{\mu}\sin\alpha = \cancel{mg}\cos\alpha \Rightarrow \mu = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha$

$\alpha = \arctan\mu = \arctan 0{,}3 = \underline{\underline{16{,}7^\circ}}$

5.36 Sökt: F för jämvikt dvs lådan glider ej – varken uppför eller utför planet.



(I) Anta glidning nedför => F_{min}

f_I hjälper att förhindra glidning $f_I^{max} = N \cdot \mu$

↖ $N - 20 \cdot 9{,}81 \cdot \cos 30° - F \sin 30° = 0$ (1)

↗ $-N \cdot \mu + F \cos 30° - 20 \cdot 9{,}81 \cdot \sin 30° = 0$ (2)

(1) $N = 169{,}9 + 0{,}5F$

(2) $-(169{,}9 + 0{,}5F) \cdot 0{,}2 + F \cos 30° = 98{,}1$

$-33{,}98 - 0{,}1F + F \cos 30° = 98{,}1$

$F_{min} = 66{,}4$ N

(II) Anta glidning uppför => F_{max}

f_{II} förhindrar glidning uppför $f_{II}^{max} = N \cdot \mu$

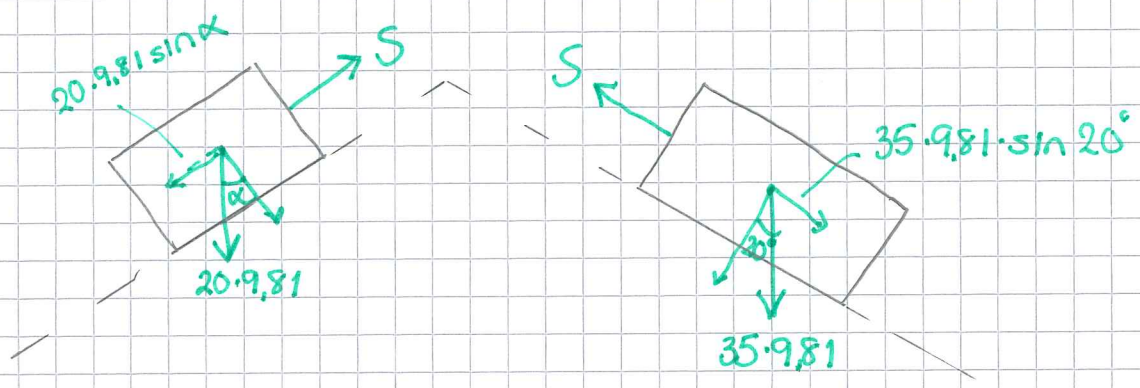
Titta på ekvationerna ovan $N\mu$ byter tecken då f byter riktning

$F_{max} = 172{,}4$ N

$66{,}4 \text{ N} \leq F \leq 172{,}4 \text{ N}$

5.37 Sökt: α vid jämvikt (ingen glidning)

Glatt => ingen friktion



$S - 20 \cdot 9{,}81 \cdot \sin\alpha = 0$

$S = 20 \cdot 9{,}81 \sin\alpha \quad (1)$

$S - 35 \cdot 9{,}81 \cdot \sin 20^\circ = 0$

$S = 35 \cdot 9{,}81 \sin 20^\circ \quad (2)$

$(1) = (2)$

$20 \cdot \cancel{9{,}81} \sin\alpha = 35 \cdot \cancel{9{,}81} \sin 20^\circ$

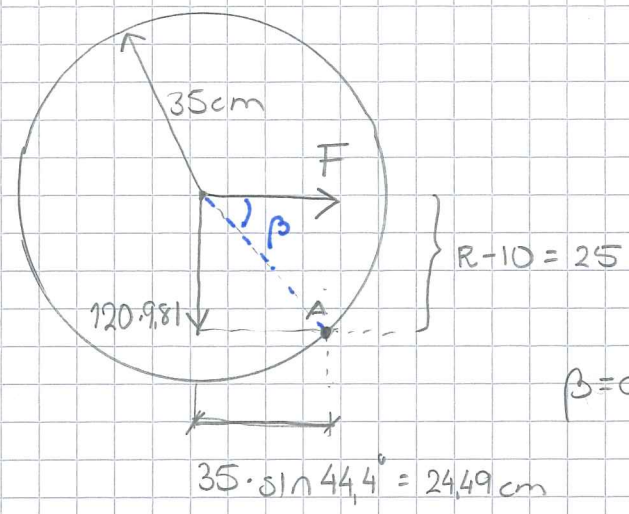
$\sin\alpha = \frac{35}{20} \sin 20^\circ \quad \Rightarrow \alpha = \arcsin 0{,}599 = \underline{\underline{36{,}7^\circ}}$

5.38

Sökt: F om a) $\alpha = 0$

b) $\alpha = 35°$

a)

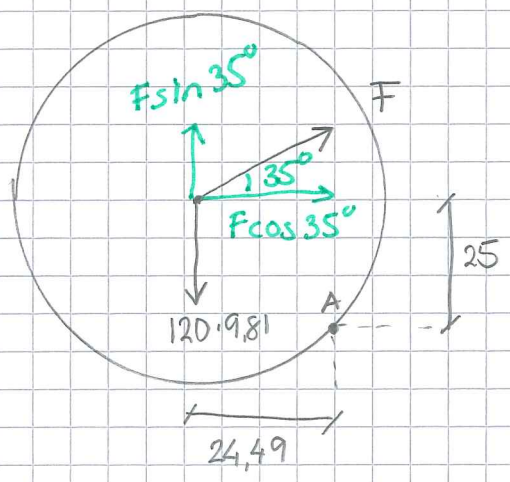


$m = 120\ kg$

$\beta = \arccos \frac{25}{35} = 44{,}4°$

$\overset{\curvearrowright}{A}$ $F \cdot 25 - 120 \cdot 9{,}81 \cdot 24{,}49 = 0 \Rightarrow \underline{\underline{F = 1153\ N}}$

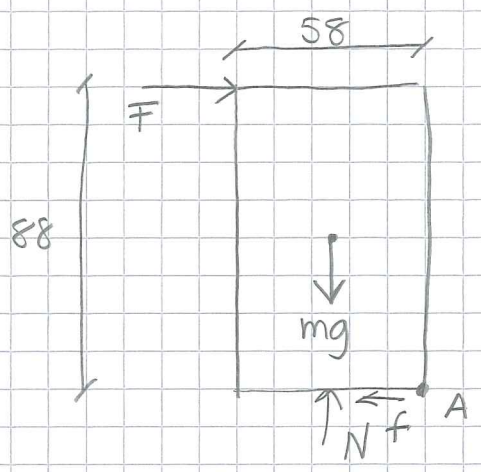
b



$\overset{\curvearrowright}{A}$ $F\cos 35° \cdot 25 + F\sin 35° \cdot 24{,}49 - 120 \cdot 9{,}81 \cdot 24{,}49 = 0$

$$F = \frac{120 \cdot 9{,}81 \cdot 24{,}49}{25\cos 35° + 24{,}49 \sin 35°} = \underline{\underline{835\ N}}$$

5.39 Sökt: μ för att stjälpning ska ske



$$\left.\begin{aligned} M_{stjälp} &= F \cdot 88 \\ M_{stabil} &= mg \cdot \frac{58}{2} \end{aligned}\right\} \quad F \cdot 88 = mg\,\frac{58}{2}$$

$$F = mg \cdot \frac{29}{88}$$

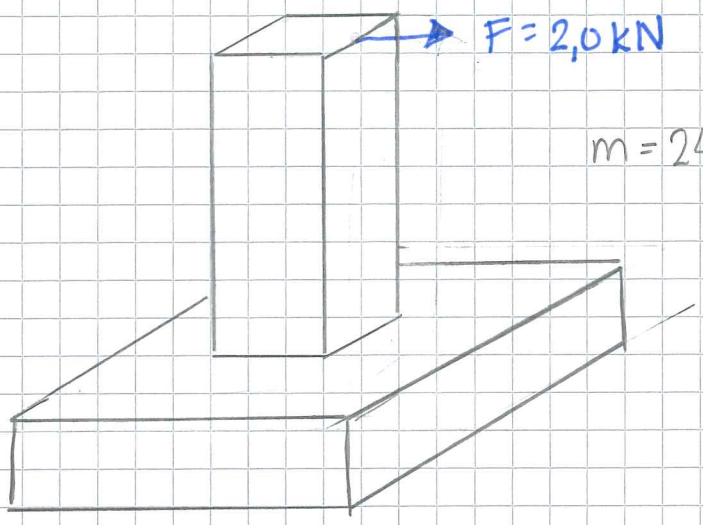
Om glidning inte ska ske måste

$$f_{max} = F$$

$$N\mu = mg \cdot \frac{29}{88}$$

$$\cancel{mg}\,\mu = \cancel{mg}\,\frac{29}{88} \quad \Rightarrow \quad \mu_{min} = \underline{\underline{0{,}33}}$$

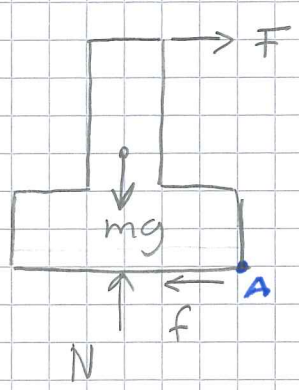
5,40 a) μ_{min} b) Säkerhet mot stjälpning n_s



$\rho = 2400\ kg/m^3$

$m = 2400(0{,}3^2 \cdot 0{,}6 + 0{,}9^2 \cdot 0{,}3)$

$m = 712{,}8\ kg$



Vid glidning: $f = N\mu$

$\uparrow\ N = mg$

$\longrightarrow\ F = f = N\mu = mg\mu = 2000$

$\mu_{min} = \frac{2000}{712{,}8 \cdot 9{,}81} = \underline{\underline{0{,}286}}$

Bestäm tyngdpunkten

$$y_{tp} = \frac{V_1 y_1^{tp} + V_2 y_2^{tp}}{V_{tot}} = \frac{0{,}3^2 \cdot 0{,}6 \cdot 0{,}6 + 0{,}9^2 \cdot 0{,}3 \cdot 0{,}15}{0{,}3^2 \cdot 0{,}6 + 0{,}9^2 \cdot 0{,}3} = 0{,}232\,m$$

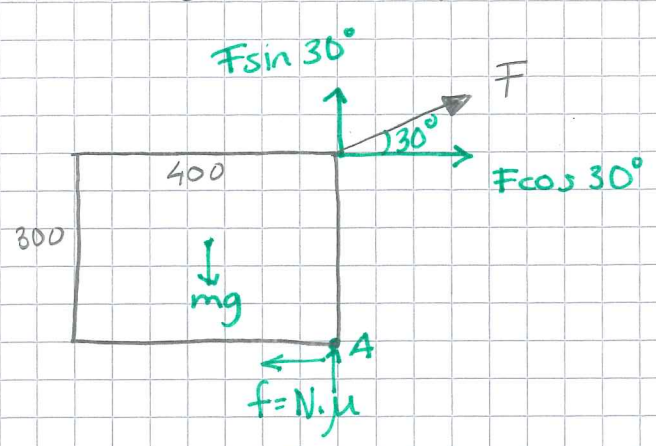
b) $M_{stjälp} = F \cdot (300 + 600) = 2000 \cdot 900 = 1800000\ Ncm$

$M_{stabil} = mg\left(\frac{900}{2}\right) = 712{,}8 \cdot 9{,}81 \cdot 450 = 3146656\ Ncm$

$$n_s = \frac{M_{stabil}}{M_{stjälp}} = \frac{3146656}{1800000} = \underline{\underline{1{,}75}}$$

5.41 Sökt: Sker glidning eller stjälpning om F ökar.

$\mu = 0,2$



$M_A = 0$ vid stjälpning

$\overset{\curvearrowright}{A}$ $mg \cdot 200 - F\cos 30^\circ \cdot 300 = 0$ $F_{stjälp} = \frac{mg \cdot 200}{30\cos 30^\circ} = mg \cdot 0,770$

Glidning då $f = N \cdot \mu$

$\uparrow$ $N - mg + F\sin 30^\circ = 0$ $\Rightarrow N = mg - F\sin 30^\circ$

$\leftarrow$ $N \cdot \mu - F\cos 30^\circ = 0$

$(mg - F\sin 30^\circ) \cdot \mu = F\cos 30^\circ$

$0,2mg = F(\cos 30^\circ + 0,2\sin 30^\circ)$

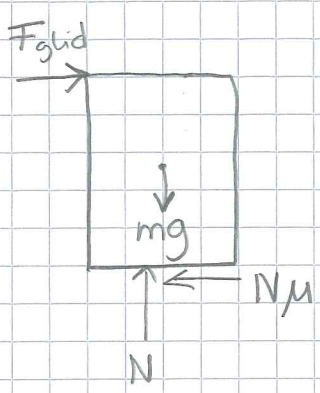
$F = F_{glid} = mg\, 0,207$

$F_{stjälp} > F_{glid}$ $\Rightarrow$ Glider.

5.42 $m = 25\ kg$ $\mu = 0{,}25$

a) F_{glid} b) $F_{stjälp}$

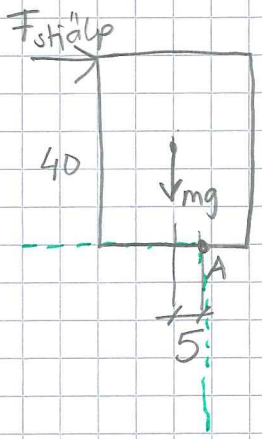
a)



$\uparrow N = mg$

$\rightarrow F = N\mu = mg\mu = 25 \cdot 9{,}81 \cdot 0{,}25 = \underline{\underline{61{,}3\ N}}$

b)



$M_A = 0 \Rightarrow F_{stjälp}$

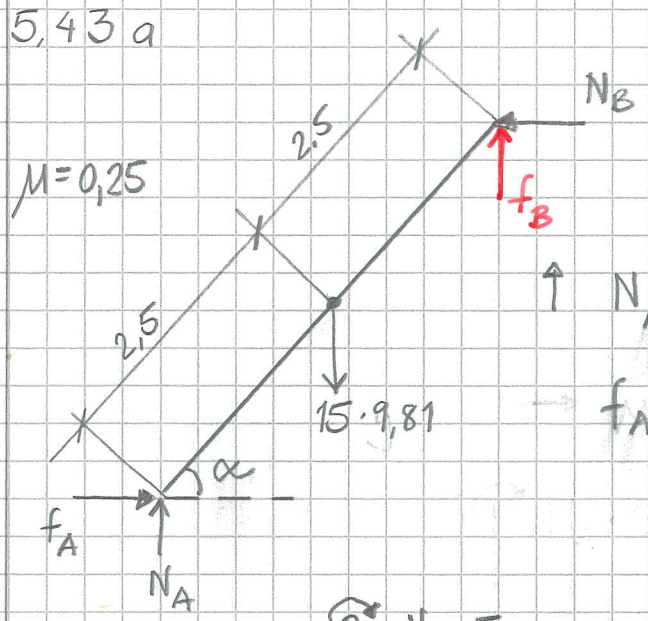
$\circlearrowleft A \quad mg \cdot 5 = F_{stjälp} \cdot 40$

$F_{stjälp} = 25 \cdot 9{,}81 \cdot \frac{5}{40} = \underline{\underline{30{,}7\ N}}$

c) Stjälper

5,43 a

Sökt: α vid glidning



glidning => $f_A = \mu N_A$

$f_B = 0$ ty glatt vägg

$\uparrow \; N_A - 15 \cdot 9{,}81 = 0 \Rightarrow N_A = 147{,}15 \text{ N}$

$f_A = 0{,}25 \cdot 147{,}15 = 36{,}79 \text{ N}$

$\circlearrowright B \; N_A \cdot 5\cos\alpha - 15 \cdot 9{,}81 \cdot 2{,}5\cos\alpha - f_A \cdot 5\sin\alpha = 0$

$(147{,}15 \cdot 5 - 15 \cdot 9{,}81 \cdot 2{,}5)\cos\alpha = 36{,}79 \cdot 5 \sin\alpha$

$\frac{\sin\alpha}{\cos\alpha} = \frac{367{,}9}{183{,}95} = \tan\alpha$

$\alpha = \arctan 2 = \underline{\underline{63{,}4^\circ}}$

---

b) strav vägg => $f_B = \mu N_B$ vid glidning

$\uparrow \; N_A - 15 \cdot 9{,}81 + \mu N_B = 0 \quad (1)$

$\rightarrow \; \mu N_A - N_B = 0 \Rightarrow N_B = \mu N_A$

$(1) \Rightarrow N_A + \mu^2 N_A = 15 \cdot 9{,}81 \quad N_A = \frac{15 \cdot 9{,}81}{1 + 0{,}25^2} = 138{,}49 \text{ N}$

$\circlearrowright B \; N_A \cdot 5\cos\alpha - 15 \cdot 9{,}81 \cdot 2{,}5\cos\alpha - f_A \cdot 5\sin\alpha = 0$

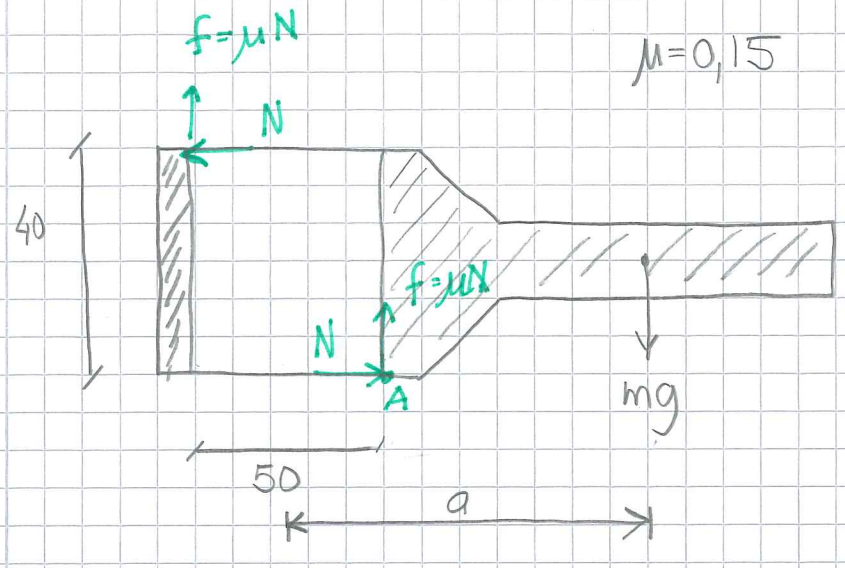
$f_A = 0{,}25 \cdot 138{,}49 = 34{,}62$

$(138{,}49 \cdot 5 - 15 \cdot 9{,}81 \cdot 2{,}5)\cos\alpha = 34{,}62 \cdot 5 \cdot \sin\alpha$

$\frac{\sin\alpha}{\cos\alpha} = \frac{324{,}57}{173{,}1} = \tan\alpha \quad \underline{\underline{\alpha = 61{,}9^\circ}}$

5.44 Sökt: a så att armen inte glider

$\mu = 0{,}15$



$\overset{\curvearrowright}{A}$ $\quad mg(a-25) + \mu N \cdot 50 - N \cdot 40 = 0$

$\uparrow \quad 2\mu N = mg \Rightarrow N = \frac{mg}{2\mu}$

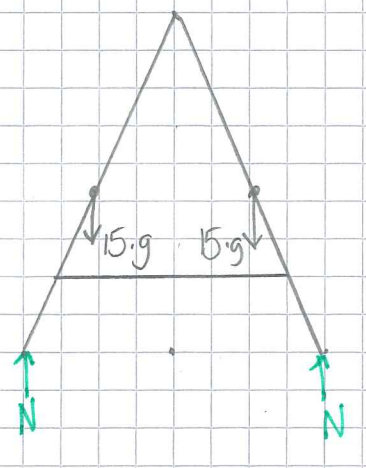
$\cancel{mg}(a-25) + \cancel{\mu}\frac{\cancel{mg}}{2\cancel{\mu}} \cdot 50 + \frac{\cancel{mg}}{2\mu} \cdot 40 = 0$

$a - 25 + 25 + \frac{20}{0{,}15} = 0 \Rightarrow \underline{a = 134\,mm}$

5.45 a) S då F=0

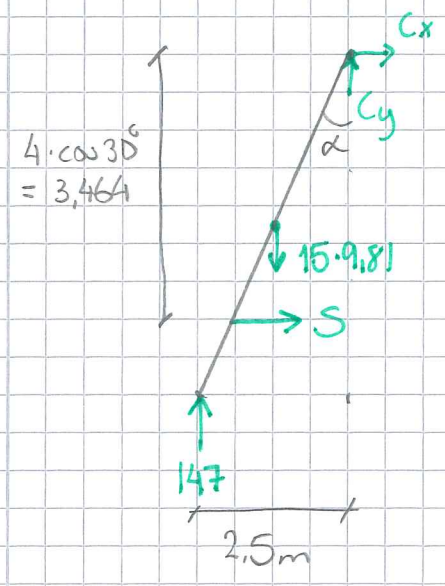
b) S då F= 75·9,81 N

a)



$\uparrow\ 2N - 2\cdot 15\cdot 9{,}81 = 0 \Rightarrow N = 147N$

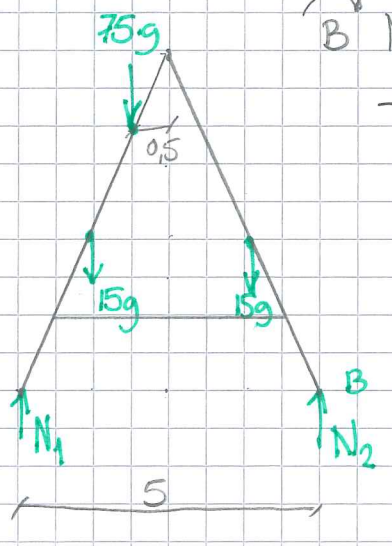
$\alpha = \arcsin\left(\frac{2{,}5}{5}\right) = 30^\circ$



$\circlearrowright C\ \ 147\cdot 2{,}5 - 15\cdot 9{,}81\cdot 1{,}25 - S\cdot 4\cos 30^\circ = 0$

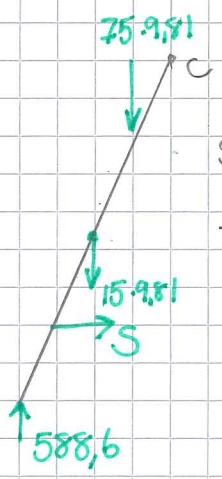
$\underline{\underline{S = 53\,N}}$

b)



$\circlearrowright B\ \ N_1\cdot 5 - 15\cdot 9{,}81\cdot 3{,}75 - 15\cdot 9{,}81\cdot 1{,}25 \ldots$

$- 75\cdot 9{,}81\cdot 3 = 0 \Rightarrow N_1 = 588{,}6\ N$



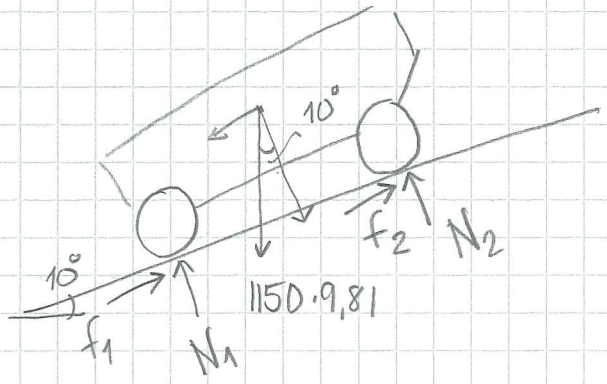
$\circlearrowright C$

$S\cdot 4\cos 30^\circ + 75\cdot 9{,}81\cdot 0{,}5 + \ldots$

$+ 15\cdot 9{,}81\cdot 1{,}25 - 588{,}6\cdot 2{,}5 = 0$

$\underline{\underline{S = 265{,}5\,N}}$

5,46



$f_1 = \mu N_1$

$f_2 = \mu N_2$

$\uparrow \quad N_1 + N_2 - 1150 \cdot 9{,}81 \cdot \cos 10^\circ = 0 \quad \Rightarrow N_1 + N_2 = 1150 \cdot 9{,}81 \cos 10^\circ$

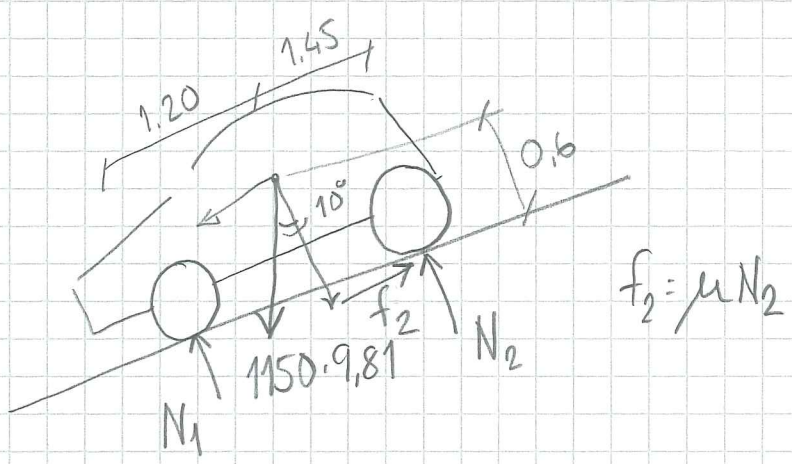
$\nearrow \quad \mu N_1 + \mu N_2 - 1150 \cdot 9{,}81 \sin 10^\circ = 0$

$\mu (N_1 + N_2) = 1150 \cdot 9{,}81 \sin 10^\circ$

$\mu \cdot \cancel{1150 \cdot 9{,}81} \cos 10^\circ = \cancel{1150 \cdot 9{,}81} \sin 10^\circ$

$\mu = \frac{\sin 10^\circ}{\cos 10^\circ} = \tan 10^\circ = 0{,}176$

b)



N_1, N_2 & μ okända => 3 ekv. krävs

$$\circlearrowleft 1 \quad N_2(1{,}20 + 1{,}45) + 0{,}6 \cdot 1150 \cdot 9{,}81 \cdot \sin 10^\circ - 1{,}2 \cdot 1150 \cdot 9{,}81 \cdot \cos 10^\circ = 0$$

$$N_2 = \frac{1150 \cdot 9{,}81}{(1{,}2 + 1{,}45)}\left(1{,}2\cos 10^\circ - 0{,}6\sin 10^\circ\right) = 4587\,N$$

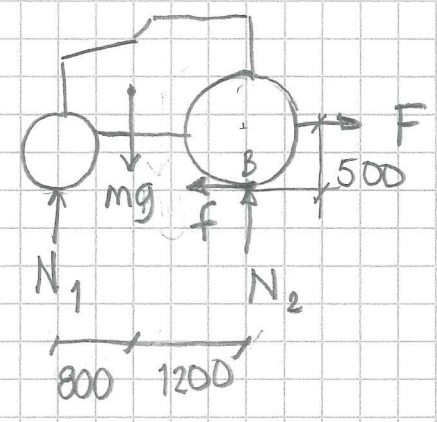
$$\nearrow \quad \mu N_2 - 1150 \cdot 9{,}81 \sin 10^\circ = 0$$

$$\mu = \frac{1150 \cdot 9{,}81 \sin 10^\circ}{4587} = \underline{\underline{0{,}427}}$$

5,47 a) F om slirning då $\mu = 0{,}5$

b) F om traktorn ska stegra vilket μ krävs.

Friläggning



$m = 3500$ kg

a) slirning $\Rightarrow f = \mu N_2 = 0{,}5 N_2$

$\uparrow \; N_1 + N_2 - mg = 0 \Rightarrow N_1 = mg - N_2$

$\leftarrow \; f - F = 0 \quad F = 0{,}5 N_2 \Rightarrow N_2 = 2F$

$\circlearrowleft B \quad mg \cdot 1200 - N_1 \cdot 2000 - F \cdot 500 = 0$

$mg \cdot 12 - (mg - 2F) \cdot 20 - F \cdot 5 = 0$

$mg\,12 - 20mg + 40F - 5F = 0$

$35F = 8mg \Rightarrow \underline{\underline{F = 7848\,N}}$

b) Stegrar då $N_1 = 0$

$\overset{\curvearrowright}{B}$ $F \cdot 500 - mg \cdot 1200 = 0$

$F = \underline{\underline{82404}}$ N

$f = F = \mu N_2$ och $N_2 = mg$

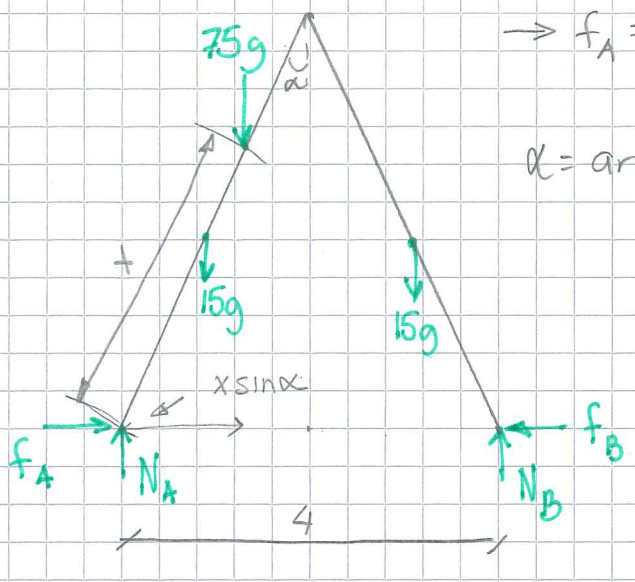
$\mu = \frac{82404}{mg} = \underline{\underline{2{,}4}}$

5,48 Sökt: x_{max}

$\mu = 0{,}25$

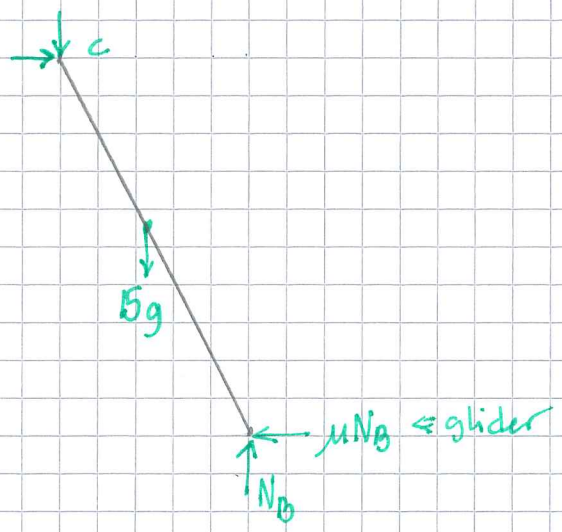
$\rightarrow f_A = f_B$

$\alpha = \arcsin\left(\frac{2}{5}\right) = 23{,}6°$



$$\overset{\curvearrowleft}{A} \quad N_B \cdot 4 - 15 \cdot 9{,}81 \cdot 3 - 15 \cdot 9{,}81 \cdot 1 - 75 \cdot 9{,}81 \cdot x \sin 23{,}6° = 0$$

$$N_B = \frac{1}{4}\left(588{,}6 + 75 \cdot 9{,}81\, x \sin 23{,}6°\right) \qquad (1)$$



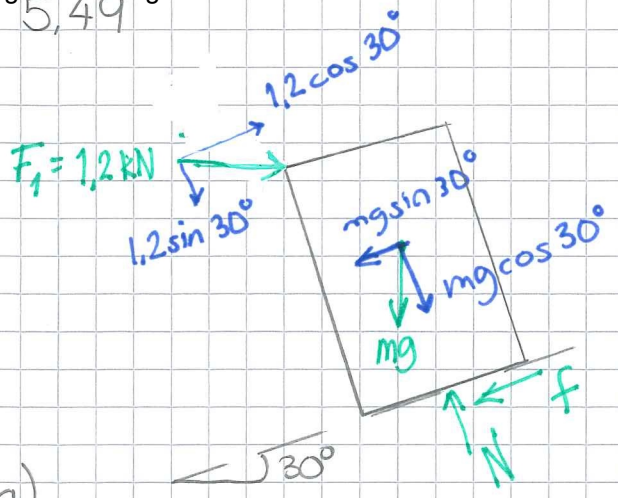
$$\overset{\curvearrowleft}{C} \quad N_B \cdot 2 - \mu N_B \cdot 5 \cos 23{,}6° - 15 \cdot 9{,}81 \cdot 1 = 0$$

$$N_B = \frac{147{,}15}{2 - 0{,}25 \cdot 5 \cos 23{,}6°} = 172{,}20 \qquad (2)$$

$$(1) \Rightarrow \frac{1}{4}\left(588{,}6 + 75 \cdot 9{,}81\, x \sin 23{,}6°\right) = 172{,}20$$

$$\underline{\underline{x = 0{,}34\,m}}$$

5,49



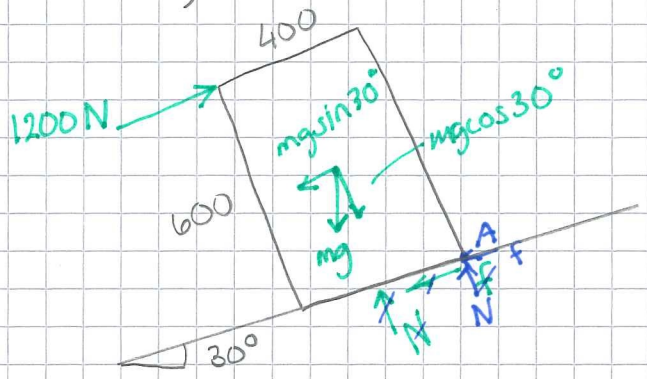
$m = 200\,kg \qquad \mu = 0{,}15$

a)

$\nwarrow \quad N - mg\cos 30^\circ - 1200\sin 30^\circ = 0 \quad \Rightarrow \quad N = 2738\,N$

$\swarrow \quad f - 1200\cos 30^\circ + mg\sin 30^\circ = 0 \quad \underline{\underline{f = 58\,N}}$

$f < N \cdot \mu = 2738 \cdot 0{,}15 = 410{,}7\,N$ glidning sker ej => info stämmer



<u>Kontrollera glidning</u>

$\nwarrow \quad N = mg\cos 30^\circ = 1699\,N \quad \Rightarrow \quad f_{friug} = 1699 \cdot 0{,}15 = 255\,N$

$\nearrow \quad 1200 - mg\sin 30^\circ - f = 0 \quad \Rightarrow \quad f = 219\,N$

$f < f_{friug} \quad \Rightarrow$ glider ej

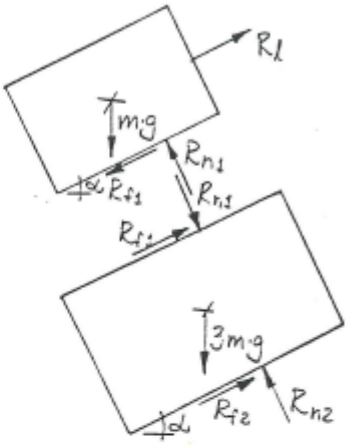
<u>Stjälpning runt A</u>

$M_{stjälp} = 1200 \cdot 600 = 720000\,Ncm$

$M_{stabil} = mg\sin 30^\circ \cdot 300 + mg\cos 30^\circ \cdot 200 = 634128\,Ncm$

$M_{stjälp} > M_{stabil}$ <u>vältar</u>

5:50	JÄMVIKT



$R_{f1} = \frac{1}{3} \cdot R_{n1}$

$R_{f2} = \frac{1}{3} \cdot R_{n2}$

LILLA KÅLAN:

$\nwarrow: R_{n1} - m \cdot g \cdot \cos\alpha = 0$

$R_{n1} = m \cdot g \cdot \cos\alpha$

STORA KÅLAN:

$\nwarrow: R_{n2} - 3 \cdot m \cdot g \cdot \cos\alpha - m \cdot g \cdot \cos\alpha = 0$

$R_{n2} = 4 \cdot m \cdot g \cdot \cos\alpha$

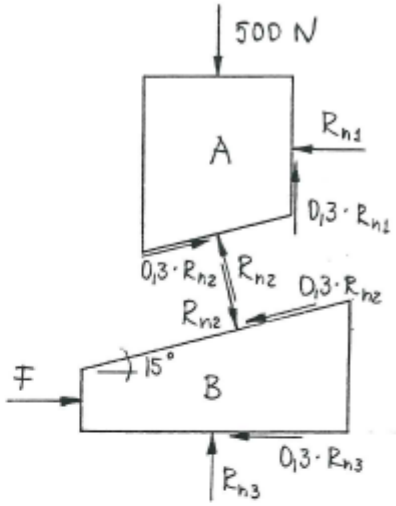
$\rightarrow: \frac{1}{3} \cdot 4 \cdot m \cdot g \cdot \cos\alpha + \frac{1}{3} \cdot m \cdot g \cdot \cos\alpha - 3 \cdot m \cdot g \cdot \sin\alpha = 0$

$\frac{5}{3} \cdot m \cdot g \cdot \cos\alpha = 3 \cdot mg \cdot \sin\alpha$

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{5}{3} \cdot m \cdot g}{3 \cdot m \cdot g} \qquad \alpha = 29{,}1^\circ$

SVAR: $\alpha = 29{,}1^\circ$

5:51 JÄMVIKT



A:

$$\begin{cases} \rightarrow: 0{,}3 \cdot R_{n2} \cdot \cos 15° - R_{n2} \cdot \sin 15° - R_{n1} = 0 & (1) \\ \uparrow: -500 + 0{,}3 \cdot R_{n1} + 0{,}3 \cdot R_{n2} \cdot \sin 15° + R_{n2} \cdot \cos 15° & (2) \end{cases}$$

(1): $R_{n1} = 0{,}3 \cdot R_{n2} \cdot \cos 15° - R_{n2} \cdot \sin 15°$

(2): $-500 + 0{,}3\,(0{,}3 \cdot R_{n2} \cdot \cos 15° - R_{n2} \cdot \sin 15°) +$

$+ 0{,}3 \cdot R_{n2} \cdot \sin 15° + R_{n2} \cdot \cos 15° = 0$

$R_{n2} = 474{,}9$ N

B:

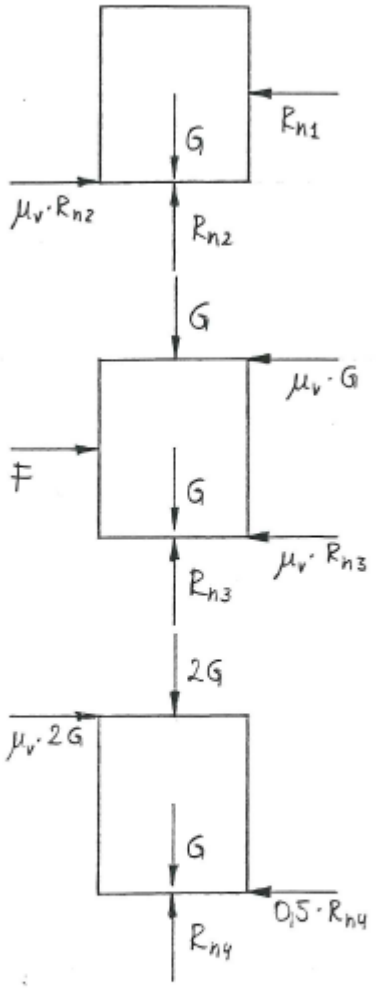
$$\begin{cases} \rightarrow: F + 474{,}9 \cdot \sin 15° - 0{,}3 \cdot 474{,}9 \cdot \cos 15° - 0{,}3 \cdot R_{n3} = 0 & (3) \\ \uparrow: R_{n3} - 474{,}9 \cdot \cos 15° - 0{,}3 \cdot 474{,}9 \cdot \sin 15° = 0 & (4) \end{cases}$$

(4): $R_{n3} = 495{,}6$ N

(3): $F = 163{,}4$ N

<u>SVAR</u>: $F = 163$ N

5:52 JÄMVIKT



$\uparrow: \; R_{n2} - G = 0$

$R_{n2} = G$

$\uparrow: \; R_{n3} - G - G = 0$

$R_{n3} = 2 \cdot G$

$\uparrow: \; R_{n4} - 2 \cdot G - G = 0$

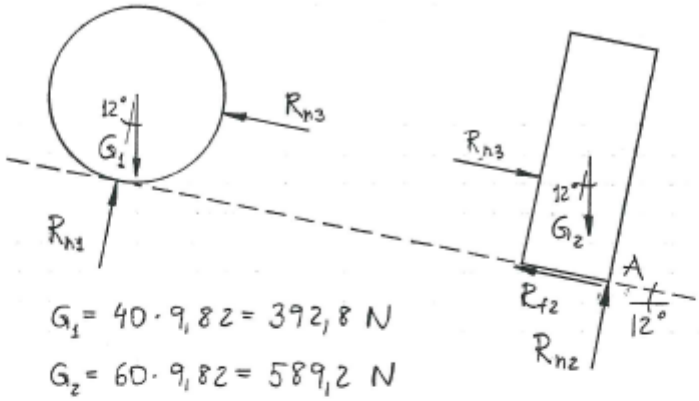
$R_{n4} = 3 \cdot G$

$\rightarrow: \; \mu_v \cdot 2G - 0{,}5 \cdot 3G = 0$

$\mu_v < 0{,}75$

SVAR: $\mu_v < 0{,}75$

5:53 JÄMVIKT



$G_1 = 40 \cdot 9{,}82 = 392{,}8$ N

$G_2 = 60 \cdot 9{,}82 = 589{,}2$ N

CYLINDERN:

$\rightarrow$: $392{,}8 \cdot \sin 12° - R_{n3} = 0$

$R_{n3} = 81{,}7$ N

KLOSSEN:

$R_{n3} = 81{,}7$ N

$\rightarrow$: $589{,}2 \cdot \sin 12° + 81{,}7 - R_{f2} = 0$ $\quad R_{f2} = 204$ N

$\uparrow$: $R_{n2} - 589{,}2 \cdot \cos 12° = 0$ $\quad R_{n2} = 576$ N

GLIDNINGSVILLKOR: $R_f < \mu \cdot R_n$

$0{,}4 \cdot 576 = 231 > 204$ INGEN GLIDNING!

STJÄLPNINGSVILLKOR: $\overset{\curvearrowright}{M_s} > \overset{\curvearrowleft}{M_m}$

$\overset{\curvearrowright}{M_s} = 81{,}7 \cdot 30 + 589{,}2 \cdot \sin 12° \cdot 40 = 7351$ Nmm

$\overset{\curvearrowleft}{M_m} = 589{,}2 \cdot \cos 12° \cdot 15 = 8645$ Nmm

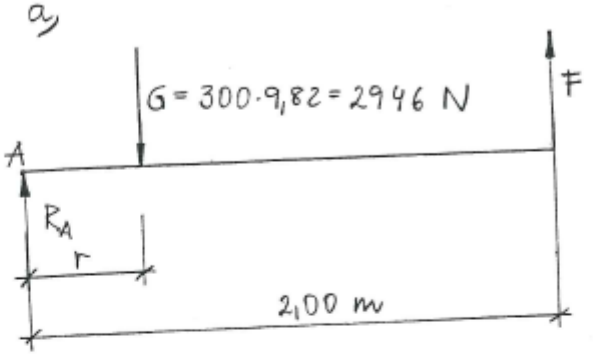
$M_m > M_s$ INGEN STJÄLPNING!

<u>SVAR</u>: a) INGEN GLIDNING

b) INGEN STJÄLPNING

5:54	JÄMVIKT

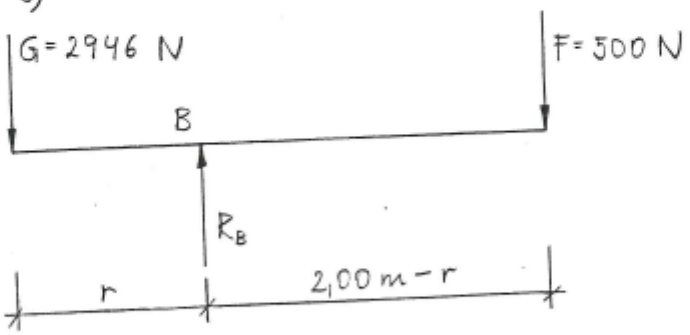
a)



$\circlearrowleft A$: $2946 \cdot r - 500 \cdot 2 = 0$

$r = 0{,}3394$ m

b)



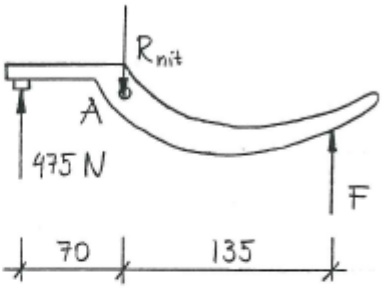
$\circlearrowleft B$: $-2946 \cdot r + 500\,(2{,}00 - r) = 0$

$r = 0{,}2902$ m

<u>SVAR:</u> a) r = 339 mm

b) r = 290 mm

5:55	JÄMVIKT



$$\begin{cases} \uparrow: & 475 - R_{nit} + F = 0 \quad (1) \\ \circlearrowleft A: & 475 \cdot 70 - F \cdot 135 = 0 \quad (2) \end{cases}$$

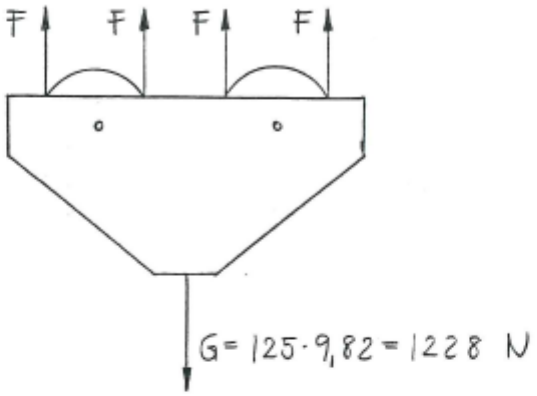
(2): $F = 246{,}3$ N

(1): $R_{nit} = 721{,}3$ N

<u>SVAR:</u> a) $F = 246$ N

b) $R_{nit} = 721$ N

5:56	JÄMVIKT

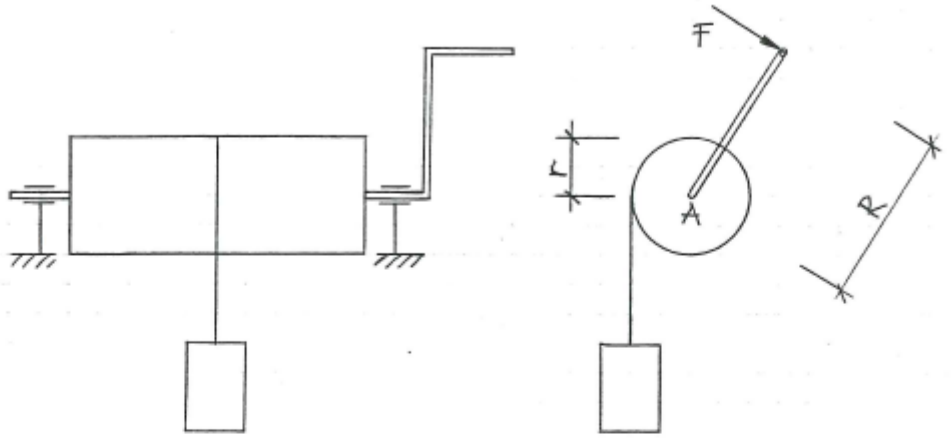


$\uparrow$: $4 \cdot F - 1228 = 0$

$F = 306{,}9$ N

<u>SVAR</u>: $F = 307$ N.

5:57 JÄMVIKT



a)

$\circlearrowleft A: \; F \cdot 310 - 50 \cdot 9{,}82 \cdot 107{,}5 = 0$

$F = 170{,}3 \text{ N}$

b)

$U = \frac{z_1}{z_2} = \frac{60}{12} = 5$

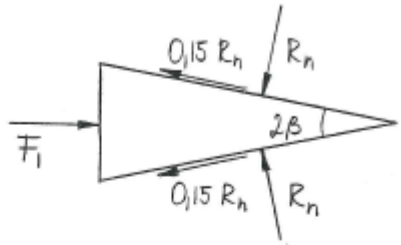
$\eta = 0{,}80$

$F_1 = \frac{F}{U \cdot \eta} = \frac{170{,}3}{5 \cdot 0{,}80} = 42{,}58 \text{ N}$

SVAR: a) $F = 170$ N

b) $F = 42{,}6$ N

5:58 JÄMVIKT

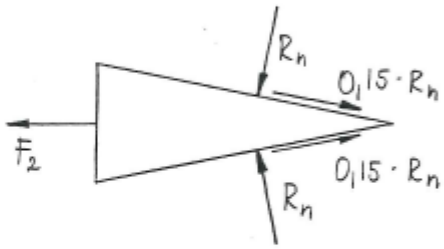


$$\tan\beta = \frac{20}{200}$$

$$2\beta = 11{,}42^\circ$$

$$\rightarrow: \quad F_1 - 2 \cdot R_n \cdot \sin\frac{11{,}42^\circ}{2} - 2 \cdot 0{,}15\ R_n \cdot \cos\frac{11{,}42^\circ}{2} = 0$$

$$F_1 = 0{,}4975 \cdot R_n$$



$$\rightarrow: \quad -F_2 - 2 \cdot R_n \cdot \sin\frac{11{,}42^\circ}{2} + 2 \cdot 0{,}15 \cdot R_n \cdot \cos\frac{11{,}42^\circ}{2} = 0$$

$$F_2 = 0{,}09952 \cdot R_n$$

$$\frac{F_1}{F_2} = \frac{0{,}4975}{0{,}09952} = 5{,}000$$

<u>SVAR:</u> 5x