Teknologi B
Hållfasthetslära
Lösningar

I detta dokument finns lösningsförslag till samtliga uppgifter i boken ”Teknologi B – Hållfasthetslära”.

På YouTube kanalen, ”Lösningar Teknologi B Hållfasthetslära”, finns inspelade lösningsförslag till många av uppgifterna. Det finns även en spellista med föreläsningar som är anpassad till boken (dock ej kombinerade lastfall och knäckning).

Jag använder mig att Karl Björks ”Formler och Tabeller för Mekanisk konstruktion” när jag löser uppgifterna. Boken är äldre och utvecklades tillsammans med en formelsamling som inte finns längre. Det kan därför vara lite förvirrande när det nämns hållfasthetsklasser då information måste hämtas från denna gamla formelsamling.

Om du inte har en formelsamling i maskinkonstruktion så rekommenderar jag starkt Karl Björks formelsamling. Denna finns att beställa på bjorksforlag.se

En komplett översikt av kanalen och materialet finns på edume.nu

/Madeleine

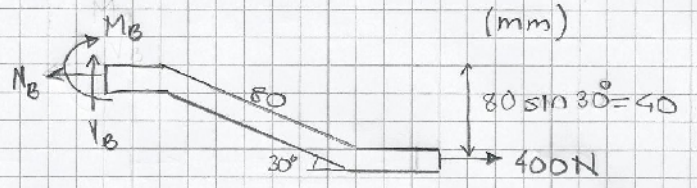
YouTube SE https://www.youtube.com/channel/UCZWty6uAUlkab9XyHQIAu9Q

Madeleine Hermann

EduME – Education and Mechanical Engineering

Lösningar till ”Teknologi B – Hållfasthetslära” Kapitel 2

2.2 Snitt B



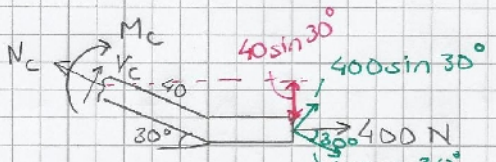
$\leftarrow$ $N_B - 400 = 0$ $\quad N_B = 400\,N$

$\uparrow$ $V_B = 0$

$\circlearrowleft_B$ $M_B - 400 \cdot 40 = 0 \Rightarrow M_B = 16000\,Nmm$

$M_B = 16\,Nm$

Snitt C

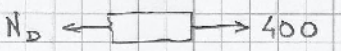


$\nwarrow$ $N_C - 400\cos 30° = 0 \Rightarrow N_C = 346\,N$

$\nearrow$ $V_C + 400\sin 30° = 0 \Rightarrow V_C = -200\,N$

$\circlearrowleft_C$ $M_C - 400 \cdot 0{,}04\sin 30° = 0 \Rightarrow M_C = 8\,Nm$

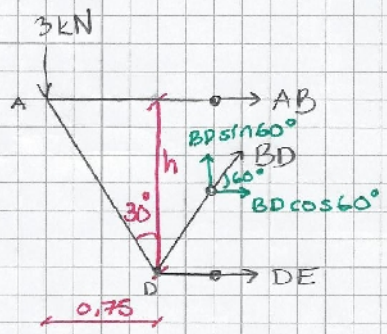
Snitt D



$\rightarrow$ $N_D = 400\,N$

Lösningar till ”Teknologi B – Hållfasthetslära” Kapitel 2

2,3



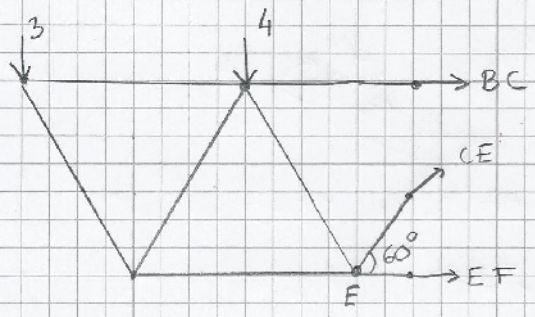
$h = 1{,}5 \cos 30^\circ = 1{,}3\ m$

$\circlearrowleft_D \quad AB \cdot 1{,}3 - 3 \cdot 0{,}75 = 0 \quad \Rightarrow \quad AB = 1{,}73\ kN$

$\uparrow \quad BD \sin 60^\circ - 3 = 0 \quad \Rightarrow \quad BD = 3{,}46\ kN$

$\rightarrow \quad AB + DE + BD \cos 60^\circ = 0$

$DE = -1{,}73 - 3{,}46 \cos 60^\circ = -3{,}46\ kN$



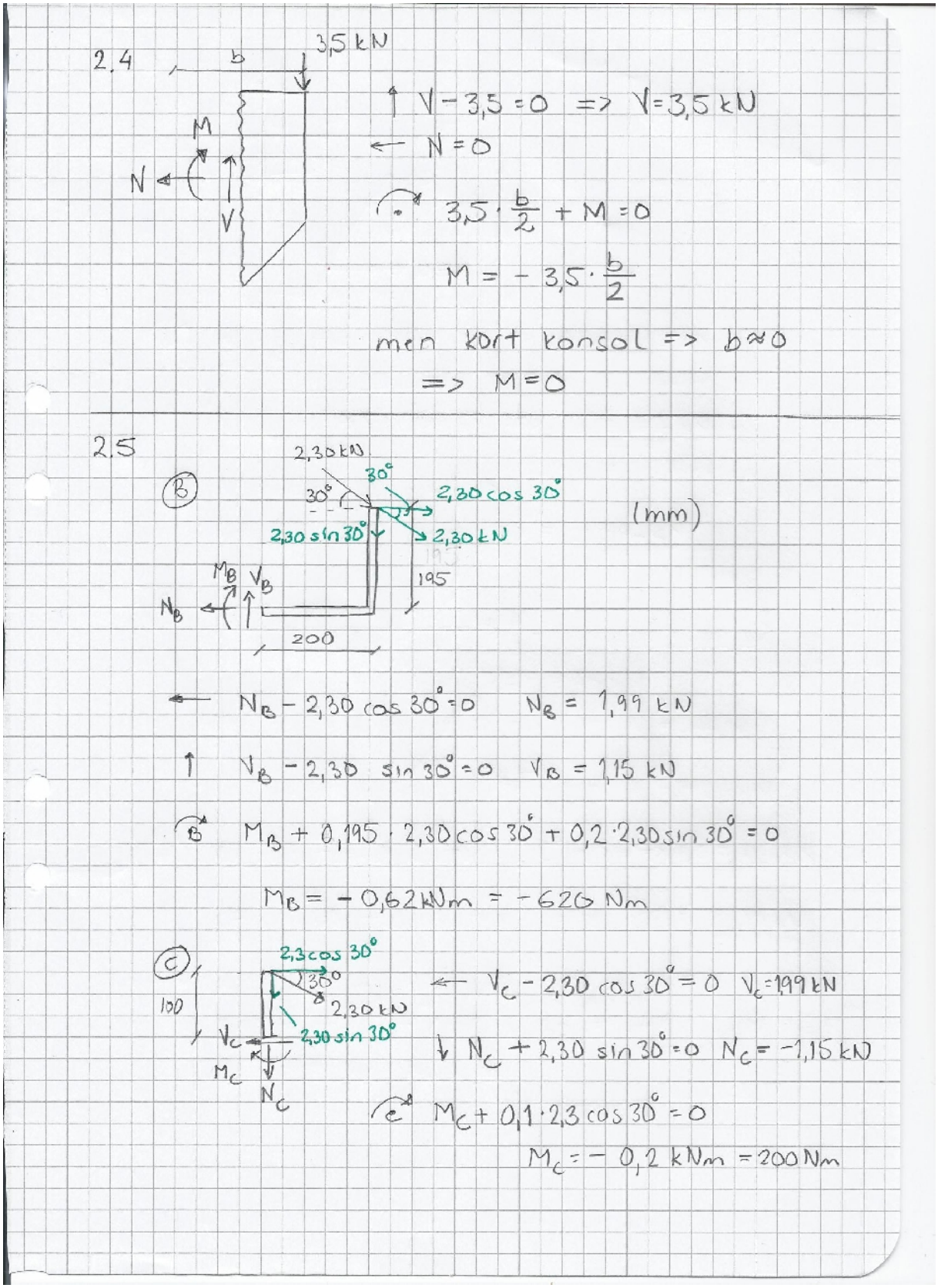
$\circlearrowleft_E \quad BC \cdot 1{,}3 - 4 \cdot 0{,}75 - 3 \cdot 2{,}25 = 0 \quad \Rightarrow \quad BC = 7{,}5\ kN$

$\uparrow \quad CE \cdot \sin 60^\circ - 3 \cdot 4 = 0 \quad \Rightarrow \quad CE = 8{,}08\ kN$

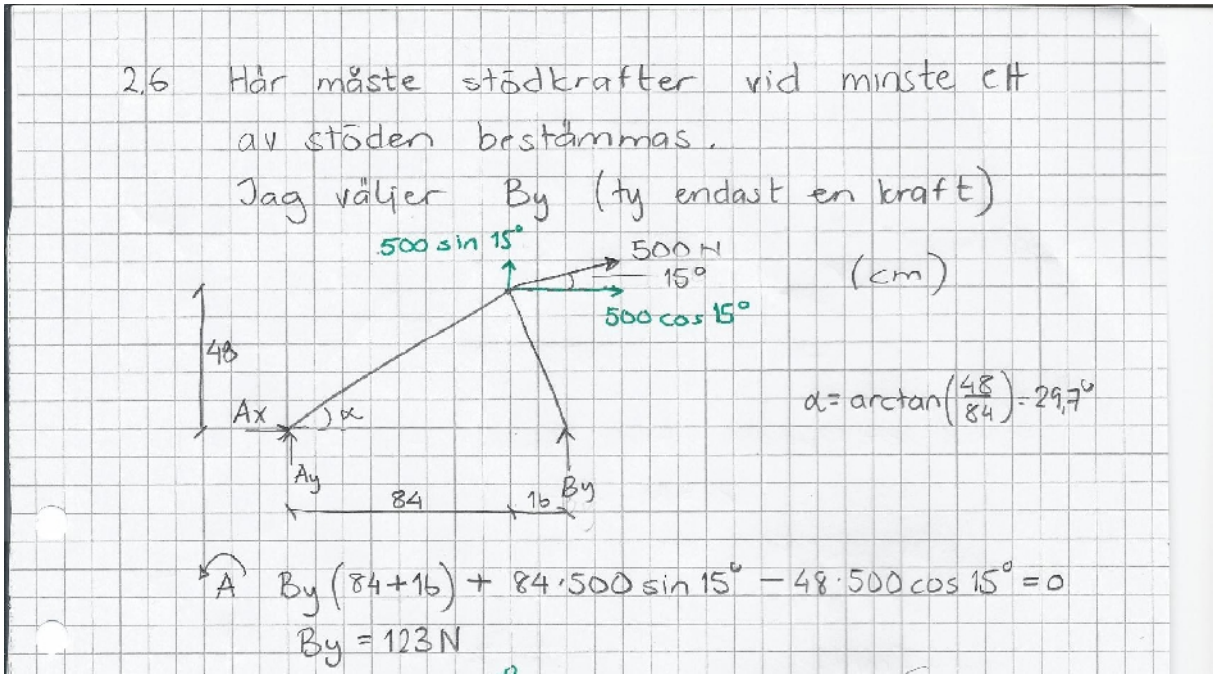
$\rightarrow \quad BC + EF + CE \cos 60^\circ = 0$

$EF = -7{,}5 - 8{,}08 \cos 60^\circ = -11{,}54\ kN = EF$

Lösningar till ”Teknologi B – Hållfasthetslära” Kapitel 2

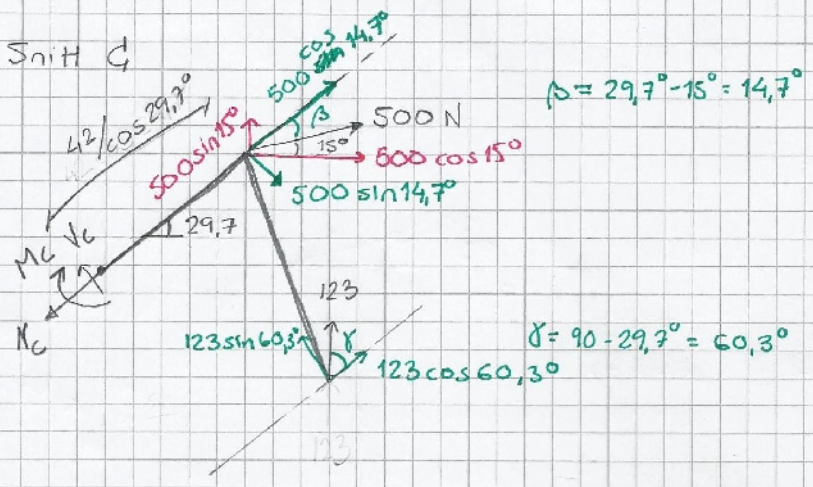


Lösningar till ”Teknologi B – Hållfasthetslära” Kapitel 2



Lösningar till ”Teknologi B – Hållfasthetslära” Kapitel 2

2,6 Snitt C



$\swarrow \quad N_C - 500\cos 14{,}7^\circ - 123\cos 60{,}3^\circ = 0$

$N_C = 545\,N$

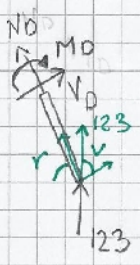
$\uparrow \quad V_C - 500\sin 14{,}7^\circ + 123\sin 60{,}3^\circ = 0$

$V_C = 20\,N$

$\circlearrowleft C \quad M_C + 24\cdot 500\cos 15^\circ - 42\cdot 500\sin 15^\circ - 123\cdot 58 = 0$

$M_C = 978\,Ncm = 9{,}78\,Nm$

Snitt D



$v = \arctan\left(\frac{48}{76}\right) = 71{,}6^\circ$

$\nwarrow \quad N_D + 123\sin 71{,}6^\circ = 0$

$N_D = -117\,N$

$\nearrow \quad V_D + 123\cos 71{,}6^\circ = 0$

$V_D = -38{,}8\,N$

$\circlearrowleft D \quad M_D - 123\cdot 8 = 0$

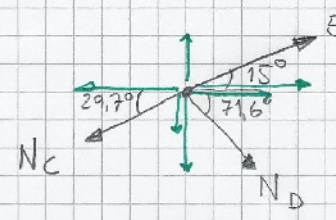
$M_D = 984\,Ncm = 9{,}84\,Nm$

2.7 Detta tal är lättare än 2:6 då profilerna är ledat infästa och endast tar upp drag/tryck i profilens riktning (2st två-krafts kroppar)

$V_D = V_C = 0$

$M_D = M_C = 0$

Normalkrafterna N_C o N_D bestäms enklast genom frilägging av övre knutpunkt



vinklar se 2:6

(1) $\uparrow \quad 500\sin 15° - N_D \sin 71{,}6° - N_C \sin 29{,}7° = 0$

(2) $\rightarrow \quad 500\cos 15° + N_D \cos 71{,}6° - N_C \cos 29{,}7° = 0$

(1) $N_D = \frac{500\sin 15°}{\sin 71{,}6°} - N_C \frac{\sin 29{,}7°}{\sin 71{,}6°} = 136{,}38 - N_C \cdot 0{,}522$

(2) $500\cos 15° + (136{,}38 - 0{,}522 N_C)\cos 71{,}6° = N_C \cos 29{,}7°$

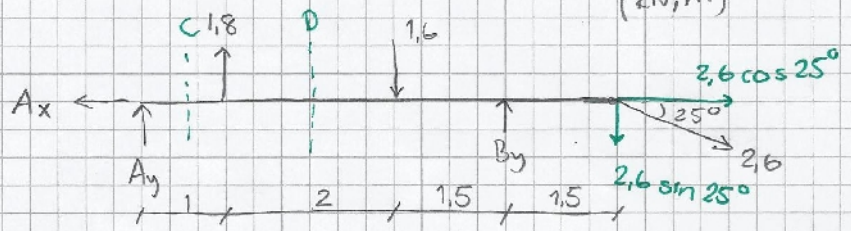
$500\cos 15° + 136{,}38 \cdot \cos 71{,}6 = N_C \cos 29{,}7° + 0{,}522 \cdot \cos 71{,}6° \cdot N_C$

$N_C = 509\,N$

$N_D = 136{,}38 - 509 \cdot 0{,}522 = -129\,N$ (innebär tryck)

Lösningar till ”Teknologi B – Hållfasthetslära” Kapitel 2
Uppgift 2.1-2.10 Inre krafter och moment

2:8 Bestäm stödkrafter (kN, m)



$\circlearrowleft_A \quad 1{,}8 \cdot 1 - 1{,}6 \cdot 3 + B_y \cdot 4{,}5 - 6 \cdot 2{,}6 \sin 25^\circ = 0$

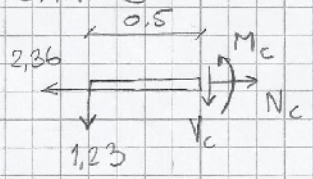
$B_y = 2{,}13 \text{ kN}$

$\uparrow \quad A_y + B_y + 1{,}8 - 1{,}6 - 2{,}6 \sin 25^\circ = 0$

$A_y = 1{,}6 - 1{,}8 + 2{,}6 \sin 25^\circ - 2{,}13 = -1{,}23 \text{ kN}$

$\rightarrow \quad 2{,}6 \cos 25^\circ - A_x = 0 \quad \Rightarrow \quad A_x = 2{,}36 \text{ kN}$

Snitt C

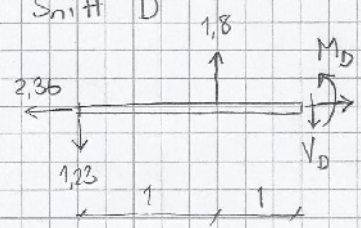


$\rightarrow \quad N_c - 2{,}36 = 0 \quad N_c = 2{,}36 \text{ kN}$

$\downarrow \quad V_c + 1{,}23 = 0 \quad V_c = -1{,}23 \text{ kN}$

$\circlearrowleft_C \quad M_c + 1{,}23 \cdot 0{,}5 = 0 \quad M_c = -0{,}615 \text{ kNm}$

Snitt D



$\rightarrow \quad N_D = 2{,}36 \text{ kN}$

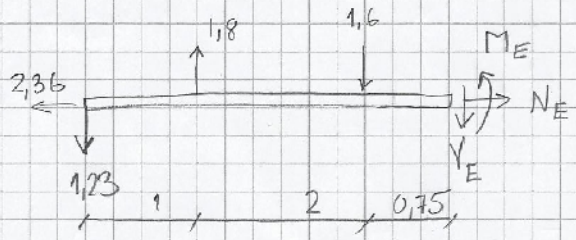
$\downarrow \quad V_D + 1{,}23 - 1{,}8 = 0 \quad V_D = 0{,}57 \text{ kN}$

$\circlearrowleft_D \quad M_D + 1{,}23 \cdot 2 - 1{,}8 \cdot 1 = 0 \quad M_D = 0{,}66 \text{ kNm}$

MD ska vara negativ.

2:8 forts

Snitt E



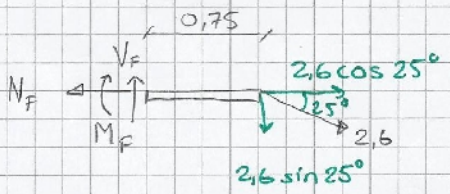
$\rightarrow \quad N_E = 2{,}36\ \text{kN}$

$\downarrow \quad V_E + 1{,}6 - 1{,}8 + 1{,}23 = 0 \qquad V_E = -1{,}03\ \text{kN}$

$\circlearrowleft_E \quad M_E + 1{,}6 \cdot 0{,}75 - 1{,}8 \cdot 2{,}75 + 1{,}23 \cdot 3{,}75 = 0$

$M_E = -0{,}86\ \text{kNm}$

Snitt F (enklare att titta höger om snittet)



$\leftarrow \quad N_F = 2{,}36\ \text{kN}$

$\uparrow \quad V_F - 2{,}6\sin 25° = 0$

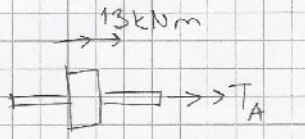
$V_F = 1{,}1\ \text{kN}$

$\circlearrowleft_F \quad M_F + 0{,}75 \cdot 2{,}6\sin 25° = 0$

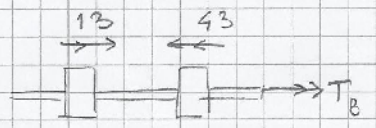
$M_F = -0{,}82\ \text{kNm}$

Lösningar till ”Teknologi B – Hållfasthetslära” Kapitel 2

2.9 Endast vridande moment => endast T som inre moment



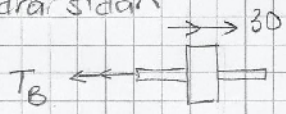
$\rightarrow 13 + T_A = 0 \Rightarrow T_A = -13\ kNm$



$\rightarrow 13 - 43 + T_B = 0$

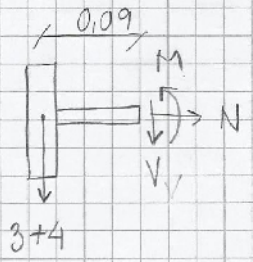
$T_B = 30\ kNm$

eller andra sidan



$\rightarrow 30 - T_B = 0 \quad T_B = 30\ kNm$

2:10

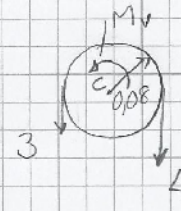


$N = 0$

$\downarrow V + 7 = 0 \Rightarrow V = -7\ kN$

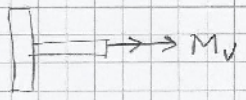
$\circlearrowleft M + 0{,}09 \cdot 7 = 0 \Rightarrow M = -0{,}63\ kNm$

Vridningen



$\circlearrowleft 4 \cdot 0{,}08 - 3 \cdot 0{,}08 - M_V = 0$

$M_V = 0{,}08\ kNm$



KB s. 18

$A = \frac{\pi d^2}{4} = \pi r^2$

KB s. 25

$\sigma = \frac{F}{A}$

3:1 $d = 15\,mm$

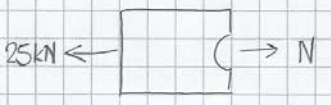


$N = 10\,kN$

$\sigma = \frac{N}{A} = \frac{10 \cdot 10^3}{\frac{\pi \cdot 0{,}015^2}{4}} = 566 \cdot 10^6\ N/m^2$

$\sigma = 56{,}6 \cdot 10^6\ Pa = \underline{\underline{56{,}6\ MPa}}$

3:2



$N = 25\,kN$

Störst spänning blir där tvärsnittsarean är minst. Här: genom hålet



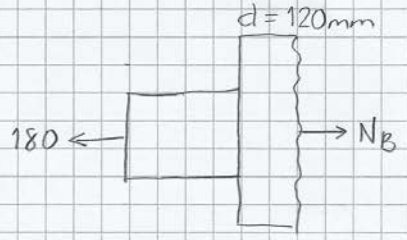
$\sigma = \frac{25 \cdot 10^3}{0{,}005\,(0{,}04 - 0{,}015)} = \underline{\underline{200\ MPa}}$

3:3 Snitt A



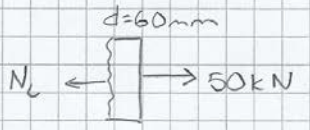
$\rightarrow N_A - 180 = 0 \Rightarrow N_A = 180\,kN$

$\sigma_A = \frac{180 \cdot 10^3 \cdot 4}{0{,}08^2 \cdot \pi} = \underline{\underline{35{,}8\ MPa}}$



$N_B = 180\,kN$

$\sigma_B = \frac{180 \cdot 10^3 \cdot 4}{0{,}120^2 \cdot \pi} = \underline{\underline{15{,}9\ MPa}}$



$N_c = 50\,kN.$

$\sigma_c = \frac{50 \cdot 10^3 \cdot 4}{0{,}06^2 \cdot \pi} = \underline{\underline{17{,}7\ MPa}}$

eller



$\rightarrow N_c + 130 - 180 = 0$

$N_c = 50\,kN.$

3:4

$$\sigma = \frac{1{,}1 \cdot 10^3}{7 \cdot \frac{0{,}001^2 \cdot \pi}{4}} = 200\ \text{MPa}$$

(7 st, $A_{tråd}$)

3:5 Se formelblad / tabell för IPE-profil

$\sigma_{max} = 100\ \text{MPa}$

$$A_{min} = \frac{F}{\sigma_{max}} = \frac{150 \cdot 10^3}{100} = 1500\ \text{mm}^2 = 15\ \text{cm}^2$$

Tänk på att $100\ \text{MPa} = 100\ \text{N/mm}^2$

detta är mkt bra att nyttja om man vill slippa decimaler

Välj IPE140 med $A = 16{,}4\ \text{cm}^2$ KB s.65

3:6 Strunta i preferensserie 2.



$\rightarrow 2F - 20 = 0 \quad F = 10\ \text{kN}$

10 kN påverkar en länkarea.

$\sigma_{max} = 75\ \text{MPa} = 75\ \text{N/mm}^2$

$$\sigma_{max} = \frac{F \cdot 4}{\pi d^2} \Rightarrow d = \sqrt{\frac{F \cdot 4}{\pi \sigma_{max}}}$$

$$d = \sqrt{\frac{10 \cdot 10^3 \cdot 4}{\pi \cdot 75}} = 13{,}03\ \text{mm}$$

Välj: 14mm

3:7 $L = 400\,mm$ stål/metall $\nu = 0{,}3$

$\Delta L = 460 \cdot 10^{-6}\,m$

$\varepsilon_l = \frac{460 \cdot 10^{-6}}{0{,}4} = 1{,}15 \cdot 10^{-3}$

KB s. 25 $\delta = \varepsilon \cdot L$

$\varepsilon_t = \nu \cdot \varepsilon_l = 0{,}3 \cdot 1{,}15 \cdot 10^{-3} = 3{,}45 \cdot 10^{-4}$

3:8 $d = 2\,mm$
$L = 1{,}2\,m$
$F = 5\,N$
$\varepsilon = 60\%$

$\Delta L = 0{,}6 \cdot 1{,}2 = 0{,}72\,m$

3:9 $d = 10\,mm$
$L = 200\,mm$
$F = 10\,kN$

Stål: $E = 210\,GPa$, $\nu = 0{,}3$ KB s. 70

Sökt: ε, ε_t, Δd

Hooks lag: $\sigma = E \cdot \varepsilon$

KB s. 24 $\varepsilon = \frac{\sigma}{E}$

$\sigma = \frac{10 \cdot 10^3 \cdot 1{,}4}{0{,}01^2 \cdot \pi} = 127{,}3\,MPa$

$\varepsilon = \frac{\sigma}{E} = \frac{127{,}3 \cdot 10^6}{210 \cdot 10^9} = 6{,}06 \cdot 10^{-4}$

$\varepsilon_t = \nu \cdot \varepsilon = -0{,}3 \cdot 6{,}06 \cdot 10^{-4} = -1{,}82 \cdot 10^{-4}$ (drag => minskning tvär => negativ ε_t)

$\varepsilon_t = \frac{\Delta d}{d} \Rightarrow \Delta d = -1{,}82 \cdot 10^{-4} \cdot 10 = 1{,}82 \cdot 10^{-3}\,mm$

$\nu = \frac{\varepsilon_t}{\varepsilon_x}$ finns ej i KB

3.10 SS 1312-00 $\Rightarrow$ $E = 210\,\text{GPa}$ (alla stål) KB 5.70

$d_y = 16\,\text{mm}$ $t = 3\,\text{mm}$

$F = 7{,}5\,\text{kN}$ $L = 1\,\text{m}$

$\Delta L = \frac{FL}{EA}$ KB. s25

$$A = \frac{0{,}016^2 \cdot \pi}{4} - \frac{0{,}010^2 \cdot \pi}{4} = \frac{\pi}{4}\left(0{,}016^2 - 0{,}01^2\right) = 1{,}225 \cdot 10^{-4}\,\text{m}^2$$

$$\Delta L = \frac{7{,}5 \cdot 10^3 \cdot 1}{210 \cdot 10^9 \cdot 1{,}225 \cdot 10^{-4}} = 0{,}291\,\text{mm}$$

3.11 $L = 70\,\text{m}$

114-trådig stållina $d = 24\,\text{mm}$ $\Delta L = 2{,}0\,\text{mm}$

Formelblad $\Rightarrow$ $A = 212\,\text{mm}^2 = 212 \cdot 10^{-6}\,\text{m}^2$ $m = 2\,\text{kg/m}$

Bestäm linans E-modul

0,5 L då tp är i mitten av linan.

$$2 \cdot 10^{-3} = \frac{70 \cdot 2 \cdot 9{,}81 \cdot 70}{2 \cdot E \cdot 212 \cdot 10^{-6}} \Rightarrow E = 113{,}5\,\text{GPa}$$

Bestäm F som ger $\Delta L = 8\,\text{mm}$

vilket ger $\Delta L_{tot} = 2 + 8 = 10\,\text{mm}$

$$0{,}008 = \frac{F \cdot 70}{113{,}5 \cdot 10^9 \cdot 212 \cdot 10^{-6}} \Rightarrow F = 2750\,\text{N}$$

3.12 $L = 100\,m$ $d = 8\,mm$ $F = 100\,N$

Längdförändring och töjning pga egenvikten samt lasten.

$$G = m \cdot 9{,}81 = A \cdot L \cdot \delta_{stål} = \frac{\pi \cdot 0{,}008^2}{4} \cdot 100 \cdot 7850 \cdot 9{,}81$$ (densitet)

$$G = 387\,N$$

$$\Delta L_G = \frac{387 \cdot 100}{2 \cdot 210 \cdot 10^9 \cdot \frac{\pi \cdot 0{,}008^2}{4}} = 1{,}83\,mm$$

halva längden då F verkar på mitten

$$\Delta L_F = \frac{100 \cdot 50}{210 \cdot 10^9 \cdot \frac{\pi \cdot 0{,}008^2}{4}} = 0{,}48\,mm$$

$$\Delta L_{tot} = 1{,}83 + 0{,}48 = \underline{\underline{2{,}31\,mm}}$$

$$\varepsilon = \frac{\Delta L}{L_0} = \frac{2{,}31 \cdot 10^{-3}}{50} = \underline{\underline{4{,}6 \cdot 10^{-5}}}$$

3.13 $L_0 = 130\,mm$ $d_0 = 13\,mm$

$F_m = 25\,kN$ $F_{eL} = 18\,kN$

KB s.25 $\sigma = \frac{F}{A}$

$$R_{eL} = \frac{18 \cdot 10^3 \cdot 4}{0{,}013^2 \cdot \pi} = 136\,MPa$$

$$R_m = \frac{25 \cdot 10^3 \cdot 4}{0{,}013^2 \cdot \pi} = 188\,MPa$$

3.14 $\Delta L = 0{,}090\,mm$ då $F = 14\,kN$ (under stråckgränsen)

$\sigma = E \cdot \varepsilon$ Hookes lag $\Rightarrow E = \frac{\sigma}{\varepsilon}$

KB s.24 $\varepsilon = \frac{\sigma}{E}$

$$\sigma = \frac{14 \cdot 10^3}{\frac{0{,}013^2 \cdot \pi}{4}} = 105\,MPa$$

$$\varepsilon = \frac{\Delta L}{L_0} = \frac{0{,}090}{130} = 6{,}92 \cdot 10^{-4}$$

$$E = \frac{105 \cdot 10^6}{6{,}92 \cdot 10^{-4}} = 152\,GPa$$

Brott förlängningen

$$A = \frac{142{,}7 - 130}{130} = 9{,}77\,\%$$

KB. s48

3.15 ur tabell för stållina $A = 53\,mm^2$

$F_m = 84\,kN$

$$R_m = \frac{84 \cdot 10^3}{53 \cdot 10^{-6}} = 1585\,MPa$$

3.16 $n_s = 3$ $n_B = 5$ KB s. 50 SS1412 $\Rightarrow$ $R_{el} = 250\,MPa$

$d = 30\,mm$ $R_m = 430\,MPa$

$$\sigma_{tiu}^s = \frac{250}{3} = 83{,}7\,MPa \qquad F_{tiu}^s = \sigma_{tiu}^s \cdot A = 83{,}3 \cdot 10^6 \cdot \frac{0{,}03^2 \cdot \pi}{4} = 58{,}9\,kN$$

$$\sigma_{tiu}^b = \frac{430}{5} = 86\,MPa \qquad F_{tiu}^b = 86 \cdot 10^6 \cdot \frac{0{,}03^2 \cdot \pi}{4} = 60{,}8\,kN$$

Svar: ↑

Boken har använt en annan tabell med $R_{el} = 260\,MPa$

3.17 $F = 23{,}3$ kN

VKR $40 \times 40 \times 3$ (KB s. 66) $\Rightarrow$ $A = 4{,}40\ cm^2 = 4{,}40 \cdot 10^{-4}\ m^2$

SS 1312 (KB s. 50) $\Rightarrow$ $R_{eL} = 220$ MPa $\quad R_m = 360$ MPa

$$\sigma = \frac{23{,}3 \cdot 10^3}{4{,}40 \cdot 10^{-4}} = 53 \text{ MPa}$$

$$n_s = \frac{220}{53} = 4{,}2$$

$$n_b = \frac{360}{53} = 6{,}8$$

3.18 $F_{max} = 30{,}0$ kN $\quad R_m = 400$ MPa

$n_b = 6$

$$\sigma_{tu} = \frac{400}{6} = 66{,}7 \text{ MPa}$$

$$\sigma_{tu} = \frac{F_{max}}{2 A_{länk}} \Rightarrow A_{länk} = \frac{30 \cdot 10^3}{2 \cdot 66{,}7 \cdot 10^6} = 2{,}249 \cdot 10^{-4}\ m^2$$

$$A_{länk} = \frac{\pi d^2}{4} \Rightarrow d = \sqrt{\frac{2{,}249 \cdot 10^{-4} \cdot 4}{\pi}} = 0{,}017\ m$$

$d = 17$ mm

3:19 Sökt: F_{till}

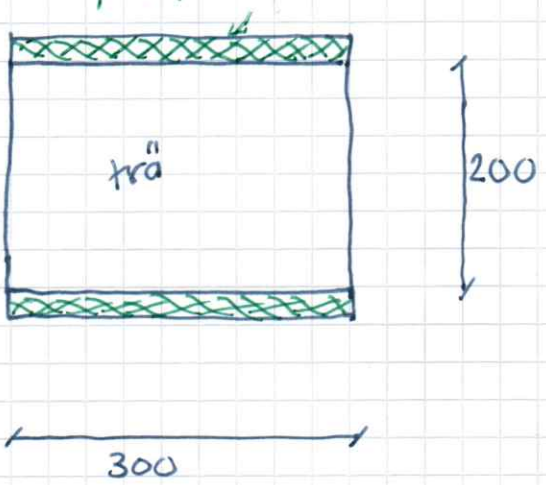
Givet:

$\sigma_{till}^{stål} = 100\ MPa$

$\sigma_{till}^{trä} = 7{,}0\ MPa$

$E_{stål} = 210\ GPa$

$E_{trä} = 9\ GPa$



$\sigma = E \cdot \varepsilon$

$\sigma = \frac{F}{A}$

$F_{till} = F_{trä} + F_{stål}$ (1)

Deformations villkor: $\varepsilon_{trä} = \varepsilon_{stål}$

Förlängningen/meter måste vara lika för plattjärnen och träbjälken

Bestäm max tillåten töjning $\varepsilon = \frac{\sigma}{E}$

$$\varepsilon_{till}^{stål} = \frac{100}{210 \cdot 10^3} = 4{,}762 \cdot 10^{-4}$$

$$\varepsilon_{till}^{trä} = \frac{7{,}0}{9{,}0 \cdot 10^3} = 7{,}778 \cdot 10^{-4}$$

$\Rightarrow \varepsilon_{till} = 4{,}762 \cdot 10^{-4}$

$\sigma_{trä} = E_{trä} \cdot \varepsilon_{till} = 9 \cdot 10^3 \cdot 4{,}762 \cdot 10^{-4} = 4{,}286\ MPa$

↑ rimlig spänning $< \sigma_{till}^{trä}$

(1) $\Rightarrow F_{till} = \sigma_{trä} \cdot A_{trä} + \sigma_{stål} \cdot A_{stål}$

$$F_{till} = 4{,}286 \cdot 200 \cdot 300 + 100 \cdot 2 \cdot 5 \cdot 300 = \underline{\underline{557{,}2\ kN}}$$

3.20 Sökt: F_{till}

Givet:

Ståltrådar

10 st med $d_s = 3\,mm$

$\sigma_{till}^{stål} = 110\,MPa$

$E_{stål} = 210\,GPa$

Kopparträdar

35st med $d_{cu} = 2\,mm$

$\sigma_{till}^{Cu} = 45\,MPa$

$E_{cu} = 120\,GPa$

$$F_{till} = F_{stål} + F_{cu} \quad (1)$$

Deformations villkor

$$\varepsilon_{stål} = \varepsilon_{cu}$$

Bestäm ε_{till}

$$\varepsilon_{till}^{stål} = \frac{110}{210 \cdot 10^3} = 5{,}238 \cdot 10^{-4}$$

$$\varepsilon_{till}^{cu} = \frac{45}{120 \cdot 10^3} = 3{,}75 \cdot 10^{-4}$$

$$\Downarrow$$

$$\varepsilon_{till} = 3{,}75 \cdot 10^{-4}$$

$\sigma = E\varepsilon$

$\sigma = \frac{F}{A}$

Kopparträdarna begränsar.

$\sigma_{cu} = 45\,MPa$ (max tillåten)

$$\sigma_{stål} = 210 \cdot 10^3 \cdot 3{,}75 \cdot 10^{-4} = 78{,}75\,MPa$$

$$F_{stål} = \sigma_{stål} \cdot A_{stål} = 78{,}75 \cdot 10 \cdot \frac{3^2 \cdot \pi}{4} = 5{,}567\,kN$$

$$F_{cu} = \sigma_{cu} \cdot A_{cu} = 45 \cdot 35 \cdot \frac{2^2 \cdot \pi}{4} = 4{,}948\,kN.$$

$$(1) \Rightarrow F_{till} = 5{,}567 + 4{,}948 = \underline{\underline{10{,}52\,kN}}$$

5:21 Sökt: σ_{betong}, $\sigma_{stål}$ om $F = 600\ kN$.

Givet:

Armeringsjärn (4st)

$E_s = 210\ GPa$

$d = 20\ mm$

$A_s = 4 \cdot \frac{20^2}{4}\pi = 400\pi$

Betong

$E_b = 32\ GPa$

$A_b = 300 \cdot 250 - 4 \cdot \frac{20^2}{4} \cdot \pi = 73743\ mm^2$

↑ minskar A med ca 2% ev. försumbart

(1) $F = \sigma_b \cdot A_b + \sigma_s \cdot A_s$ (F, A_b, A_s är kända)

Finn ett förhållande mellan σ_b & σ_s

$\sigma = \varepsilon \cdot E$

$\sigma = \frac{F}{A}$

Deformations villkor

$\varepsilon_s = \varepsilon_b \Rightarrow \frac{\sigma_s}{E_s} = \frac{\sigma_b}{E_b}$

$\sigma_s = \sigma_b \cdot \frac{E_s}{E_b} = \sigma_b \cdot \frac{210}{32} = 6{,}5625\,\sigma_b$ (2)

(1) $\Rightarrow 600 \cdot 10^3 = \sigma_b\, 73743 + 6{,}5625\,\sigma_b \cdot 400\pi$

$\sigma_b = \frac{600 \cdot 10^3}{73743 + 6{,}5625 \cdot 400\pi} = \underline{\underline{7{,}32\ MPa}}$

(2) $\Rightarrow \sigma_s = 6{,}5625 \cdot 7{,}32 = \underline{\underline{48{,}0\ MPa}}$

4.1

$\sigma_b = 21{,}5\ \text{MPa} \quad n_b = 8 \quad \sigma_{till} = \frac{21{,}5}{8} = 2{,}69\ \text{MPa}$

$\rho = 2000\ \text{kg/m}^3$ densitet betong

$\sigma_{till} = \frac{F_{max}}{A} \Rightarrow F_{max} = \sigma_{till} A \quad (1)$

$F_{max} = 9{,}81 \cdot m = 9{,}81 \cdot A \cdot h \cdot \rho \quad (2)$

(1) = (2)

$\sigma_{till} A = 9{,}81 \cdot A \cdot h \cdot \rho$

$h = \frac{\sigma_{till}}{9{,}81 \cdot \rho} = \frac{21{,}5 \cdot 10^6}{9{,}81 \cdot 2000} = 137\text{m}$

KB s.25

$\sigma_{till} = \frac{R_{eL}}{s}$

Behöver ej vara ReL kan vara R_m eller annan spänning manska jämföra mot.

s är säkerhets faktor. ofta namnges med n.

4.2 Tryckkraften är densamma i båda profilerna

KB s.25

$\Delta L = \frac{FL}{AE}$

$\Delta L_{tot} = \Delta L_{60} + \Delta L_{120}$

$\Delta L = F\left(\frac{L_{60}}{A_{60} \cdot E_{60}} + \frac{L_{120}}{A_{120} \cdot E_{120}}\right)$

$0{,}2 \cdot 10^{-3} = F\left(\frac{0{,}350}{0{,}06^2 \cdot 210 \cdot 10^9} + \frac{0{,}450}{0{,}120^2 \cdot 70 \cdot 10^9}\right)$

$0{,}2 \cdot 10^{-3} = F \cdot 9{,}0939 \cdot 10^{-10}$

$F = 220\ \text{kN}$

4.3 $d_y = 80\,mm$ $F = 50\,kN$

$d_i = 60\,mm$ $E = 120\,GPa$

$L = 300\,mm$

$$\sigma = \frac{50 \cdot 10^3}{\frac{(0{,}08^2 - 0{,}06^2)\cdot\pi}{4}} = 22{,}7\,MPa$$

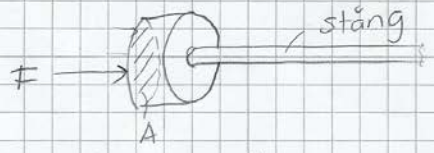
FB 5.25

$$\Delta L = \frac{FL}{AE} = \sigma \cdot \frac{L}{E} = 22{,}7 \cdot 10^6 \cdot \frac{0{,}3}{120 \cdot 10^9} = 57 \cdot 10^{-6}\,m$$

4.4 $d = 350\,mm$ SS 1312 $\sigma_{eh} = 200\,MPa$

$p = 10\,MPa$ $n_s = 3$

Tryck kraft på kolven



$$F = A \cdot p = \frac{0{,}350^2 \cdot \pi}{4} \cdot 10 \cdot 10^6 = 962\,kN$$

$$\sigma_{tiu} = \frac{200}{3} = 66{,}7\,MPa$$

$$A = \frac{F}{\sigma_{tiu}} = \frac{d^2 \cdot \pi}{4} \Rightarrow d = \sqrt{\frac{4F}{\pi \sigma_{tiu}}} = \sqrt{\frac{4 \cdot 962 \cdot 10^3}{\pi \cdot 66{,}7 \cdot 10^6}} = 0{,}136\,m$$

Svar: $d = 136\,mm$

4.5

$F = 50\ kN$

$p_{hål} = 1{,}2\ MPa$

IPE 400 => $B = 180$ ← KB s. 65

$$p = \frac{F}{B \cdot L} \Rightarrow L = \frac{F}{B \cdot p} = \frac{50 \cdot 10^3}{0{,}180 \cdot 1{,}2 \cdot 10^6} = 231\ mm$$

4.6 $F = 3\ kN$ $d_a = 35\ mm$

$d_f = 25\ mm$

$L = 30\ mm$

Trycket från horisontell kraft, $3\cos 40^\circ$

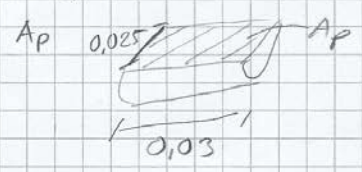


$$p = \frac{3\cos 40^\circ}{\frac{\pi}{4}(0{,}035^2 - 0{,}025^2)} = 4{,}9\ MPa$$

Hålkantstryck, σ_H, från vertikalkraft, $3\sin 40^\circ$

$$\sigma_H = \frac{3\sin 40^\circ}{0{,}03 \cdot 0{,}025} = 2{,}6\ MPa$$

KB. s. 25 Hålkantstryck



4.7 $F = 15\ kN$ $d = 20mm$

Störst hålkanttryck blir i ovan/underdelen ty $6+6 < 13$ (större A_p tar upp kraften i mittdelen)

$$\sigma_H = \frac{15 \cdot 10^3}{2(0{,}006 \cdot 0{,}02)} = 62{,}5\ MPa$$

KB s.25 Hålkantstryck

(Mittbiten

$$\sigma_H = \frac{15 \cdot 10^3}{0{,}013 \cdot 0{,}02} = 58{,}9\ MPa)$$

4.8 Hålkantstryck oslagen bit $\varnothing = 19$ => slagen nit hål 20mm

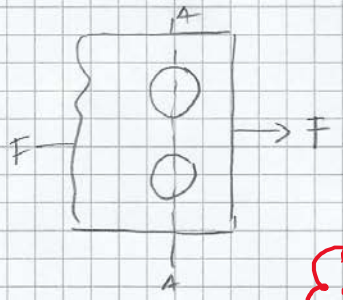
Två nitar tar upp hålkantstrycket även en plåttjocklek

$$A_p = 2 \cdot 0{,}012 \cdot 0{,}020\ m^2$$

$$\sigma_H = \frac{8 \cdot 10^3}{2 \cdot 0{,}012 \cdot 0{,}020} = 16{,}7\ MPa$$

KB s.25 Hålkantstryck

Dragspänning



Svagaste snittet genom nitarna

$$\sigma = \frac{8 \cdot 10^3}{0{,}012\,(0{,}120 - 2 \cdot 0{,}02)} = 8{,}3\ MPa$$

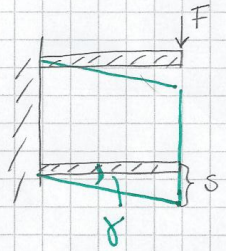
KB s.25 $\sigma = \frac{F}{A}$

5.1 $E = 210\,GPa$ (konstigt enhetsval i boken)

$\nu = 0{,}3$

$$G = \underbrace{\frac{E}{2(1+\nu)}}_{\text{formelsamling}} = \frac{210}{2(1+0{,}3)} = 80\,GPa$$

5.2



$d_y = 30\,mm$ $\quad t = 2{,}0\,mm$

$L = 20\,mm$ $\quad F = 5\,kN$

$s = 21\,\mu m = 21 \cdot 10^{-6}\,m$

Sökt: G

Hooks lag för skjuvning

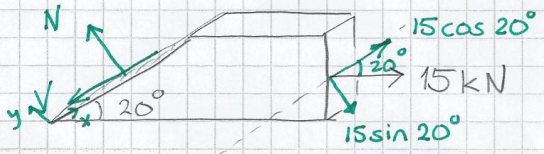
$$\boxed{\tau = G \cdot \gamma}$$

$$\tau = \frac{5 \cdot 10^3}{\frac{\pi}{4}(30^2 - 26^2)} = 28{,}4\,MPa$$

$$\gamma = \frac{s}{L} = \frac{21 \cdot 10^{-6}}{0{,}02} = 1{,}05 \cdot 10^{-3}$$

$$G = \frac{\tau}{\gamma} = \frac{28{,}4 \cdot 10^6}{1{,}05 \cdot 10^{-3}} = 26{,}7\,GPa$$

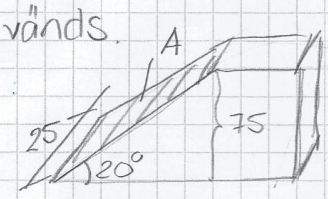
5:3



$\nwarrow$ $N - 15\sin 20^\circ = 0 \quad N = 5{,}13\ \text{kN}$

$\swarrow$ $V + 15\cos 20^\circ = 0 \quad V = 14{,}1\ \text{kN}$

Jämvikten blir mkt lättare om y x koordinat-system används.

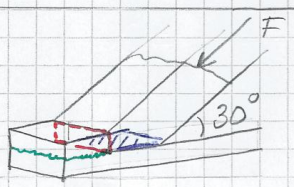


A är ytan som N & V verkar på.

$$A = \frac{75}{\sin 20^\circ} \cdot 25 = 5482\ \text{mm}^2$$

$$\tau = \frac{14{,}1 \cdot 10^3}{5482} = 2{,}6\ \text{MPa} \qquad \sigma = \frac{5{,}13 \cdot 10^3}{5482} = 0{,}93\ \text{MPa}$$

5:4



$F = 3{,}0\ \text{kN}$

Skjuvad yta, Skjuvas av $F\cos 30^\circ$

$$\tau = \frac{3 \cdot 10^3 \cdot \cos 30^\circ}{75 \cdot 75} = 0{,}46\ \text{MPa}$$

Tryck mot ytan av $F\cos 30^\circ$

$$p_1 = \frac{3 \cdot 10^3 \cdot \cos 30^\circ}{37{,}5 \cdot 75} = 0{,}92\ \text{MPa}$$

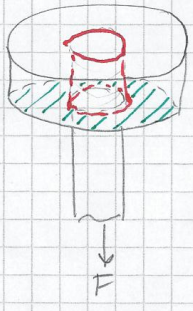
Tryck mot yta av $F\sin 30^\circ$

$$p_2 = \frac{3 \cdot 10^3 \cdot \sin 30^\circ}{85 \cdot 75} = 0{,}24\ \text{MPa}$$

5:5 $\Delta L = 0{,}1\ mm$ $E = 210\ GPa$

$$\Delta L = \frac{FL}{AE} \Rightarrow F = \frac{\Delta L A E}{L} = \frac{0{,}1 \cdot 10^{-3} \cdot 0{,}01^2 \cdot \pi \cdot 210 \cdot 10^9}{0{,}2 \cdot 4}$$

$$F = 8247\ N$$



Stången skjuvar skallen med kraften $F = 8247\ N$ och över arean (mantelytan)

$$A_s = 10 \cdot \pi \cdot 15 = 150\pi\ mm^2$$

$$\tau = \frac{8247}{150\pi} = 17{,}5\ MPa$$

Kontakttryck "grönt"

$$p = \frac{8247}{\frac{\pi}{4}(40^2 - 10^2)} = 7\ MPa$$

5:6 $\tau_B = 220\ MPa$

$$\tau = \frac{F}{A} \Rightarrow F = 220 \cdot 10^6 \cdot 4 \cdot 0{,}025 \cdot 0{,}005 = 110\ kN$$

$$\sigma_{stans} = \frac{F}{A_{stans}} = \frac{110 \cdot 10^3}{0{,}025^2} = 176\ MPa$$

5:7 $\tau_B = 250\ MPa$ $F_{stans}^{max} = 350\ kN$ $t = 6mm$

$$\tau_B = \frac{F}{A} = \frac{F}{d\pi t} \Rightarrow d = \frac{F}{\tau_B \pi t}$$

$$d = \frac{350 \cdot 10^3}{250 \cdot 10^6 \cdot \pi \cdot 0{,}006} = 74mm$$

5:8 $F_{stans} = 100\,kN$ $t = 4mm$

$\tau_B = 220\,MPa$ $\sigma_{tiu} = 425\,MPa$

Största rondell

$$\tau_B = \frac{F_{stans}}{d\pi t} \Rightarrow d = \frac{100 \cdot 10^3}{220 \cdot 10^6 \cdot \pi \cdot 0{,}004} = 36mm$$

Minsta rondell

$$\sigma_{tiu} = \frac{F}{A_{stans}} = \frac{F}{\frac{\pi d^2}{4}} \Rightarrow d = \sqrt{\frac{4F}{\pi \sigma_{tiu}}}$$

$$d = \sqrt{\frac{100 \cdot 10^3 \cdot 4}{\pi \cdot 425 \cdot 10^6}} = 17{,}3mm$$

5:9 $t = 2mm$ $\alpha = 10^\circ$ $\tau_B = 220\,MPa$

$$F = \tau_B \frac{t^2}{2\tan\alpha} = 220 \cdot 10^6 \cdot \frac{0{,}002^2}{2 \cdot \tan 10^\circ} = 2{,}5kN$$

5:10 $F = 5kN$ $\alpha = 12^\circ$ $\tau_B = 250\,MPa$

$$t = \sqrt{\frac{2F\tan\alpha}{\tau_B}} = \sqrt{\frac{2 \cdot 5 \cdot 10^3 \cdot \tan 12^\circ}{250 \cdot 10^6}} = 2{,}9mm$$

5:11 $\tau_{lim} = 12\,MPa$ SS1412 $R_{el} = 250\,MPa$

$L = 60\,mm$ $n_s = 1{,}5$

$\sigma_{tiu} = \frac{250}{1{,}5} = 167\,MPa$

$b = 50\,mm$

$t = 5\,mm$

F_{max} för limfog

$\tau = \frac{F}{A_{lim}} = \frac{F}{L \cdot b}$

$F_{max} = 12 \cdot 10^6 \cdot 0{,}06 \cdot 0{,}05 = 36\,kN$

F_{max} för plåt

$\sigma_{tiu} = \frac{F}{A_{pl}} = \frac{F}{b \cdot t}$

Svar: $F_{max} = 36\,kN$

$F_{max} = 167 \cdot 10^6 \cdot 0{,}05 \cdot 0{,}005 = 41{,}7\,kN$

5:12 $\tau = \frac{F}{2A_{lim}} = \frac{33.6 \cdot 10^3}{2 \cdot 0{,}03 \cdot 0{,}02} = 28\,MPa$

5:13 $F = 20\,kN$ $50 \times 5\,mm$ av AL 4054-14

$\tau_{tiu} = 10\,MPa$ $n_s = 1{,}5$

$R_{el} = 185\,MPa$

$\tau_{tiu} = \frac{F}{b \cdot L}$ $\sigma_{tiu} = \frac{185}{1{,}5} = 123\,MPa$

$L = \frac{20 \cdot 10^3}{0{,}05 \cdot 10 \cdot 10^6} = 0{,}04\,m$

$\sigma = \frac{F}{b \cdot t} = \frac{20 \cdot 10^3}{0{,}05 \cdot 0{,}005} = 80\,MPa$

$80 < 123$ OK!

5.14 Stumsvets => räkna på grundmaterialet

$a = t = 5\,mm$

$l = L - 2a = 50 - 2 \cdot 5 = 40\,mm$

$R_{el} = 200\,MPa \qquad n_s = 1{,}5$

$\sigma_{tillåt} = \frac{200}{1{,}5} = 133\,MPa$

$F = \sigma_{tillåt} \cdot A = 133 \cdot 5 \cdot 40 = 26{,}7\,kN$

5.15 Sökt: τ_b

$F = 60\,kN$

$a = 4\,mm$

$l = L - 2a = 60$ (ej hänsyn)

$\tau_b = \frac{60 \cdot 10^3}{2 \cdot 60 \cdot 4} = 125\,MPa$

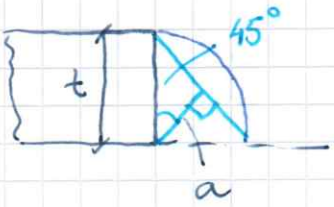
2 svetsar

5:16 Sökt: L

Givet: $F = 40\,kN$

$\tau_{tillåt} = 104\,MPa$

Bestäm maximalt a-mått



$a = t\cos 45° = 5\cos 45° = 3{,}53 = 3{,}5\,mm$

l: effektiv svetslängd

$\tau = \frac{F}{2al}$ (2 svetsar)

$l = \frac{F}{2a\tau} = \frac{40 \cdot 10^3}{2 \cdot 3{,}5 \cdot 104} = 54{,}95\,mm$

$L = l + 2a = 54{,}95 + 2 \cdot 3{,}5 = 61{,}9\,mm = 62\,mm$

5.17 $t = 12\,mm$
$B = 130\,mm$
3 nitar $d = 22\,mm$
$F = 100\,kN$

Ta ej hänsyn till slagen ej oslagen nit
$d = 22\,mm$

en-skärigt förband => 1 A_{skjuv}/nit

$$\tau = \frac{F}{3A_{nit}} = \frac{100 \cdot 10^3 \cdot 4}{3 \cdot 22^2 \cdot \pi} = 87{,}7\,MPa$$

$$\sigma = \frac{F}{(B-3d)\cdot t} = \frac{100 \cdot 10^3}{(130-3\cdot 22)\cdot 12} = 130\,MPa$$

$$p_H = \frac{F}{3 \cdot d \cdot t} = \frac{100 \cdot 10^3}{3 \cdot 22 \cdot 12} = 126\,MPa$$

5.18 $t_p = 8\,mm$ $t_s = 6\,mm$
$B = 50\,mm$
$d = 17\,mm$

$\tau_{hu} = 100\,MPa$
$\sigma_{hu} = 140\,MPa$
$p_{H,hu} = \sigma_{H,hu} = 260\,MPa$

2-skärigt förband $2A_{nit}$/nit

Kontrollera nit

$$\tau_{hu} = \frac{F_{hu}}{2A_{nit}} \Rightarrow F_{hu} = 100 \cdot 2 \cdot \frac{17^2 \cdot \pi}{4} = 45369\,N$$

Kontrollera plåtar

$t_p = 8\,mm$ $t_s = 6\,mm$ (2 st)
$8 < 2 \cdot 6$ => max spänning i mittplåten

$$\sigma_{hu} = \frac{F_{hu}}{(B-d)\,t_p} \Rightarrow F_{hu} = 140 \cdot 8 \cdot (50-17) = 36960\,N$$

Kontrollera hålkant
$8 < 2 \cdot 6$ max hålkantstryck i mittplåten

$$p_{H,hu} = \frac{F}{d \cdot t_p} \Rightarrow F_{hu} = 260 \cdot 17 \cdot 8 = \boxed{35360\,N}$$

F_{hu} ty lägst värde

5.19 $F = 110\ kN$

2-skärigt och 3 nitar =>

$A_{skjuv} = 2 \cdot 3 A_{nit} = 6 A_{nit}$

$\tau_{tillåt} = 120\ MPa$

$\tau_{tillåt} = \frac{F}{6A_{nit}} = \frac{F}{6 r_{nit}^2 \cdot \pi}$

$r_{nit} = \sqrt{\frac{110 \cdot 10^3}{120 \cdot 6 \cdot \pi}} = 6{,}97\ mm \Rightarrow d = 13{,}9$

$\underline{\underline{d = 14\ mm}}$

$\sigma_{tillåt} = 260\ MPa$

Max tryck i mittplåten ty $12 < 8+8$

$\sigma_H = \frac{F}{3A_{proj}}$ $\qquad A_{proj} = d \cdot \underset{t}{12}$

$260 = \frac{110 \cdot 10^3}{3 \cdot d \cdot 12} \Rightarrow d = \frac{110 \cdot 10^3}{260 \cdot 3 \cdot 12} = 11{,}75\ mm$

σ_{max} i plattstål ty $12 < 8+8$

$\sigma_{till} = 140\ MPa$

$\sigma = \frac{F}{A} = \frac{F}{12(b-3{,}14)} \Rightarrow b = \frac{110\,10^3}{12 \cdot 140} + 3{,}14 = \underline{108\ mm}$

5.2 D Sökt: F_{max}

s. 13 KB 3/4-10 UNC => d = 19,05 mm (ytterdiameter)

8.8 => $R_m = 800$ MPa

s. 16 KB $R_{eL} = 640$ MPa $\tau_{Hill} = 0{,}6 \cdot 640 = 384$ MPa

s. 25 KB

Kontroll skjuvning

en-skärigt förband } => $2A_{skjuv}$
2 skruvar

$$384 = \frac{F_{max} \cdot 4}{2 \cdot 19{,}05^2 \cdot \pi} \Rightarrow F_{max} = 219 \text{ kN}$$

Kontroll hålkantstryck

s. 35 KB 1312 => $p_h^{Hill} = 250$ MPa

↑ 1311 fanns ej

$A_{proj} = d \cdot t \cdot 2$

$$250 = \frac{F_{max}}{19{,}05 \cdot 10 \cdot 2}$$

$F_{max} = 95$ kN

5.21 Sökt: d samt $\sigma_{plattstål}$

Givet: F = 70 kN.
Plattjärn 110 × 10 mm
SS1311
Hållfasthetsklass
skruv 4.6
$n_s = 1,5$

Ta ej hänsyn till förbandsklass S2.

Skruven: $R_m = 400\,MPa$ $R_{eL} = 400 \cdot 0,6 = 240\,MPa$
5.4.6 KB

$\tau = 0,6 \cdot 240 = 144\,MPa$ (KB s. 25)

Förbandet => 2-skärigt, 2 skruvar } 4 A_{skruv}.

$$144 = \frac{70 \cdot 10^3 \cdot 4}{4 d^2 \cdot \pi} \Rightarrow d = \sqrt{\frac{70 \cdot 10^3}{\pi \cdot 144}} = 12,4\,mm$$

Kontroll skjuvning

Välj: d = 13 mm

Håkantstryck (SS1311)

$p_H^{riu} = 250\,MPa$ KB. s. 35 (1312)

Trycket blir störst på mittplåten med 4 hålkanter

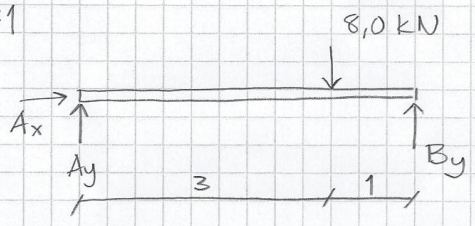
$$p_H = \frac{70 \cdot 10^3}{4 \cdot 13 \cdot 10} = 135\,MPa < 250\,MPa \quad OK.$$

$$\sigma_{max} = \frac{70 \cdot 10^3}{(110 - 2 \cdot 13) \cdot 10} = 84\,MPa$$

1311 => $R_{eL} = 210\,MPa$ $n = 1,5$

$\sigma_{riu} = \frac{210}{1,5} = 140\,MPa$ 84 < 140 OK

6:1



$\rightarrow A_x = 0$

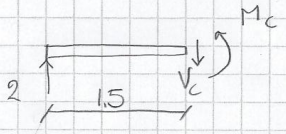
$\curvearrowleft_A \; 8 \cdot 3 - B_y \cdot 4 = 0$

$B_y = 6\,kN$

$\uparrow A_y + B_y - 8 = 0$

$A_y = 8 - 6 = 2\,kN$

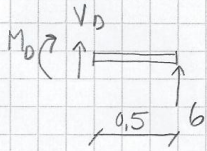
Snitt C



$\downarrow V_C - 2 = 0 \Rightarrow V_C = 2\,kN$

$\curvearrowleft_C \; M_C - 2 \cdot 1{,}5 = 0 \quad M_C = 3\,kNm$

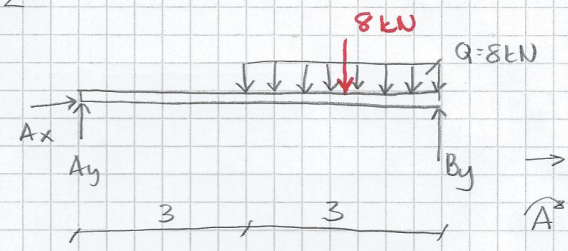
Snitt D



$\uparrow V_D + 6 = 0 \Rightarrow V_D = -6\,kN$

$\curvearrowleft_D \; M_D - 6 \cdot 0{,}5 = 0 \Rightarrow M_D = 3\,kNm$

6:2



Q utbredd ersätts med punktlast (röd)

$\rightarrow A_x = 0$

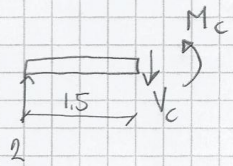
$\curvearrowright A: \quad 8 \cdot 4{,}5 - 6 \cdot B_y = 0$

$B_y = 6\,kN$

$\uparrow \quad A_y + B_y - 8 = 0$

$A_y = 8 - 6 = 2\,kN$

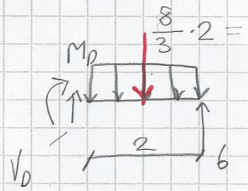
Snitt C



$\downarrow \quad V_c - 2 = 0 \Rightarrow V_c = \underline{2\,kN}$

$\curvearrowleft C: \quad M_c - 2 \cdot 1{,}5 = 0 \Rightarrow M_c = \underline{3\,kNm}$

Snitt D



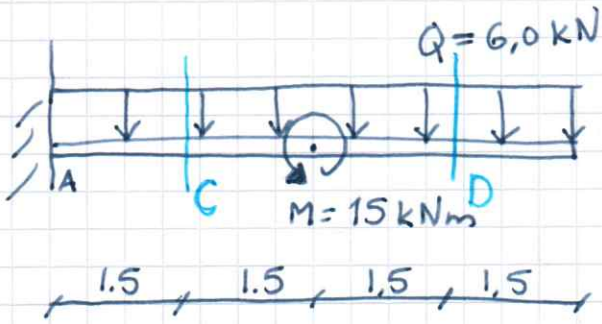
$\uparrow \quad V_D - \frac{8}{3} \cdot 2 + 6 = 0$

$V_D = \frac{16}{3} - \frac{18}{3} = -\frac{2}{3}\,kN = \underline{0{,}67\,kN}$

$\curvearrowright D: \quad M_D + \frac{8}{3} \cdot 2 \cdot 1 - 6 \cdot 2 = 0$

$M_D = 12 - \frac{16}{3} = \frac{20}{3} = \underline{6{,}67\,kNm}$

6.3 Sökt: Snittstorheter i C & D.



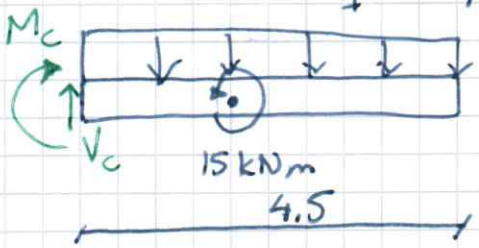
Enklast || Stödkrafter i A behövs ej då man kan välja att frilägga till höger om snittet.

Bestäm q (last/meter)

$$q = \frac{Q}{L_{tot}} = \frac{6}{6} = 1\ kN/m$$

Snitt C

Inga normalkrafter, N, ty inga krafter i x-led

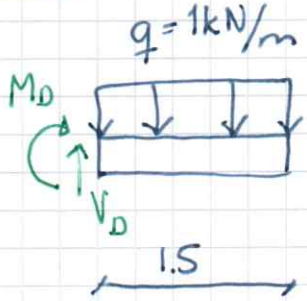


$\uparrow V_C - 1 \cdot 4{,}5 = 0 \Rightarrow \underline{V_C = 4{,}5\ kN}$

$\circlearrowright M_C + \underbrace{1 \cdot 4{,}5}_{Q} \cdot \underbrace{\frac{4{,}5}{2}}_{\text{hävarm}} - 15 = 0$

$\underline{M_C = 15 - \frac{4{,}5^2}{2} = 4{,}9\ kNm}$

Snitt D



$\uparrow V_D - 1 \cdot 1{,}5 = 0 \quad \underline{V_D = 1{,}5\ kN}$

$\circlearrowright M_D + 1 \cdot 1{,}5 \cdot \frac{1{,}5}{2} = 0$

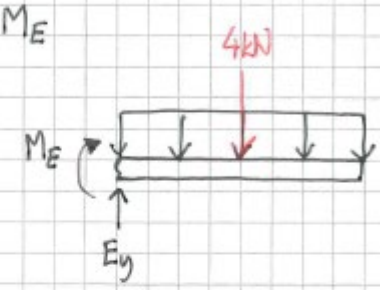
$\underline{M_D = -\frac{1{,}5^2}{2} = -1{,}1\ kNm}$

6.4 Sökt: M_A, M_B, M_C, M_D, M_E

$M_B = 0$ ty led

$M_C = 0$ ty fri ände

M_E

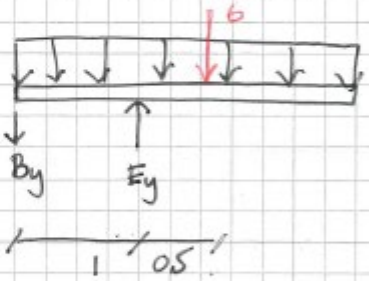


$Q = 6\,kN$ över 3 m

$Q_{2m} = 4\,kN$ över 2 m

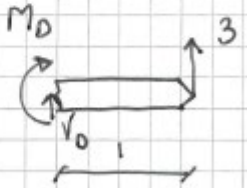
E $M_E + 4 \cdot 1 = 0 \Rightarrow M_E = -4\,kNm$

Kraft i B krävs för $M_D \leq M_A$



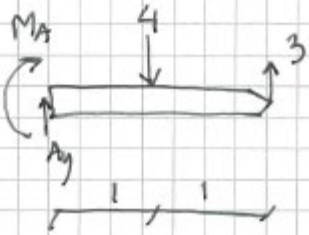
E $6 \cdot 0{,}5 - B_y \cdot 1 = 0$

$B_y = 3\,kN$



D $M_D - 3 \cdot 1 = 0$

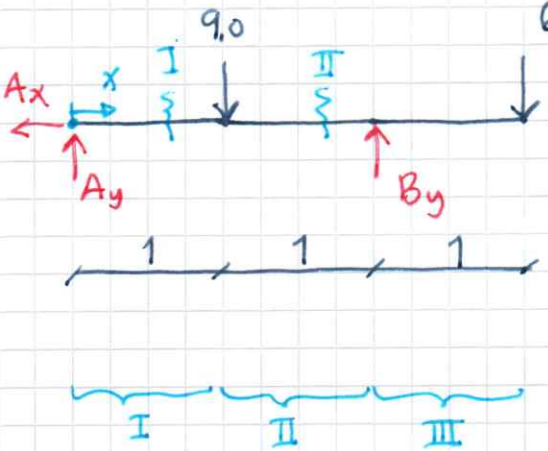
$M_D = 3\,kNm$



A $M_A + 4 \cdot 1 - 3 \cdot 2 =$

$M_A = 2\,kNm$

6.5 Sökt: V- & M-diagram med snittmetoden



(kN, m)

$\leftarrow A_x = 0$

$\curvearrowleft_A \; B_y \cdot 2 - 9 \cdot 1 - 6 \cdot 3 = 0$

$B_y = 13{,}5\,kN$

$\uparrow A_y + B_y - 9 - 6 = 0$

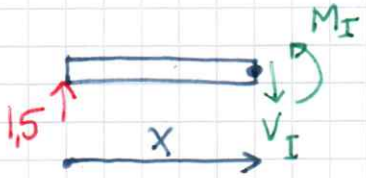
$A_y = 9 + 6 - 13{,}5 = 1{,}5\,kN$

Tre delområden (I, II, III) $\Rightarrow$ Tre snitt med frilägging och jämvikt.

Alla ekvationer ska vara en funktion av x.

Koordinatsystem långt till vänster.

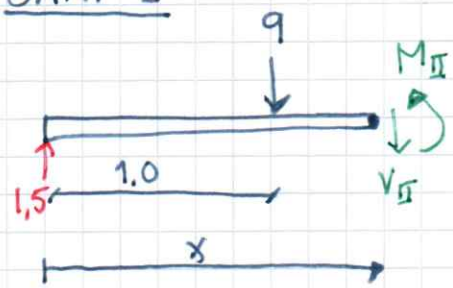
Snitt I



$\downarrow V_I - 1{,}5 = 0 \quad V_I(x) = 1{,}5\,kN$

$\curvearrowleft M_I - 1{,}5x = 0 \quad M_I(x) = 1{,}5x \;\; kNm$

Snitt II

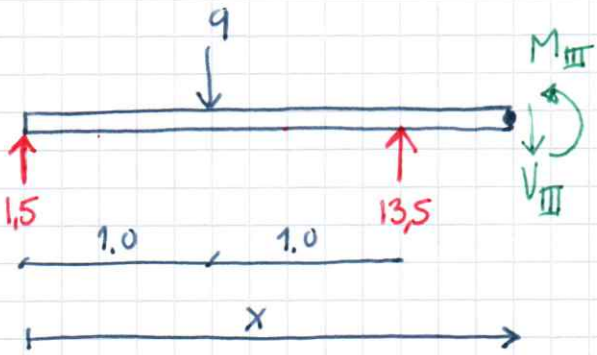


$\downarrow V_{II} + 9 - 1{,}5 = 0 \quad V_{II}(x) = -7{,}5\,kN$

$\curvearrowleft M_{II} + 9(x-1) - 1{,}5x = 0$

$M_{II}(x) = 1{,}5x - 9x + 9 = -7{,}5x + 9 \;\; kNm$

6.5 forts



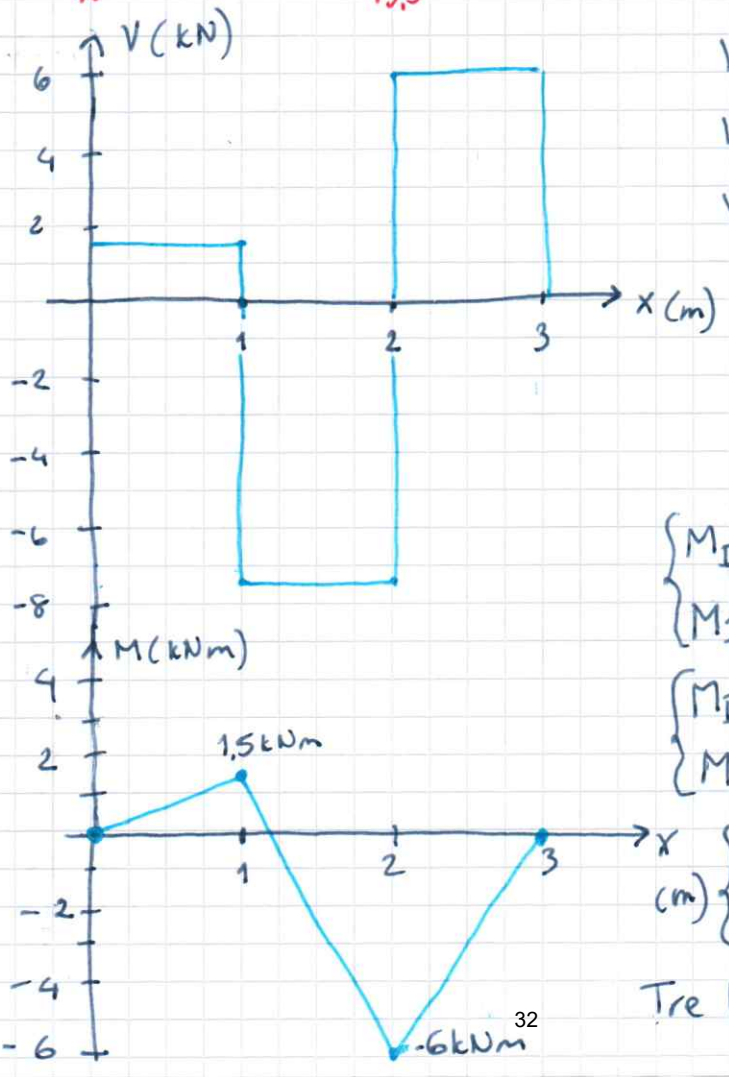
$\downarrow\ V_{III}(x) + 9 - 1{,}5 - 13{,}5 = 0 \qquad V_{III} = 6\ kN$

$\circlearrowleft\ M_{III}(x) + 9(x-1) - 1{,}5x - 13{,}5(x-2) = 0$

$M_{III}(x) + 9x - 9 - 1{,}5x - 13{,}5x + 27 = 0$

$M_{III}(x) = 6x - 18 \quad kNm$





Tvärkraftdiagram

$V_I = 1{,}5\ kN \quad 0 \le x \le 1$

$V_{II} = -7{,}5\ kN \quad 1 \le x \le 2$

$V_{III} = 6\ kN \quad 2 \le x \le 3$

$\begin{cases} M_I(x) = 1{,}5x \quad 0 \le x \le 1 \\ M_I(0) = 0 \quad M_I(1) = 1{,}5 \end{cases}$

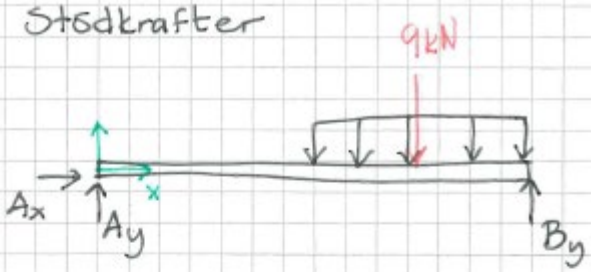
$\begin{cases} M_{II}(x) = -7{,}5x + 9 \quad 1 \le x \le 2 \\ M_{II}(1) = 1{,}5 \quad M_{II}(2) = -6 \end{cases}$

$\begin{cases} M_{III} = 6x - 18 \quad 2 \le x \le 3 \\ M_{III}(2) = -6 \quad M_{III}(3) = 0 \end{cases}$

Tre linjära funktioner $y = kx + m$

6.6 Bestäm T- & M-diagram

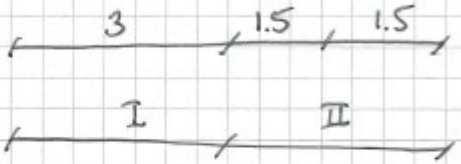
Stödkrafter



$A_x = 0$

$\curvearrowright_A \quad 9 \cdot 4{,}5 - B_y \cdot 6 = 0$

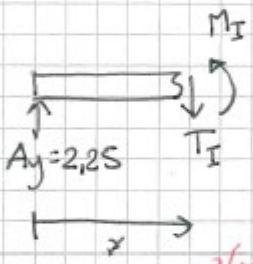
$B_y = 6{,}75 \text{ kN}$



$\uparrow \quad A_y + B_y - 9 = 0$

$A_y = 9 - 6{,}75 = 2{,}25 \text{ kN}$

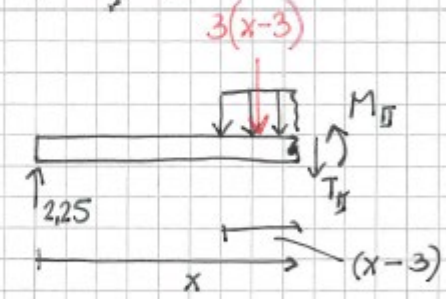
Ⓘ



$\downarrow \quad T_I - 2{,}25 = 0 \qquad T_I = 2{,}25 \text{ kN}$

$\curvearrowright \quad M_I - 2{,}25x = 0 \qquad M_I(x) = 2{,}25x$

Ⓘⓘ



Utbredd last per meter

$$q = \frac{Q}{L} = \frac{9}{3} = 3 \text{ kN/m}$$

$\downarrow \quad T_{II} + 3(x-3) - 2{,}25 = 0$

$T_{II}(x) = 2{,}25 + 9 - 3x = -3x + 11{,}25$

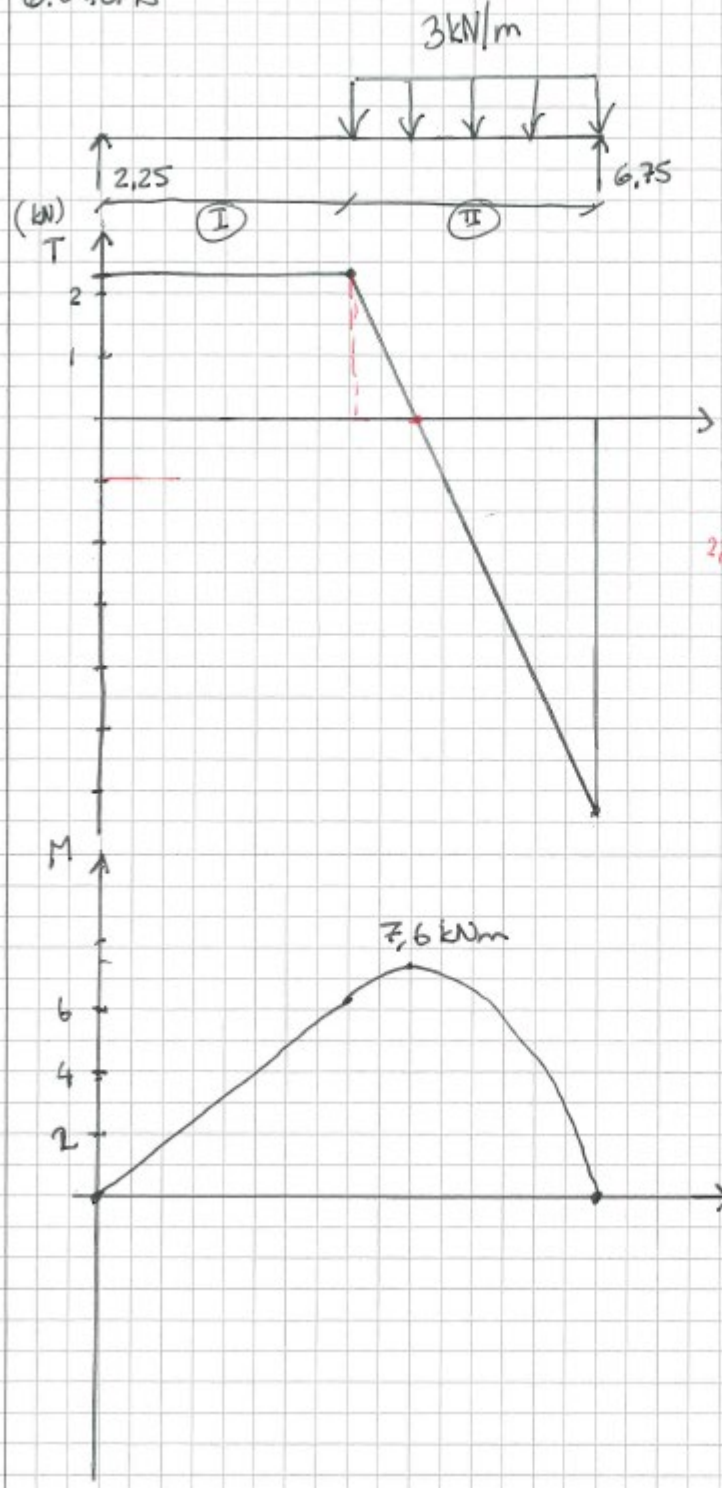
$\curvearrowright \quad M_{II} + 3(x-3) \cdot \frac{(x-3)}{2} - 2{,}25 \cdot x = 0$

$$M_{II} = 2{,}25x - \frac{3}{2}(x-3)^2 = 2{,}25x - \frac{3}{2}(x^2 - 6x + 9)$$

$$M_{II}(x) = -1{,}5x^2 + 2{,}25x + 9x - 13{,}5$$

$$M_{II}(x) = -1{,}5x^2 + 11{,}25x - 13{,}5$$

6.6 forts



$T_I = 2,25$

$T_{II} = -3x + 11,25$

$T_{II}(3) = -9 + 11,25 = 2,25$

$T_{II}(6) = -3 \cdot 6 + 11,25 = -6,75$

$\frac{2,25}{x} = \frac{9}{3}$

$x = 0,75$ m

$M_I(x) = 2,25x$

$M_I(3) = 2,25 \cdot 3 = 6,75$

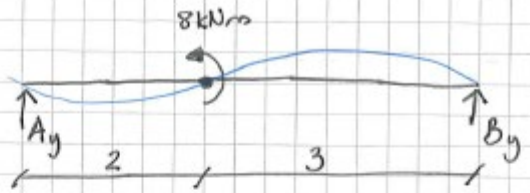
$M_{II}(x) = -1,5x^2 + 11,25x - 13,5$

$M_{II}(3) = 6,75$

$M(3,75) = 7,6$ kNm

$T = 0$

6.7

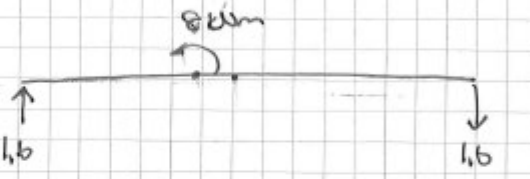


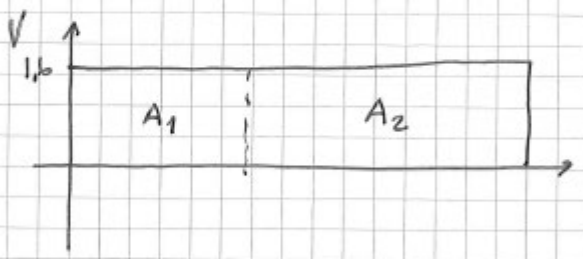
$\circlearrowleft_A \quad 8 + B_y \cdot 5 = 0$

$B_y = -1{,}6\ \text{kN}$

$A_y + B_y = 0 \quad A_y = -B_y$

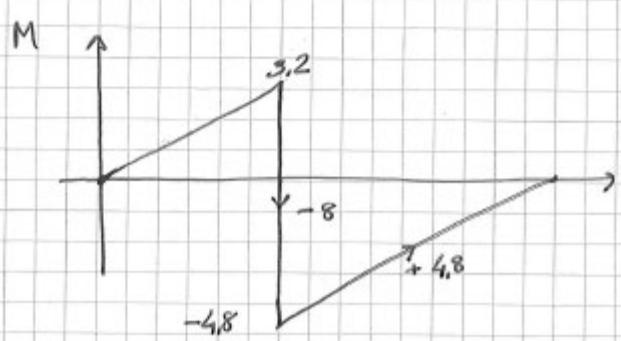
$A_y = 1{,}6\ \text{kN}$



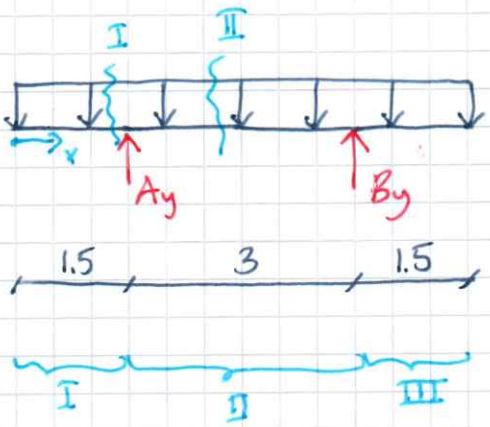


$A_1 = 2 \cdot 1{,}6 = 3{,}2$

$A_2 = 3 \cdot 1{,}6 = 4{,}8$



6.8 "Snittmetoden" Finns även lösning med förenklade metoden efter denna lösning. Förenklade metoden börjar efter upp. 6:13 men 6:5-6:12 kan också lösas med den metoden.



Lasten, G, är balkens egenvikt.

IPE 100 => m = 8,1 kg/m *B s.65

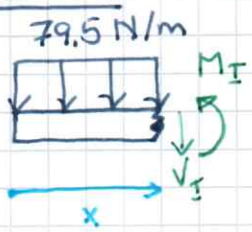
G = 8,1 · 6 · 9,81 = 476,8 N

Symmetri => $A_y = B_y = \frac{476,8}{2} = 238$ N

Symmetri => Delområde I ≙ III är lika.

q = 8,1 · 9,81 = 79,5 N/m

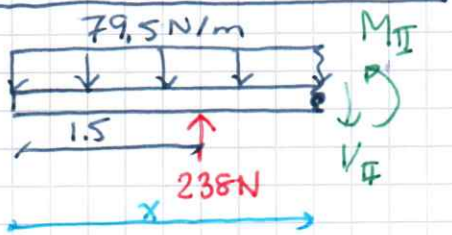
Snitt I $0 \leq x \leq 1,5$



$\downarrow \; V_I + 79,5x = 0 \quad V_I(x) = -79,5x$ N

$\circlearrowleft \; M_I(x) + 79,5x \cdot \frac{x}{2} = 0 \Rightarrow M_I(x) = -39,7x^2$ Nm

Snitt II $1,5 \leq x \leq 4,5$



$\downarrow \; V_{II}(x) - 238 + 79,5x = 0$

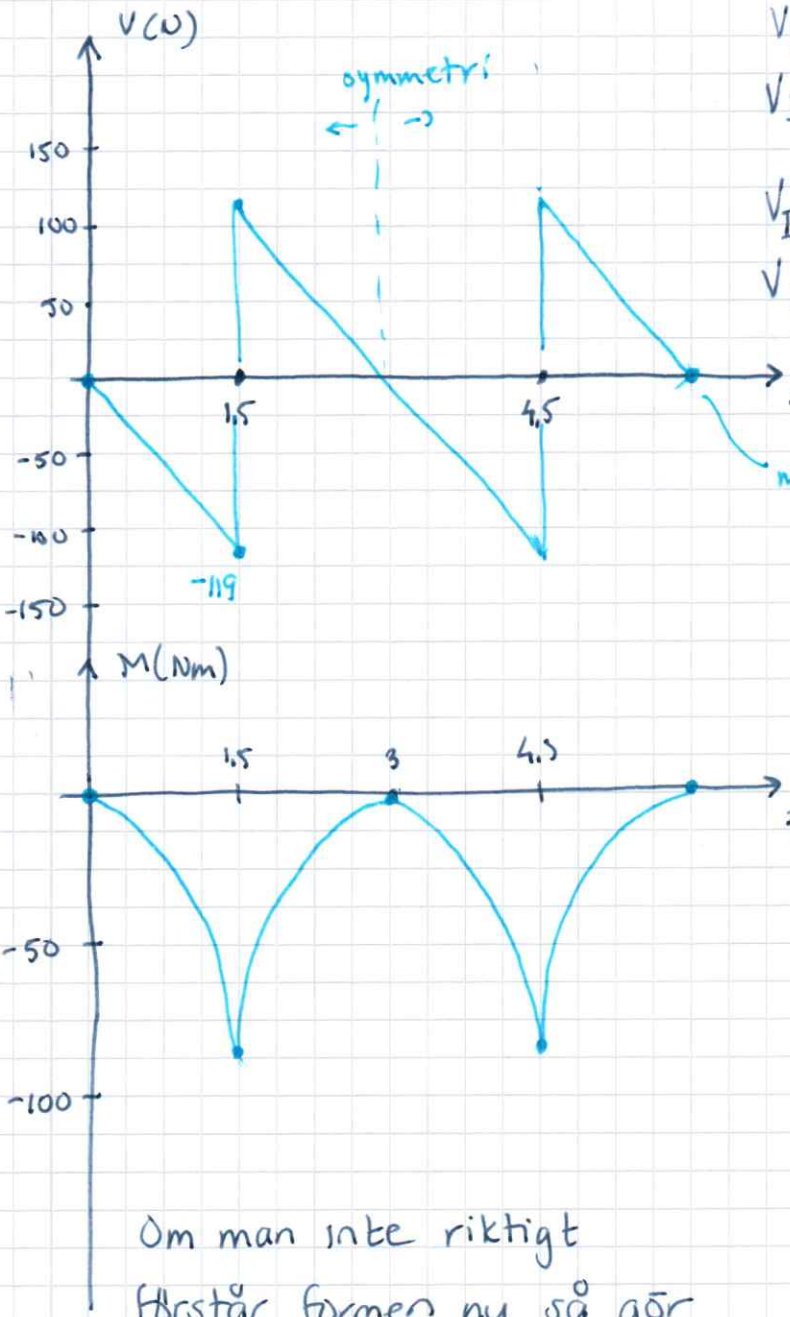
$V_{II}(x) = -79,5x + 238$ N

$\circlearrowleft \; M_{II}(x) + 79,5\frac{x^2}{2} - 238(x-1,5) = 0$

$M_{II}(x) = -39,7x^2 + 238x - 357$ Nm

6.8 Snittmetoden forts





$V_I(x) = -79,5x$ N

$V_I(0) = 0$ $V_I(1,5) = -119$ N

$V_{II}(x) = -79,5x + 238$ N

$V_{II}(1,5) = 119$ N $V_{II}(4,5)$

måste sluta på noll!

$M_I(x) = -39,7x^2$

x(m) $M_I(0) = 0$ $M_I(1,5) = -89$ Nm

$M_{II}(x) = -39,7x^2 + 238x - 357$

$M_{II}(1,5) = -89$ Nm

$M_{II}(4,5) = -89$ Nm

$M_{II}(3) = 0$

Om man inte riktigt förstår formen nu så gör man (förhoppningsvis) det när man lärt sig den förenklade metoden.

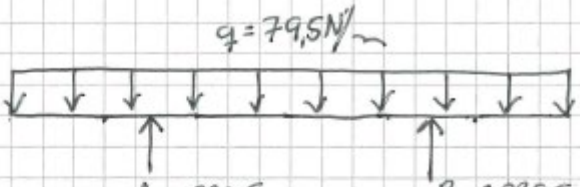
6.8 med förenklade metoden

IPE 100 väger 8,1 kg/m

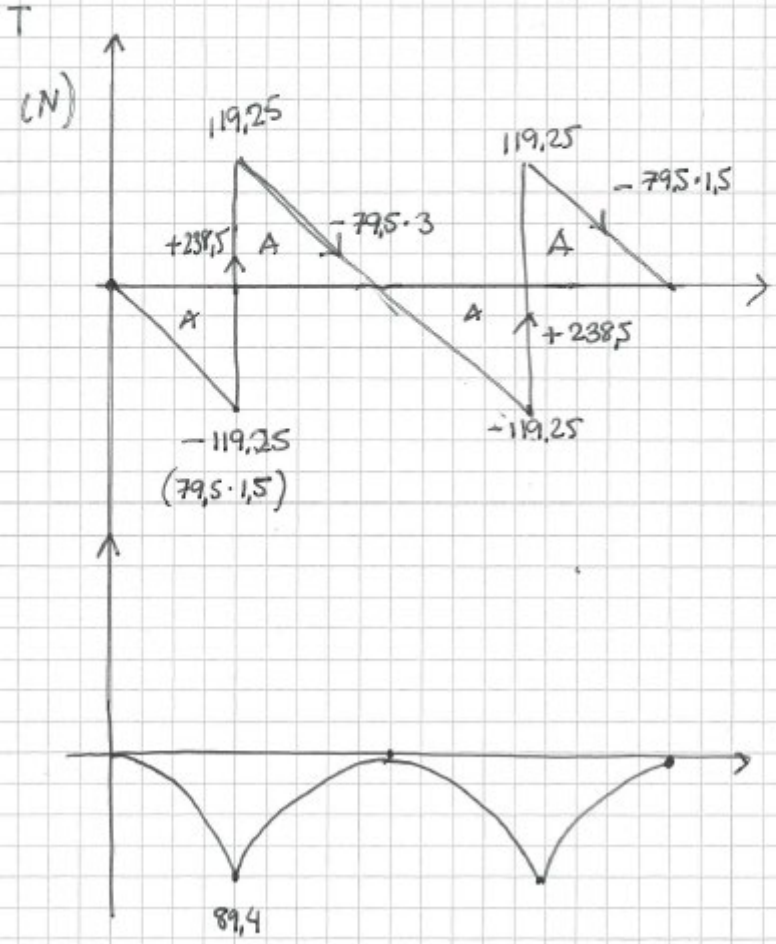
6m => 6·8,1 = 48,6 kg

$Q = 48,6 \cdot 9,81 = 477\,N$

$q = \frac{477}{6} = 79,5\,N/m$



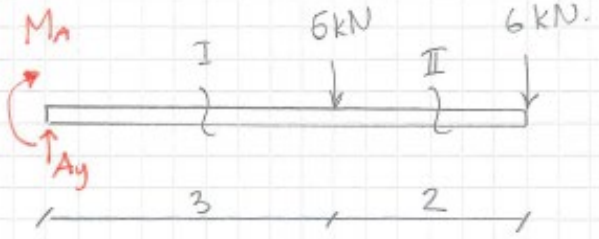
$A_y = B_y = \frac{477}{2} = 238,5\,N$



$A = \frac{119,25 \cdot 1,5}{2}$

$A = 89,4$

6.9 Snittmetoden

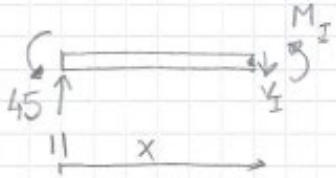


$\curvearrowleft_A \quad M_A + 5\cdot 3 + 6\cdot 5 = 0$

$M_A = -45\ \text{kNm}$

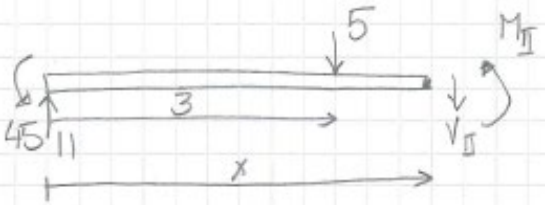
$\uparrow \quad A_y - 5 - 6 = 0$

$A_y = 11\ \text{kN}$



$\downarrow \quad V_I - 11 = 0 \qquad V_I(x) = 11\ \text{kN}$

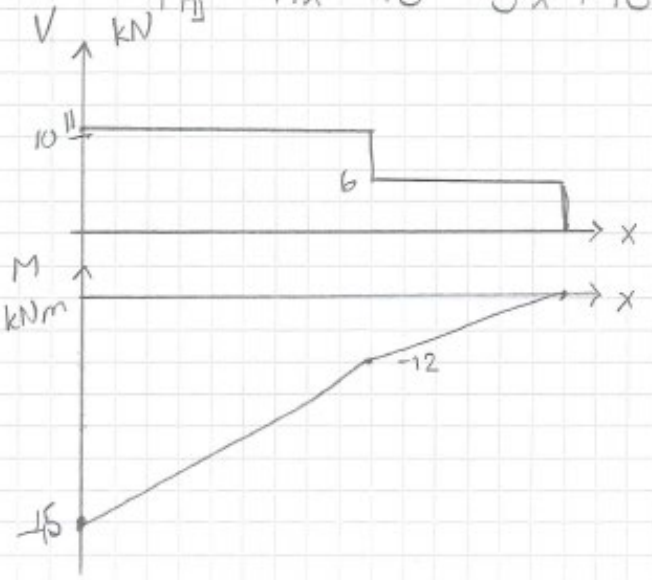
$\circlearrowleft \quad M_I + 45 - 11x = 0 \Rightarrow M_I(x) = 11x - 45$



$\downarrow \quad V_{II} + 5 - 11 = 0 \Rightarrow V_{II}(x) = 6\ \text{kN}$

$\circlearrowleft \quad M_{II} + 5(x-3) - 11x + 45 = 0$

$M_{II} = 11x - 45 - 5x + 15 = 6x - 30$

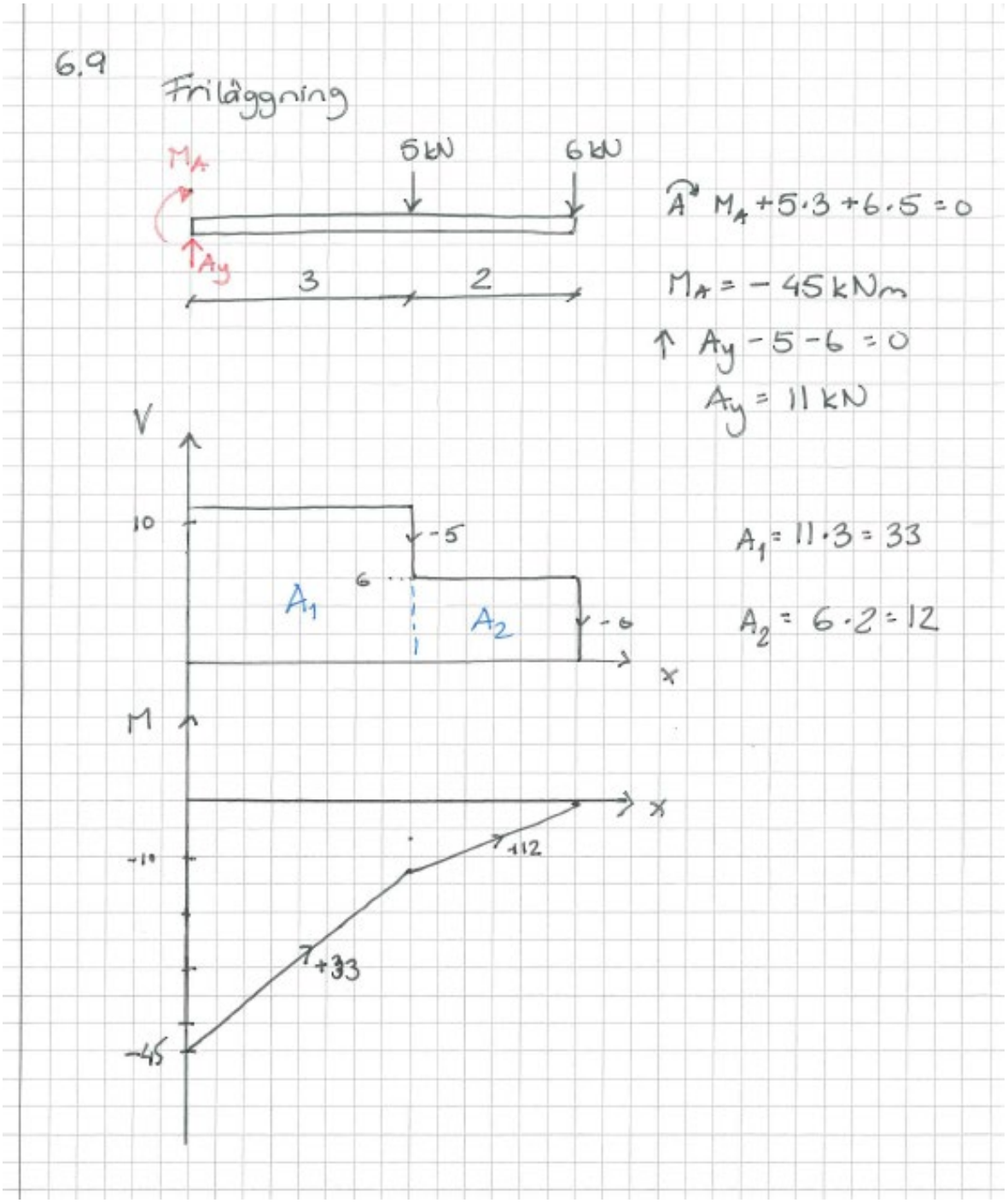


$M_I(3) = -12$

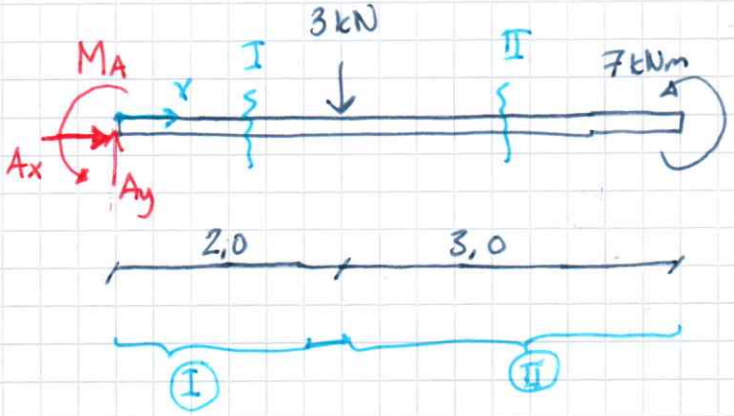
$M_{II}(3) = -12$ ok

$M_{II}(5) = 0$

Förenklad metod



6.10 Sökt: V- o M-diagram med snittmetoden



Stödkrafter

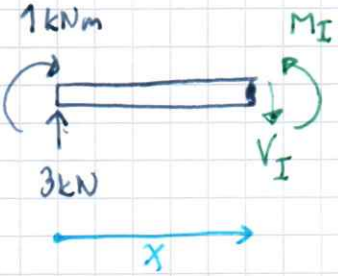
$\rightarrow A_x = 0$

$\uparrow A_y - 3 = 0 \quad A_y = 3\,kN$

$\circlearrowleft M_A + 7 - 3 \cdot 2 = 0$

$M_A = -1\,kNm$

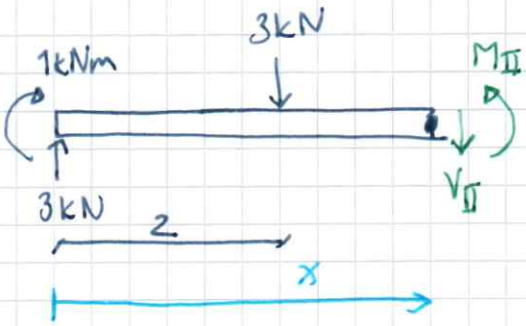
Snitt I $\quad 0 \leq x \leq 2$



$\downarrow V_I(x) - 3 = 0 \quad V_I(x) = 3\ kN$

$\circlearrowleft M_I(x) - 1 - 3x = 0 \Rightarrow M_I(x) = 3x + 1$ kNm

Snitt II $\quad 2 \leq x \leq 5$

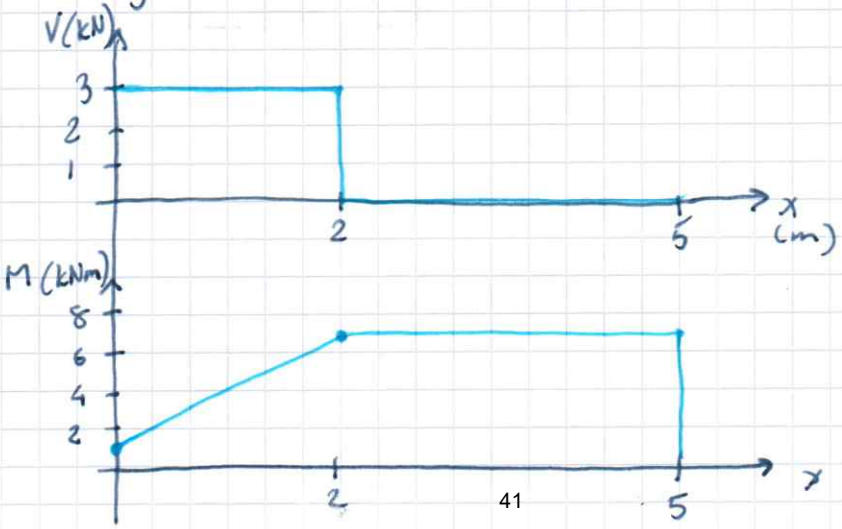


$\downarrow V_{II} - 3 + 3 = 0 \quad V_{II}(x) = 0$

$\circlearrowleft M_{II}(x) + 3(x-2) - 3x - 1 = 0$

$M_{II}(x) = 7\ kNm$





$V_I(x) = 3$

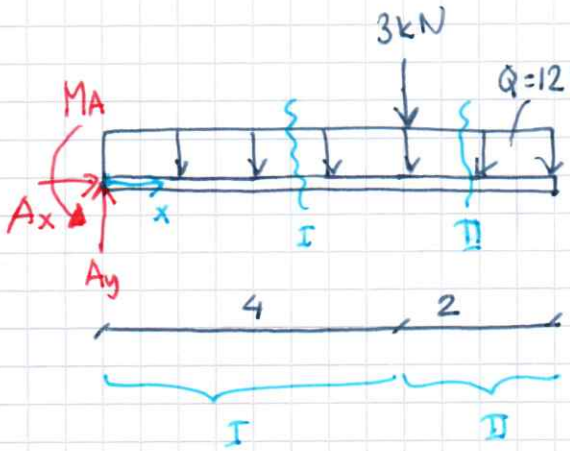
$V_{II}(x) = 0$

$M_I(x) = 3x + 1$

$M_I(0) = 1 \quad M_I(2) = 7$

$M_{II}(x) = 7$

6.11 Sökt: V- o M-diagram med snittmetoden



Stödkrafter

$\rightarrow A_x = 0$

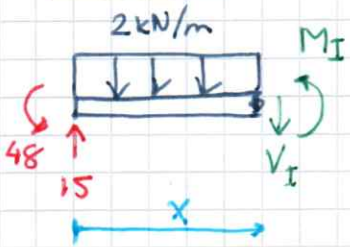
$\uparrow A_y - 3 - 12 = 0 \quad A_y = 15\ kN$

$\curvearrowleft_A \; M_A - 3\cdot 4 - 12\cdot 3 = 0$

$M_A = 48\ kNm$

$q = \frac{12}{6} = 2\ kN/m$

Snitt I $0 \le x \le 4$

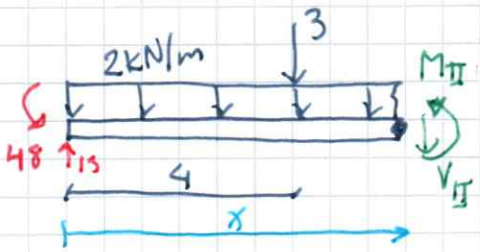


$\downarrow V_I(x) + 2\cdot x - 15 = 0 \quad V_I(x) = -2x + 15\ kN$

$\curvearrowleft_{\bullet} M_I(x) + 48 + \underbrace{2\cdot x}_{Q}\cdot\underbrace{\frac{x}{2}}_{\text{hävarm}} - 15x = 0$

$M_I(x) = -x^2 + 15x - 48\ kNm$

Snitt II $4 \le x \le 6$



$\downarrow V_{II} + 3 - 15 + 2\cdot x = 0$

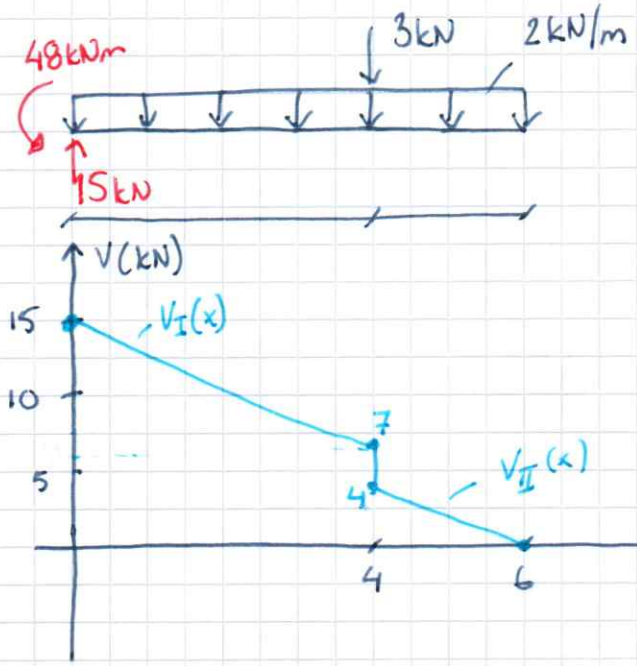
$V_{II} = -2x + 12\ kN$

$\curvearrowleft_{\bullet} M_{II}(x) + 3(x-4) - 15x + 2\cdot\frac{x^2}{2} + 48 = 0$

$M_{II}(x) + 3x - 12 - 15x + x^2 + 48 = 0$

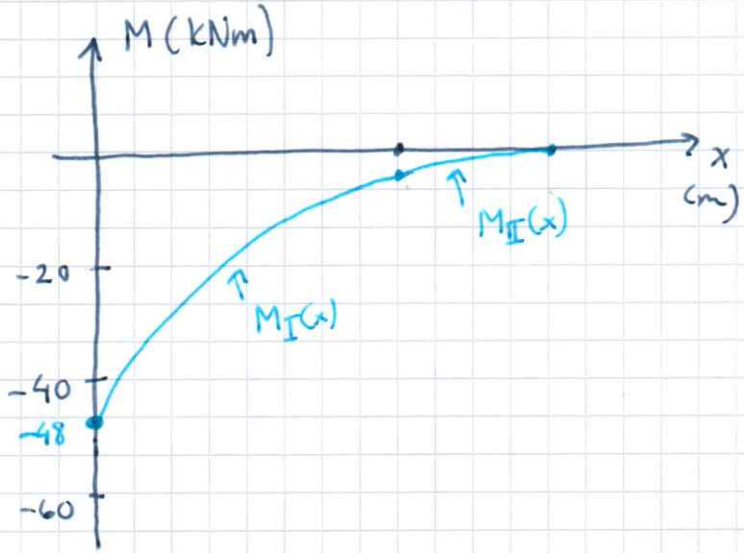
$M_{II}(x) = -x^2 + 12x - 36\ kNm$

6.11 forts.



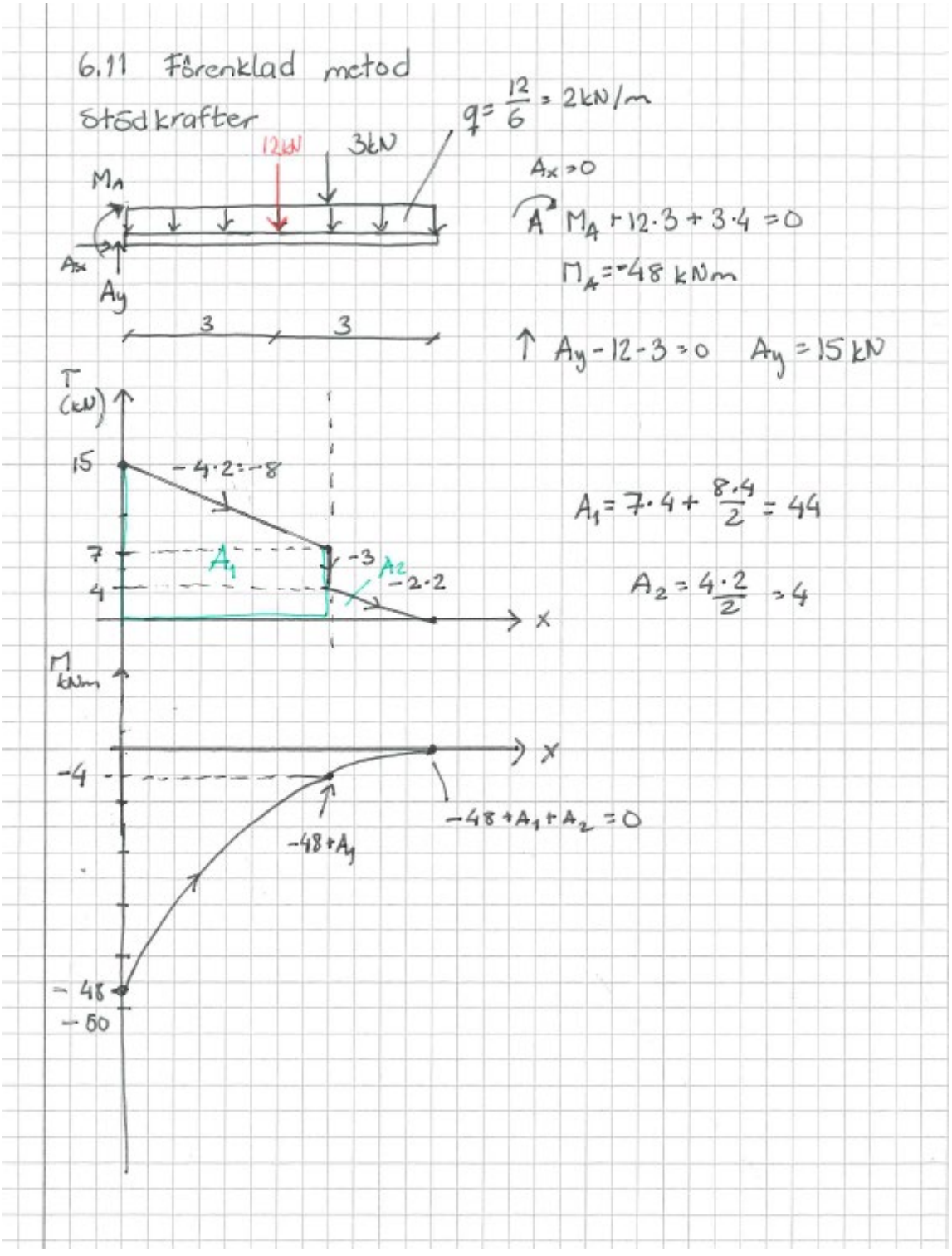
$$\begin{cases} V_I(x) = -2x + 15 \text{ kN} \\ V_I(0) = 15 \text{ kN} \\ V_I(4) = 7 \text{ kN}. \end{cases}$$

$$\begin{cases} V_{II}(x) = -2x + 12 \text{ kN} \\ V_{II}(4) = 4 \text{ kN} \\ V_{II}(6) = 0 \end{cases}$$



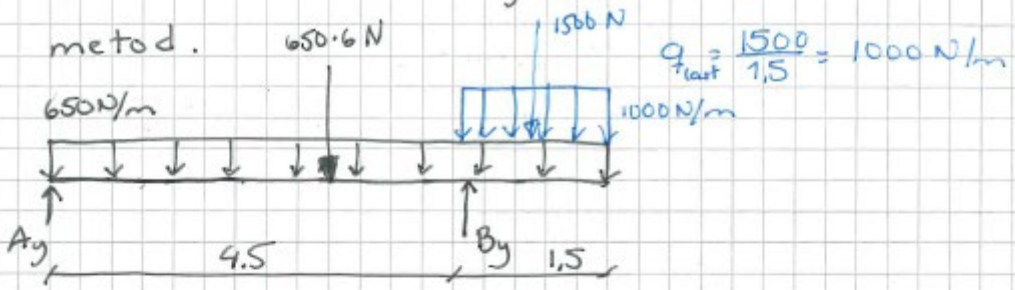
$$\begin{cases} M_I(x) = -x^2 + 15x - 48 \text{ kNm} \\ M_I(0) = -48 \text{ kNm} \\ M_I(4) = -4 \text{ kNm} \end{cases}$$

$$\begin{cases} M_{II}(x) = -x^2 + 12x - 36 \text{ kNm} \\ M_{II}(4) = -4 \text{ kNm} \\ M_{II}(6) = 0 \end{cases}$$


6.11 Förenklad metod
Stödkrafter
12kN
3kN
$q = \frac{12}{6} = 2$ kN/m
M_A
A_x
A_y
3
3
$A_x = 0$
$\curvearrowright A$ $M_A + 12 \cdot 3 + 3 \cdot 4 = 0$
$M_A = -48$ kNm
$\uparrow$ $A_y - 12 - 3 = 0$ $A_y = 15$ kN
T (kN)
15
$-4 \cdot 2 = -8$
7
4
A_1
A_2
-3
$-2 \cdot 2$
x
$A_1 = 7 \cdot 4 + \frac{8 \cdot 4}{2} = 44$
$A_2 = \frac{4 \cdot 2}{2} = 4$
M kNm
x
-4
$-48 + A_y$
$-48 + A_1 + A_2 = 0$
-48
-50

6.12 IPE400 => 66,3 kg/m => 66,3 · 9,81 N/m = 650 N/m

Bestämmer V- & M-diagram med förenklad metod.

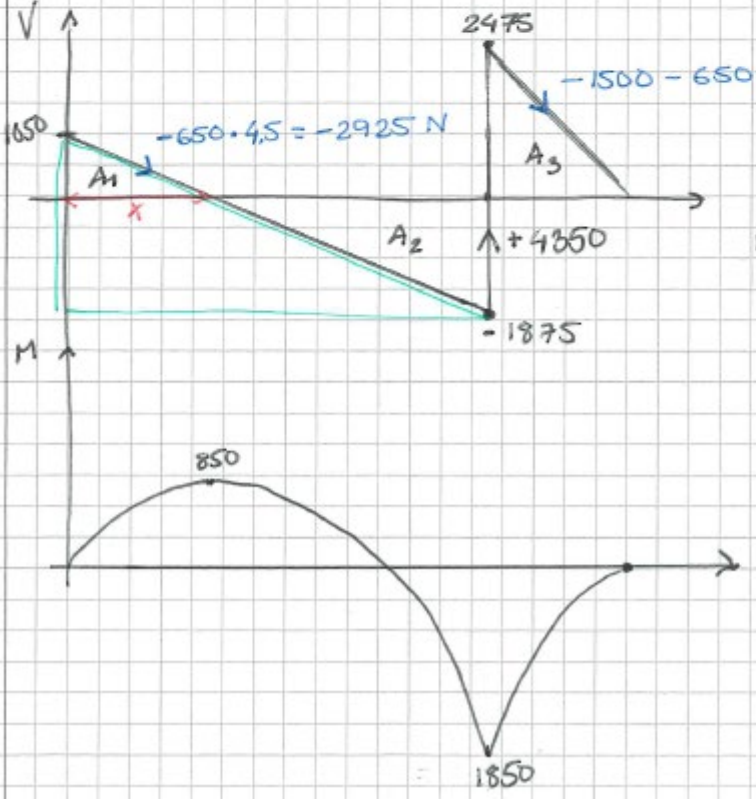


$\curvearrowleft A$ $4,5 \cdot B_y - 650 \cdot 6 \cdot 3 - 1500 \cdot 5,25 = 0$

$B_y = 4350$ N

$\uparrow$ $A_y + B_y - 650 \cdot 6 - 1500 = 0$

$B_y = 1050$ N



Δ-likformighet

$\frac{x}{1050} = \frac{4,5}{2925}$

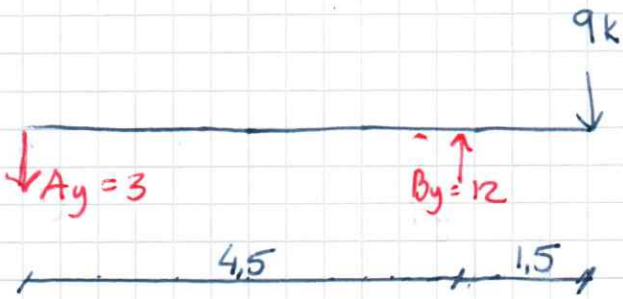
$x = 1,62$ m

$A_1 = \frac{1050 \cdot 1,62}{2} = 850,5$

$A_2 = \frac{1875 \cdot 2,88}{2} = 2700$

$A_3 = \frac{2475 \cdot 1,5}{2} = 1856$

6.13 Sökt: V- o M-diagram – förenklad metod

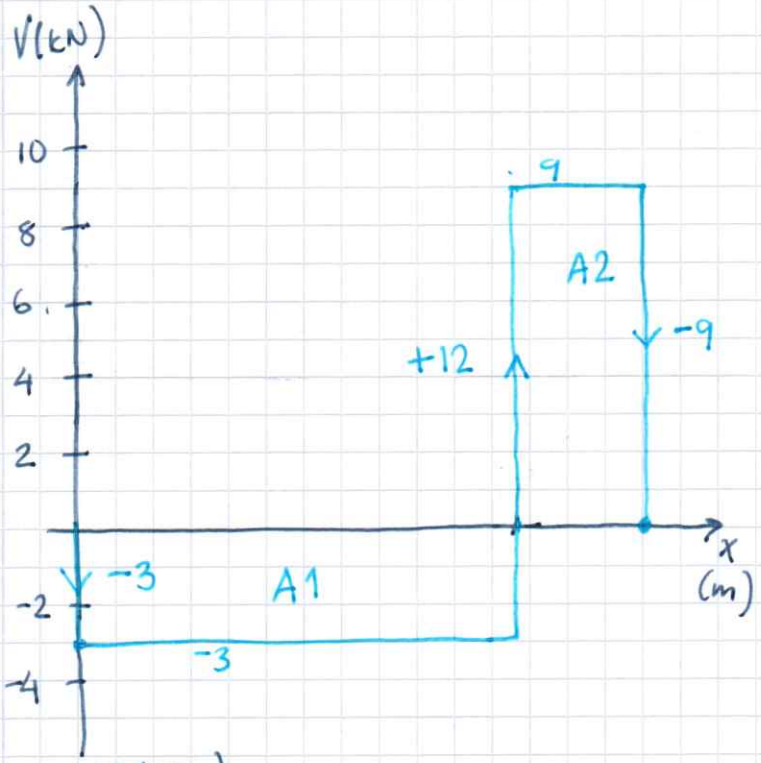


$\curvearrowleft A \quad B_y \cdot 4{,}5 - 9 \cdot 6 = 0$

$B_y = 12\ kN$

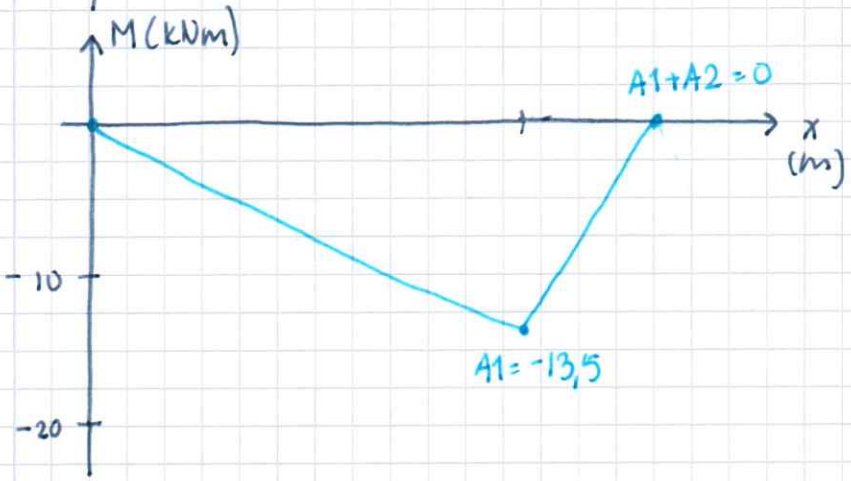
$\uparrow \quad -A_y + B_y - 9 = 0$

$A_y = 12 - 9 = 3\ kN$

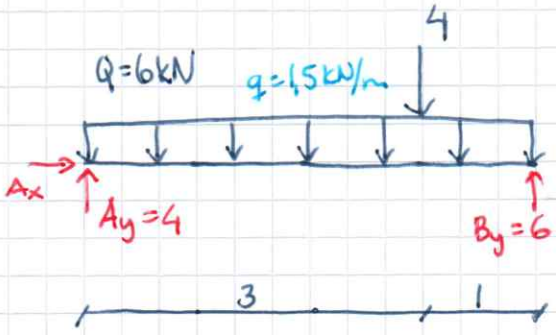


$A1 = -3 \cdot 4{,}5 = -13{,}5$

$A2 = 9 \cdot 1{,}5 = 13{,}5$



6.14 Sökt: V- o M-diagram - förenklade metoden



Stödkrafter

$\rightarrow A_x = 0$

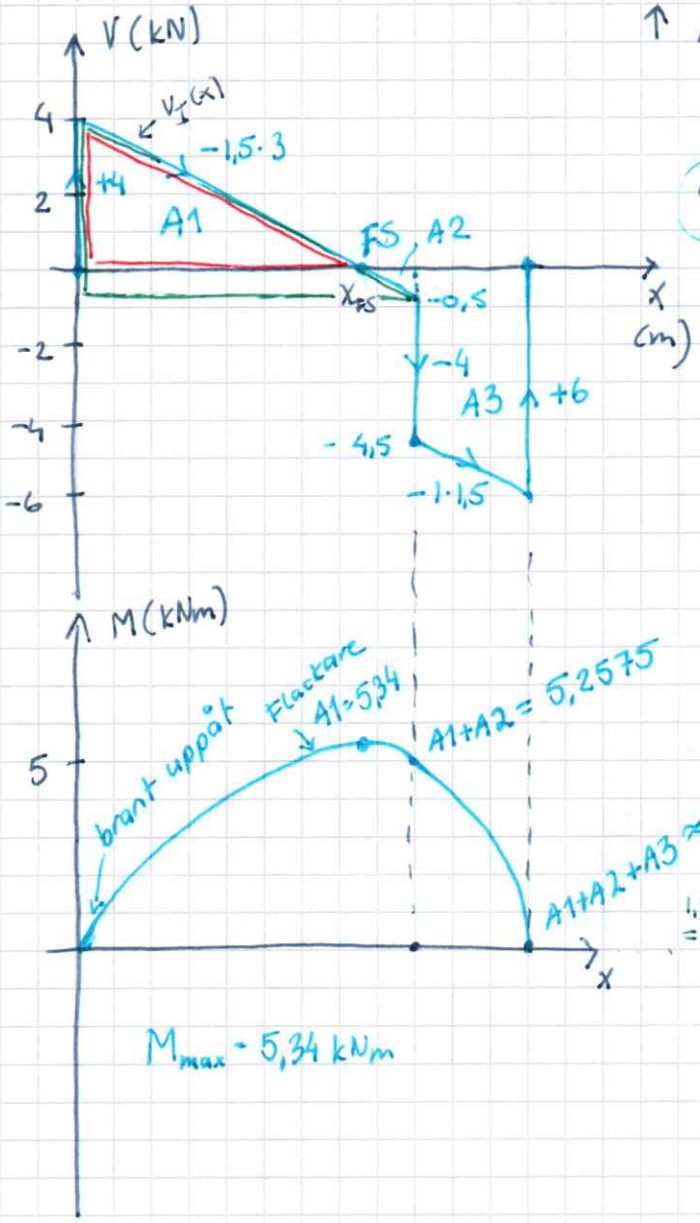
$\circlearrowleft_A \; B_y \cdot 4 - 4 \cdot 3 - 6 \cdot \frac{4}{2} = 0$

$B_y = 6 \text{ kN}$

$\uparrow A_y + B_y - 6 - 4 = 0$

$A_y + 10 - 6 = 4 \text{ kN}$

$q = \frac{6}{4} = 1{,}5 \text{ kN/m}$



Bestäm x_{FS}

Alt. 1

Bestäm $V_I(x)$ $(y = kx + m)$

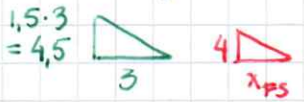
$V_I(x) = -1{,}5x + 4$

$V_I(x) = 0 \Rightarrow x_{FS}$

$-1{,}5 x_{FS} + 4 = 0 \quad x_{FS} = 2{,}67 \text{ m}$

Alt. 2

triangel likformighet



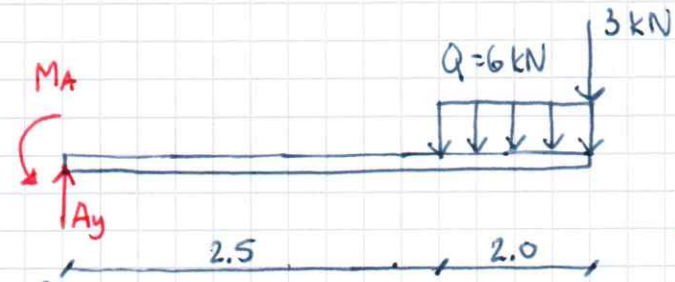
$\frac{4{,}5}{3} = \frac{4}{x_{FS}} \Rightarrow x_{FS} = 2{,}67 \text{ m}$

När x_{FS} är känt kan A1, A2 o A3 bestämmas.

$A1 = \frac{4 \cdot 2{,}67}{2} = 5{,}34 \qquad A2 = -\frac{0{,}5 \cdot (3 - 2{,}67)}{2} = -0{,}0825$

$A3 = -\left(6 \cdot 1 - \frac{1{,}5 \cdot 1}{2}\right) = -5{,}25$

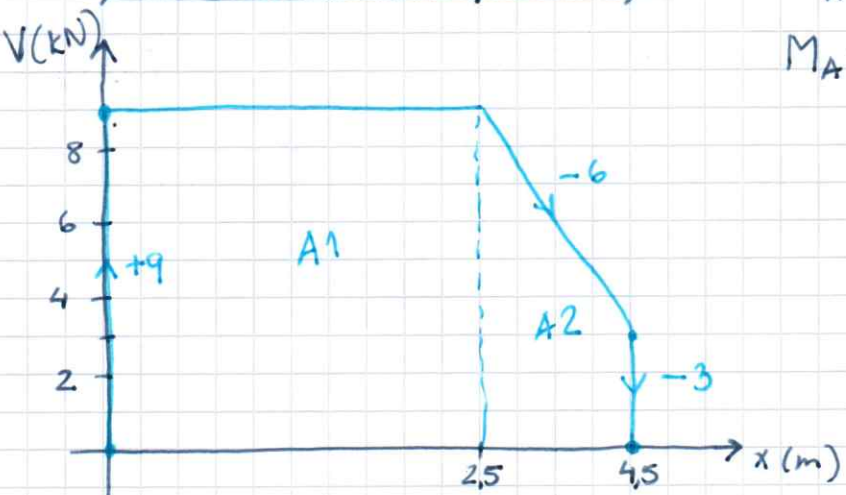
6:15 Sökt: V- & M-diagram Förenklad metod



$\uparrow A_y - 6 - 3 = 0 \quad A_y = 9\,kN$

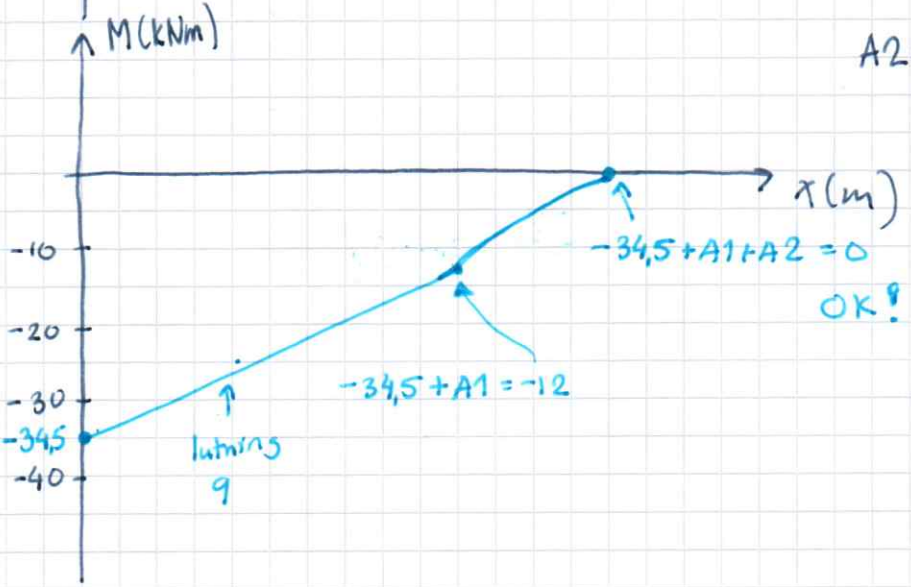
$\circlearrowleft_A \; M_A - 3 \cdot 4{,}5 - 6(2{,}5+1) = 0$

$M_A = 34{,}5\,kNm$



$A1 = 9 \cdot 2{,}5 = 22{,}5$

$A2 = 9 \cdot 2 - \frac{6 \cdot 2}{2} = 12$

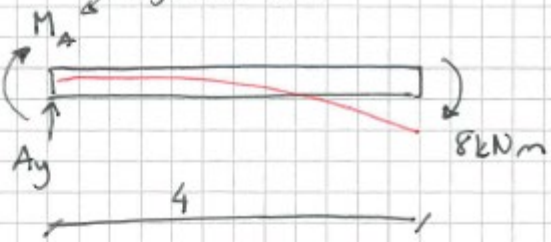


Momentdiagrammet

- $M(4{,}5) = 0$ "fri ände"
- För att hamna på $M(4{,}5) = 0$ måste $M(0) = -34{,}5\,kNm$ då A1 & A2 är positiva.
-  en ledsen böjform ger alltid ett negativt moment.

6:16 reglerna bestämmer medurs

Utan balkens egenvikt

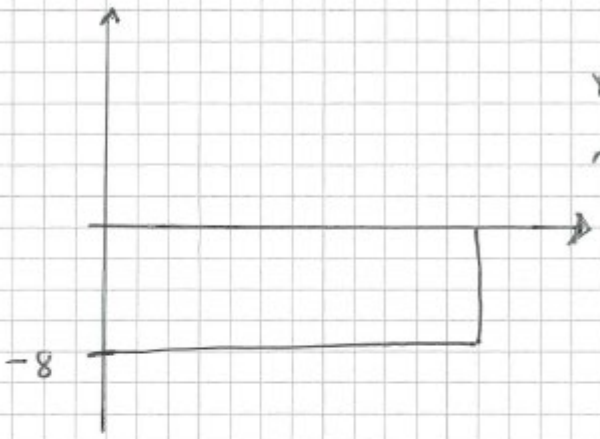


$\circlearrowright_A \quad M_A + 8 = 0$

$M_A = -8\ kNm$

$\uparrow \quad A_y = 0$

$V = 0$



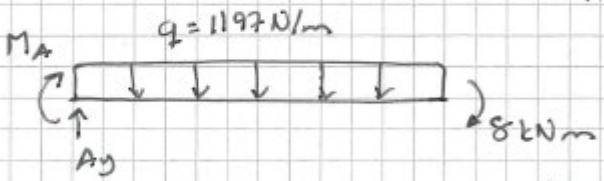
Negativt konstant moment => ledsen balk.

med balkens egenvikt

IPE600 => 122 kg/m

$m = 4 \cdot 122 = 488\ kg$

$q = 122 \cdot 9{,}81 = 1197\ N/m$

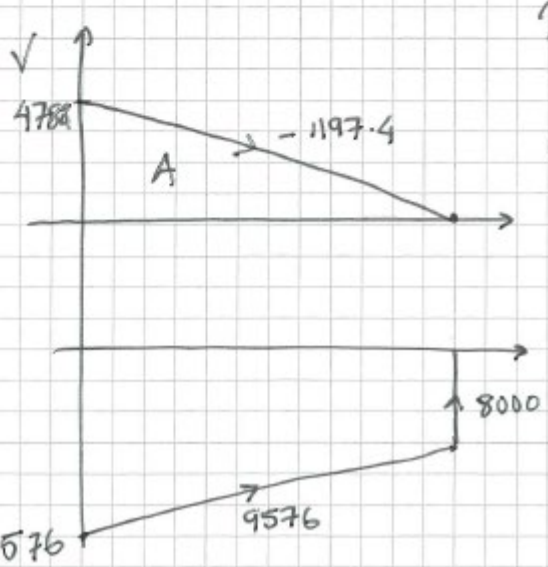


$\circlearrowright_A \quad M_A + 8000 + 1197 \cdot 4 \cdot 2 = 0$

$M_A = -17576\ Nm$

$\uparrow \quad A_y - 1197 \cdot 4 = 0$

$A_y = 4788\ N$



$A = \frac{4788 \cdot 4}{2} = 9576$

6:17

$M_{max} = 3\,kNm$

$$\sigma_{max} = \frac{M_{max}}{W_b} = \frac{3 \cdot 10^3}{1{,}33 \cdot 10^{-3}} = 2{,}25\,MPa$$

Formelsamling $\Rightarrow$ $W = \frac{bh^2}{6}$

$$W = \frac{0{,}2 \cdot 0{,}2^2}{6} = 1{,}33 \cdot 10^{-3}\ m^3$$

6:18 $M_{max} = 15\,kNm$ $\quad n_s = 1{,}5$

tab. $\sigma_s = 210\,MPa$

$$\sigma_{hu} = \frac{\sigma_s}{n_s} = \frac{210}{1{,}5} = 140\,MPa$$

$$\sigma_{max} = \frac{M_{max}}{W_b} \Rightarrow W_b = \frac{M_{max}}{\sigma_{max}} = \frac{15 \cdot 10^3}{140 \cdot 10^6} = 1{,}07 \cdot 10^{-4}\ m^3$$

IPE-balk från tab. enhet cm^3

$1{,}07 \cdot 10^{-4} \cdot 100^3 = 107\,cm^3$

$100\,cm = 1\,m$

Välj: IPE160 med $W_x = 109\,cm^3$

6.19

Bestäm M_{max} genom M-diagram

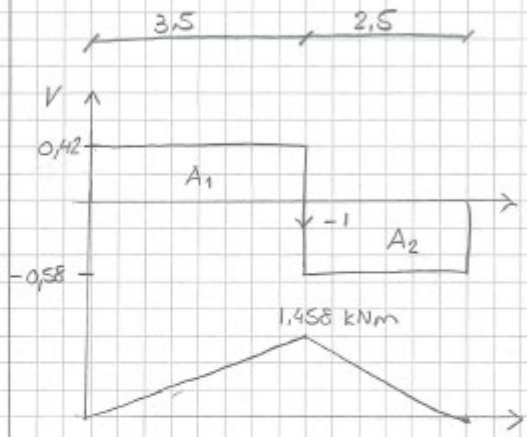


$\curvearrowright A: \quad 1 \cdot 3{,}5 - B_y \cdot 6 = 0$

$B_y = 0{,}58 \text{ kN}$

$\uparrow \quad A_y + B_y - 1 = 0$

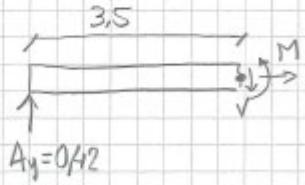
$A_y = 0{,}42$



$A_1 = 0{,}42 \cdot 3{,}5 = 1{,}458$

$M_{max} = 1458 \text{ Nm}$

eller med snitt (inse att maxmoment är under kraften ty störst nedböjning)



$\curvearrowleft \quad M - 0{,}42 \cdot 3{,}5 = 0$

$M = 1{,}458 \text{ kN}$

eller med Karl Björk s.29 Fall 6

$M_{max} = F \cdot \frac{ab}{L}$

$F = 1\text{kN}$, $a = 3{,}5$, $b = 2{,}5$, $L = 6$

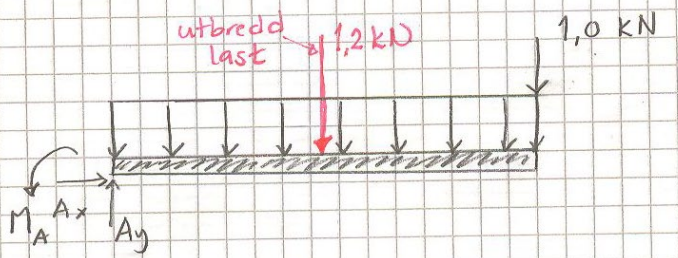
$M_{max} = 1 \cdot \frac{3{,}5 \cdot 2{,}5}{6} = 1{,}458 \text{ kNm}$

IPE 200 $\Rightarrow W_x = 194 \text{cm}^3 = 194 \cdot 10^3 \text{ mm}^3$

$\sigma = \frac{M_{max}}{W_x} = \frac{1458 \cdot 10^3}{194 \cdot 10^3} = 7{,}5 \text{ MPa}$

6.20 Sökt: n_s

Givet: VKR 100×50×5
$I_x = 170\ cm^4$
$W_x = 34\ cm^3 = 34 \cdot 10^{-6}\ m^3$
Stål 1412 $\sigma_s = 250\ MPa$
} Karl Björk FS



$\curvearrowleft A \quad M_A - 1,2 \cdot 0,75 - 1,0 \cdot 1,5 = 0$

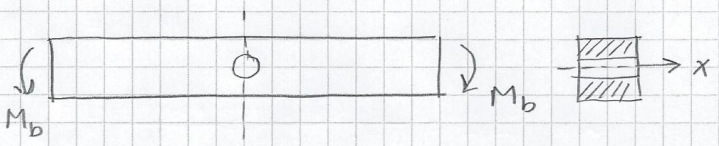
$M_A = 2,4\ kNm$ ↷

$$\sigma_{max} = \frac{M_A}{W_x} = \frac{2,4 \cdot 10^3}{34 \cdot 10^{-6}} = 70,6\ MPa$$

$$n_s = \frac{250}{70,6} = 3,54$$

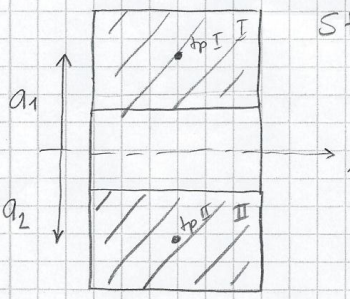
Svar $n_s = 3,54$

6.21



Böjning enl bild ovan => runt x-axel

$M_b = 200\,Nm$



Steiners satz:

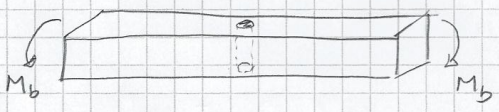
$A_I = 0{,}03 \cdot 0{,}013 = 3{,}9 \cdot 10^{-4}\ m^2 = A_{II}$

$a_1 = 0{,}0135\,m = a_2$

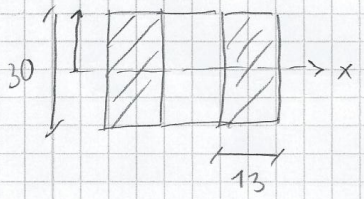
$I_{xI} = \frac{0{,}03 \cdot 0{,}013^3}{12} = 5{,}4925 \cdot 10^{-9}\ m^4$

$I = \left(5{,}4925 \cdot 10^{-9} + 3{,}9 \cdot 10^{-4} \cdot 0{,}0135^2\right) 2 = 1{,}5314 \cdot 10^{-7}\ m^4$

$\sigma = \frac{200}{1{,}5314 \cdot 10^{-7}} \cdot 0{,}02 = \underline{\underline{26\ MPa}}$



Spänning om balken vänds 1/4 varv.



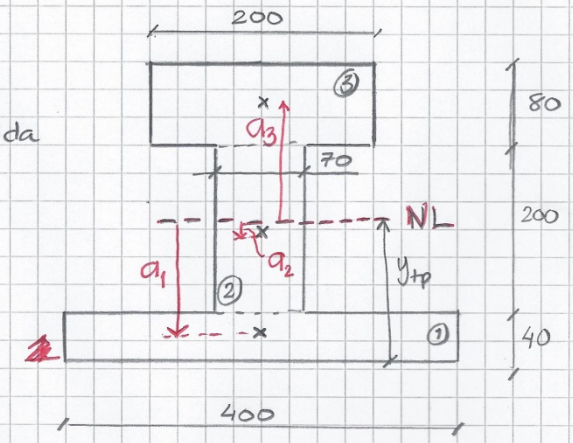
$a_1 = a_2 = 0 \Rightarrow I_{tot} = I_I + I_{II}$

$I = \frac{0{,}013 \cdot 0{,}03^3}{12} \cdot 2 = 5{,}85 \cdot 10^{-8}\ m^4$

$\sigma = \frac{200}{5{,}85 \cdot 10^{-8}} \cdot 0{,}015 = \underline{\underline{51{,}3\ MPa}}$

6,22 Sökt: σ^-_{max} σ^+_{max}

↑ undersida ↑ ovansida



Givet: M = 40 kNm

Bestäm neutrallagret dvs tyngdpunkten y_{tp}

$$y^{tot}_{tp} = \frac{A_1 y^1_{tp} + A_2 y^2_{tp} + A_3 y^3_{tp}}{A_1 + A_2 + A_3}$$

$y^1_{tp} = 20$ $A_1 = 40 \cdot 400$

$y^2_{tp} = 140$ $A_2 =$ 70*200

$y^3_{tp} = 280$ $A_3 = 200 \cdot 80$

$y^{tot}_{tp} = 147\,mm$

Bestäm I_x

$$I_x = I_1 + A_1 a_1^2 + I_2 + A_2 a_2^2 + I_3 + A_3 a_3^2$$

$a_1 = 147 - 20 = 127\,mm$

$a_2 = 147 - 140 = 7\,mm$

$a_3 = 280 - 147 = 133\,mm$

$$I_1 = \frac{400 \cdot 40^3}{12} \qquad I_2 = \frac{70 \cdot 200^3}{12} \qquad I_3 = \frac{200 \cdot 80^3}{12}$$

$$I_x = \frac{400 \cdot 40^3}{12} + 40 \cdot 400 \cdot 127^2 + \frac{70 \cdot 200^3}{12} + 70 \cdot 200 \cdot 7^2 + \frac{200 \cdot 80^3}{12} + 200 \cdot 80 \cdot 133^2$$

$I_x =$ $599 * 10^6\,mm^4$

$$\sigma^+_{max} = \frac{M}{I} \cdot y_{max} = \frac{40 \cdot 10^3 \cdot 10^3}{599 * 10^6} \cdot (320 - 147) =$$ 11,6 MPa drag

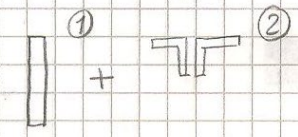
$$\sigma^-_{max} = \frac{M}{I} y_{min} = \frac{40 \cdot 10^3 \cdot 10^3}{599 * 10^6} \cdot 147 =$$ 9,81MPa tryck

6.23

Sökt: σ_{max}^{drag} σ_{max}^{tryck}

90x90x3 Data finns i KB s.62 (A, $tp(c) = I_x$)

Bestäm tyngdpunkten för tvärsnittet



$A_1 = 200 \cdot 12$ $y_{tp1} = 100$

$A_2 = 2 \cdot 15{,}5\ cm^2 = 2 \cdot 1550\ mm^2$

$y_{tp2} = 200 - 25{,}4 = 174{,}6\ mm$

$$y_{tp} = \frac{200 \cdot 12 \cdot 100 + 2 \cdot 1550 \cdot 174{,}6}{200 \cdot 12 + 2 \cdot 1550} = 142\ mm$$

$a_1 = 142 - 100 = 42\ mm$

$a_2 = 174{,}6 - 142 = 32{,}55\ mm$

$I_1 = \frac{12 \cdot 200^3}{12}$

$I_2 = 116\ cm^4 = 116 \cdot 10^4\ mm^4$

$$I_x = \frac{12 \cdot 200^3}{12} + 200 \cdot 12 \cdot 42^2 + 2\left(116 \cdot 10^4 + 1550 \cdot 32{,}55^2\right)$$

$I_x = 17{,}838 \cdot 10^6\ mm^4$

$$\sigma_{max}^{tryck} = -\frac{18 \cdot 10^6}{17{,}838 \cdot 10^6} \cdot 58 = 58{,}4\ MPa$$

r

$$\sigma_{max}^{drag} = \frac{-18 \cdot 10^6}{17{,}838 \cdot 10^6} \cdot 142 = 143\ MPa$$

6.24

Sökt: F_{max}

Givet:

$\sigma_{tillåten} = 120\ MPa$

$d = 20\ mm$

$M_{max} = F_{max} \cdot 0{,}110$

$$\sigma = \frac{M}{W_x} = \frac{F_{max} \cdot 0{,}110}{\frac{\pi \cdot 0{,}02^3}{32}}$$

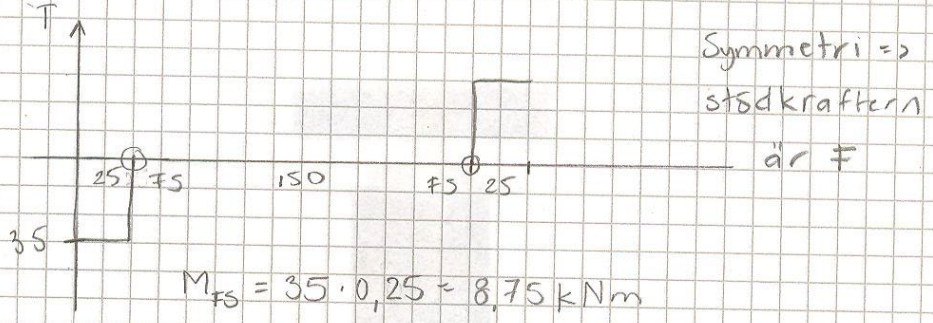
$$F_{max} = \frac{120 \cdot 10^6 \cdot \pi \cdot 0{,}02^3}{0{,}110 \cdot 32} = 856\ N$$

Svar: $F_{max} = 856\ N$

6.25

Sökt: d

Givet: $\sigma_{tillåten} = 80\ MPa$ $F = 35\ kN$



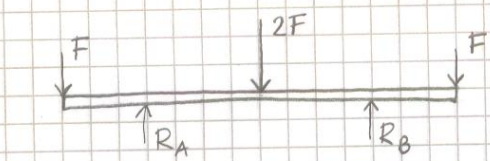
$M_{FS} = 35 \cdot 0{,}25 = 8{,}75\ kNm$

$$\sigma_b = \frac{M_b}{W_b} = \frac{8{,}75 \cdot 10^3}{\frac{\pi d^3}{32}} = 80 \cdot 10^6$$

$$d = \sqrt[3]{\frac{8{,}75 \cdot 10^3 \cdot 32}{80 \cdot 10^6 \cdot \pi}} = 0{,}104\ m$$

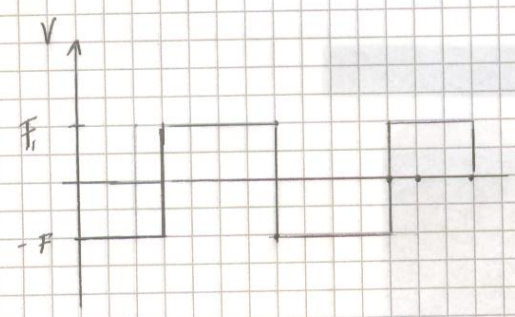
Svar: $d = 104\ mm$

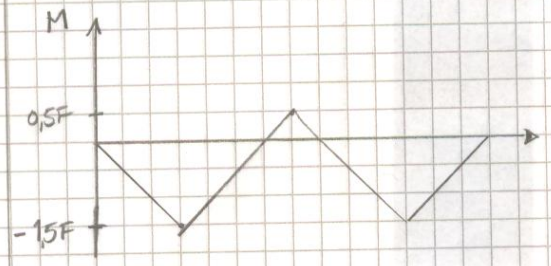
6.26 Sökt: F_{max} $\sigma_{drag,tillåt} = 40\,MPa$ $\sigma_{tryck,tillåt} = 160\,MPa$



$R_A = R_B = 2F$

ty symmetri

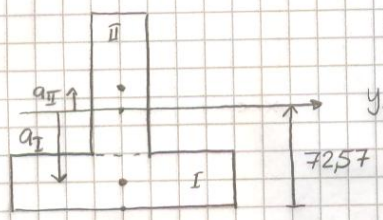




$\Rightarrow |M_{max}| = 1{,}5F$

ovansida => drag

nedansida => tryck



x-axel ges av ytp

$$y_{tp} = \frac{220 \cdot 60 \cdot 30 + 150 \cdot 60 \cdot 135}{220 \cdot 60 + 150 \cdot 60} = 72{,}57\,mm$$

$a_I = 72{,}57 - 30 = 42{,}57\,mm$

$a_{II} = 135 - 72{,}57 = 62{,}43\,mm$

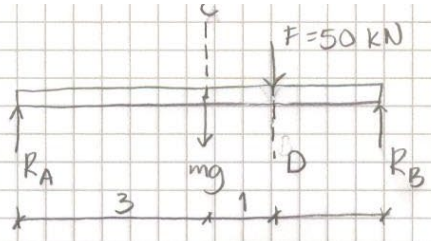
$$I_x = \frac{220 \cdot 60^3}{12} + 220 \cdot 60 \cdot 42{,}57^2 + \frac{60 \cdot 150^3}{12} + 60 \cdot 150 \cdot 62{,}43^2$$

$I_x = 79{,}83 \cdot 10^6\,mm^4$

Drag $\quad 40 = \frac{1{,}5F \cdot 10^3}{79{,}83 \cdot 10^6} \cdot (210 - 72{,}57) \Rightarrow F = 15491\,N$

tryck $\quad 160 = \frac{1{,}5F \cdot 10^3}{79{,}83 \cdot 10^6} \cdot 72{,}57 \Rightarrow F = 117338\,N$

6.27

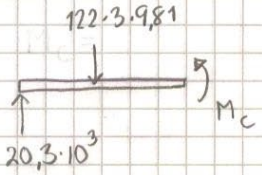


IPE 600
$m = 122\ kg/m$
$t_{liv} = 12\ mm$
$d = 500\ mm$

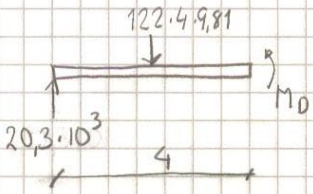
$\curvearrowleft A \quad R_B \cdot 6 - 122 \cdot 6 \cdot 9{,}81 \cdot 3 - 50 \cdot 10^3 \cdot 4 = 0$

$R_B = 36{,}9\ kN$

$\uparrow \quad R_A + R_B - 50 \cdot 10^3 - 122 \cdot 6 \cdot 9{,}81 = 0 \Rightarrow R_A = 20{,}3\ kN$



$M_C = 20{,}3 \cdot 10^3 \cdot 3 - 122 \cdot 3 \cdot 9{,}81 \cdot 1{,}5 = 55{,}5\ kNm$



$M_D = 20{,}3 \cdot 10^3 \cdot 4 - 122 \cdot 4 \cdot 9{,}81 \cdot 2 = 71{,}6\ kNm$

Snitt C

$$I_x = I_o - \frac{d^3 t_{liv}}{12} = 92080 \cdot 10^4 - \frac{12 \cdot 500^3}{12} = 795{,}8 \cdot 10^6\ mm^4$$

$$\sigma_c = \frac{55{,}5 \cdot 10^3 \cdot 10^3}{795{,}8 \cdot 10^6} \cdot 300 = 21\ MPa$$

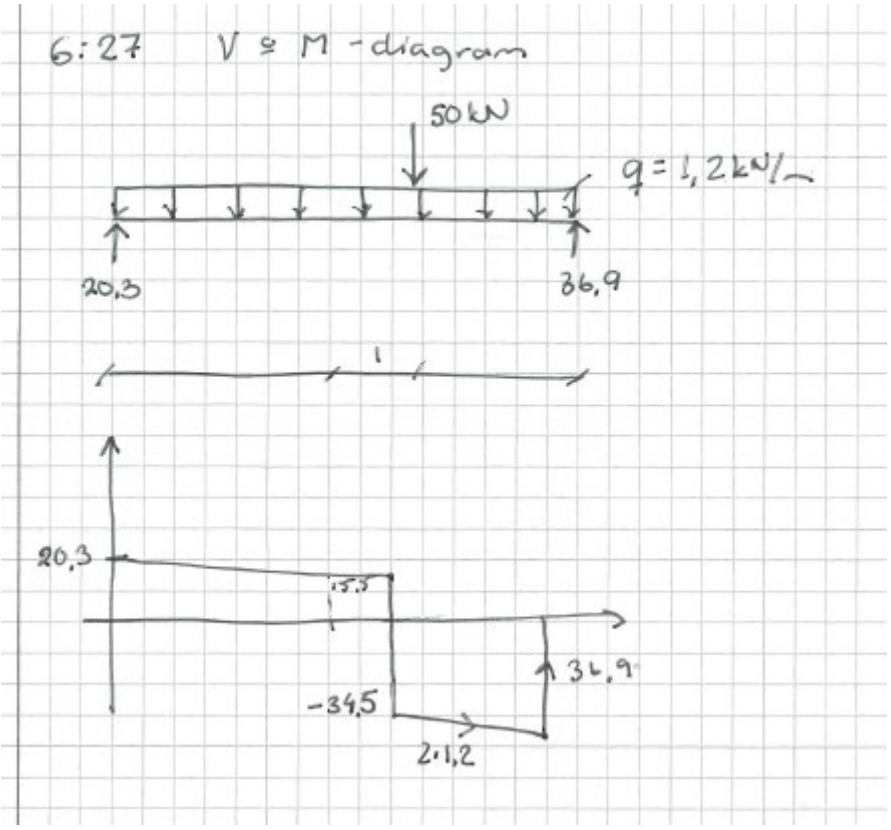
Snitt D

$$\sigma_D = \frac{71{,}6 \cdot 10^3 \cdot 10^3}{3070 \cdot 10^3} = 23{,}3\ MPa$$

Svar: $\sigma_{max} = 23{,}3\ MPa$

Nedan finns tvärkraftsdiagrammet för balken. Tvärkraften = 0 ger Farligt snitt. Areorna kan ge momenten i C och D om man inte vill snitta enligt ovan.

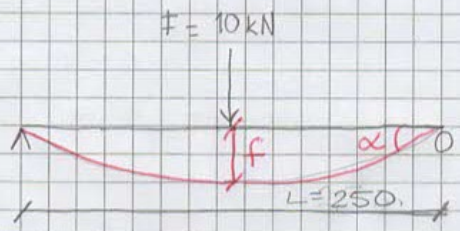
6:27 V & M - diagram



6.28 Stål => E = 210 GPa

d = 35 mm => $I = \frac{\pi \cdot 0{,}035^4}{64} = 7{,}366 \cdot 10^{-8}\ m^4$

upplagd på lager => momentfri inspänt (gångjärn)



Anta: mm

K.B Fall 5

$f = \frac{FL^3}{48EI} = \frac{10 \cdot 10^3 \cdot 0{,}25^3}{48 \cdot 210 \cdot 10^9 \cdot 7{,}366 \cdot 10^{-8}} = 2{,}104 \cdot 10^{-4}\ m = 0{,}21\ mm$

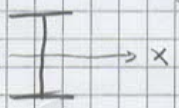
$\alpha = (\varphi_a) = \frac{FL^2}{16EI} = \frac{10 \cdot 10^3 \cdot 0{,}25^2}{16 \cdot 210 \cdot 10^9 \cdot 7{,}366 \cdot 10^{-8}} = 2{,}52 \cdot 10^{-3}\ rad$

6.29

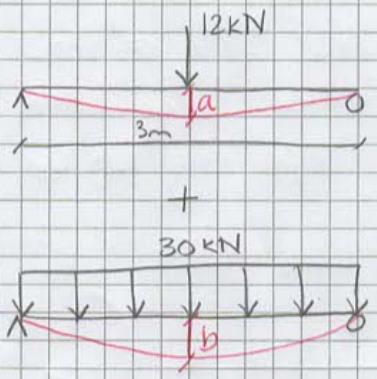
$\delta_{max} = 0{,}0025 \cdot 3 = 7{,}5 \cdot 10^{-3}\ m = 7{,}5\ mm$

Bestäm I_x för att kunna välja lämplig IPE-profil. Anta att böjning sker kring x-axel

$E = 210\ GPa$



Superposition ger max utböjning



Fall 5 (KB)

$a = \frac{FL^3}{48EI} = \frac{12 \cdot 10^3 \cdot 3^3}{48 \cdot 210 \cdot 10^9 \cdot I_x} = \frac{3{,}214 \cdot 10^{-8}}{I_x}$

Fall 10 KB

$b = (f) = \frac{5QL^3}{384EI} = \frac{5 \cdot 30 \cdot 10^3 \cdot 3^3}{384 \cdot 210 \cdot 10^9 \cdot I_x} = \frac{5{,}022 \cdot 10^{-8}}{I_x}$

$\delta_{max} = a + b$

$7{,}5 \cdot 10^{-3} = \frac{3{,}214 \cdot 10^{-8}}{I_x} + \frac{5{,}022 \cdot 10^{-8}}{I_x} = \frac{10^{-8}}{I_x}(3{,}214 + 5{,}022)$

$I_x = \frac{10^{-8}}{7{,}5 \cdot 10^{-3}}(3{,}214 + 5{,}022) = 1{,}0981 \cdot 10^{-5}\ m^4 =$

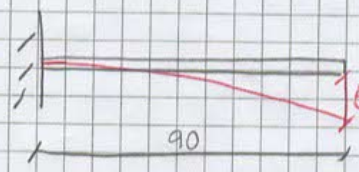
$= 1{,}0981 \cdot 10^{-5} \cdot 100^4 = 1098\ cm^4$

s. 65 (KB) $\Rightarrow$ IPE 180 med $I_x = 1317\ cm^4$

6.30

(mm)

$E = 210\,GPa$



$$I = \frac{20 \cdot 2^3}{12} = 13{,}3\,mm^4 = 13{,}3 \cdot 0{,}001^4 = 1{,}33 \cdot 10^{-11}\,m^4$$

Fall 2 (KB)

$$f = \frac{FL^3}{3EI} \Rightarrow F = \frac{f \cdot 3EI}{L^3} = \frac{0{,}005 \cdot 3 \cdot 210 \cdot 10^9 \cdot 1{,}33 \cdot 10^{-11}}{0{,}09^3}$$

$$F = 57{,}5\,N$$

7.1 Sökt: W_v, τ_{vmax}

Givet: $M_v = 4{,}0\ kNm$

$d = 90\ mm$

$$W_v = \frac{\pi \cdot d^3}{16} = \frac{\pi \cdot 0{,}09^3}{16} = 1{,}431 \cdot 10^{-4}\ m^3$$

$$\tau_{vmax} = \frac{M_v}{W_v} = \frac{4 \cdot 10^3}{1{,}431 \cdot 10^{-4}} = 27{,}9\ MPa$$

7.2 Sökt: τ_{vmax}, ... % lättare med rör

Givet: $D = 60\ mm$

$d = 40\ mm$

$M_v = 800\ Nm$

$$W_{v,rör} = \frac{\pi(D^4 - d^4)}{16D} = \frac{\pi(0{,}06^4 - 0{,}04^4)}{16 \cdot 0{,}06} =$$

$$W_{v,rör} = 3{,}403 \cdot 10^{-5}\ m^3$$

$$\tau_{vmax} = \frac{M_v}{W_{v,rör}} = 23{,}5\ MPa$$

$W_{v,rör} = W_{stång}$

$$\frac{\pi \cdot d_s^3}{16} = 3{,}403 \cdot 10^{-5} \Rightarrow d_s = \sqrt[3]{\frac{3{,}403 \cdot 10^{-5} \cdot 16}{\pi}}$$

$d_s = 56\ mm$

$m_s = L \cdot A_s \cdot \rho \qquad m_{rör} = L \cdot A_{rör} \cdot \rho$

$$\frac{m_{rör}}{m_s} = \frac{A_{rör}}{A_s} = \frac{\pi(D^2 - d^2) \cdot 4}{4 \cdot d_s^2 \cdot \pi} = \frac{60^2 - 40^2}{56^2} = 0{,}64$$

Röret är 36% lättare

7.3 Sökt: M_v, d

Givet: $\tau_{tu} = 25$ MPa

$P = 10$ kW

$n = 1200$ rpm

$$M_v = \frac{P \cdot 30}{\pi \cdot n} = \frac{10 \cdot 10^3 \cdot 30}{\pi \cdot 1200} = 79{,}6 \text{ Nm}$$

$$\tau_{tu} = \frac{M_v}{W_v}$$

$$\frac{\pi d^3}{16} = \frac{M_v}{\tau_{tu}} \Rightarrow d = \sqrt[3]{\frac{M_v \cdot 16}{\tau_{tu} \cdot \pi}}$$

$$d = \sqrt{\frac{79{,}6 \cdot 16}{25 \cdot 10^6 \cdot \pi}} = 0{,}0253$$

$$d = 25{,}3 \text{ mm}$$

7.4 Sökt: τ vid d_i & d_y

Givet: $d_y = 25$ mm

$t = 1{,}5$ mm $\Rightarrow d_i = 22$ mm

$P = 7{,}5$ kW

$n = 900$ rpm

$$M_v = \frac{P \cdot 30}{\pi \cdot n} = \frac{7{,}5 \cdot 10^3 \cdot 30}{\pi \cdot 900} = 79{,}58 \text{ Nm}$$

$$I_v = \frac{\pi (d_y^4 - d_i^4)}{32} = \frac{\pi (0{,}025^4 - 0{,}022^4)}{32} =$$

$$= 1{,}535 \cdot 10^{-8} \text{ m}^4$$

$$\tau_{v,d_i} = \frac{M_v}{I_v} \cdot \frac{d_i}{2} = \frac{79{,}58}{1{,}535 \cdot 10^{-8}} \cdot \frac{0{,}022}{2} = 57 \text{ MPa}$$

$$\tau_{v,d_y} = \frac{79{,}58}{1{,}535 \cdot 10^{-8}} \cdot \frac{0{,}025}{2} = 64{,}8 \text{ MPa}$$

7.5 Sökt M_v, d_y, d_i

Givet. $D = 60\,mm$

$d_y/d_i = 1{,}25 \Rightarrow d_y = 1{,}25 \cdot d_i$

$\tau_{hu} = 90\,MPa$

$$M_v = \tau_{hu} \cdot W_v = 90 \cdot 10^6 \cdot \frac{\pi \cdot 0{,}06^3}{16} = 3817\,Nm$$

Rör

$$W_{v,rör} = \frac{M_v}{\tau_{hu}} = \frac{\pi((1{,}25 d_i)^4 - d_i^4)}{16 \cdot 1{,}25 d_i} =$$

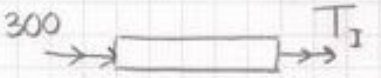
$$= \frac{\pi \cdot d_i^4 (1{,}25^4 - 1)}{16 \cdot 1{,}25 \cdot d_i} = \frac{\pi \cdot d_i^3 (1{,}25^4 - 1)}{20}$$

$$d_i^3 = \frac{20 M_v}{\pi \cdot (1{,}25^4 - 1) \cdot \tau_{hu}} = \frac{20 \cdot 3817}{\pi (1{,}25^4 - 1) \cdot 90 \cdot 10^6}$$

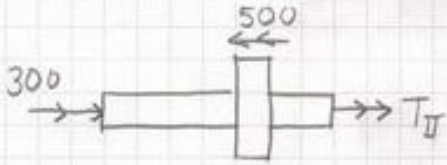
$d_i = 57\,mm \quad d_y = 1{,}25 \cdot 57 = 72\,mm$

7.6 Sökt: T-diagram, $\tau_{v,max}$

Frilägging $d = 40mm$

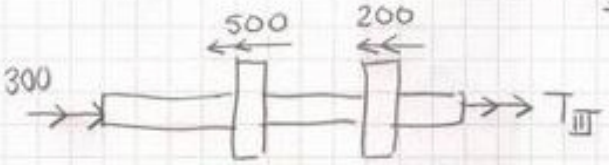


$\rightarrow T_I + 300 = 0 \quad T_I = -300\ Nm$



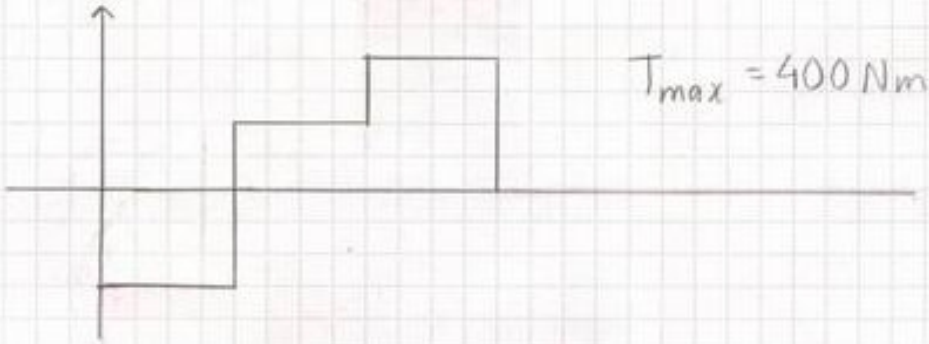
$\rightarrow T_{II} + 300 - 500 = 0$

$T_{II} = 200\ Nm$



$\rightarrow T_{III} + 300 - 500 - 200 = 0 \quad T_{III} = 400\ Nm$





$$\tau_{v,max} = \frac{T_{max}}{W_v} = \frac{400}{\frac{\pi \cdot 0{,}04^3}{16}} = 31{,}8\ MPa$$

7.7 Sökt: $\tau_{v,max}$, θ

Givet: $P = 100\,kW$, $n = 1910\,rpm$

$d = 50\,mm$ $L = 500\,mm$ $G = 80\,GPa$.

$$M_v = \frac{P \cdot 30}{\pi \cdot n} = \frac{100 \cdot 10^3 \cdot 30}{\pi \cdot 1910} = 500\,Nm$$

$$\tau_{v,max} = \frac{M_v}{W_v} = \frac{500}{\frac{\pi \cdot 0{,}05^3}{16}} = 20{,}4\,MPa$$

$$\theta = \frac{M_v \cdot L}{G \cdot I_p} = \frac{500 \cdot 0{,}5}{80 \cdot 10^9 \cdot \frac{\pi \cdot 0{,}05^4}{32}} = 5{,}09 \cdot 10^{-3}\,rad$$

$$I_p = \frac{\pi d^4}{32}$$

$$\theta = 5{,}09 \cdot 10^{-3} \cdot \frac{180}{\pi} = 0{,}29^\circ$$

7.8 Sökt M_v

Givet $d = 350\ mm$ $\quad G = 80\ GPa$

$\tau_{tiu} = 60\ MPa$

$\varphi_{tiu} = 0{,}2°/m$

Sätt axellängd till 1m

Kontrollera om spänning eller förvridning är dimensionerande.

Förvridning

$$\theta = \frac{M_v L}{G \cdot I_v} \Rightarrow M_v = \frac{\theta G I_v}{L}$$

Sätt $L = 1 \Rightarrow \theta = 0{,}2° = 0{,}2 \cdot \frac{\pi}{180}$ rad

$$I_v = \frac{\pi d^4}{32}$$

$$M_v = \frac{0{,}2 \cdot \pi}{180} \cdot 80 \cdot 10^9 \cdot \frac{\pi \cdot 0{,}35^4}{32} = 411\ kNm$$

Spänning

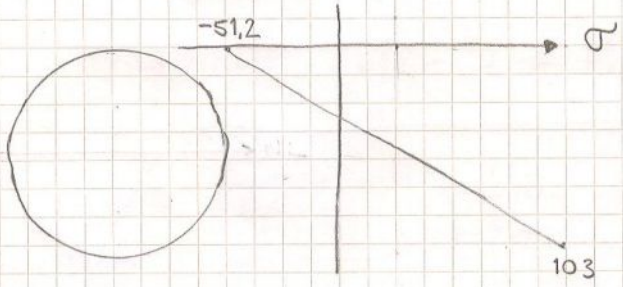
$$M_v = \tau_{tiu} \cdot W_v = 60 \cdot 10^6 \cdot \frac{\pi \cdot 0{,}35^3}{16} = 505\ kNm$$

Svar: $M_{v,max} = 411\ kNm$

8.1 Sökt: spänningsfördelningen σ_{drag}^{max}, σ_{tryck}^{max}

$$\sigma_d = \frac{25 \cdot 10^3 \cdot 4}{\pi \cdot 35^2} = 25{,}98 \text{ MPa}$$

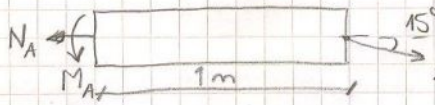
$$\sigma_b = \pm \frac{325 \cdot 10^3 \cdot 32}{\pi\, 35^3} = \pm 77{,}21 \text{ MPa} \qquad W_b = \frac{\pi d^3}{32}$$



$$\sigma_{drag}^{max} = 25{,}98 + 77{,}21 = 103 \text{ MPa}$$

$$\sigma_{tryck}^{max} = 25{,}98 - 77{,}21 = -51{,}23 \text{ MPa}$$

8.2 Sökt: σ_{drag}^{max}, σ_{tryck}^{max}



$\leftarrow \quad N_A - 20\cos 15^\circ = 0 \Rightarrow N_A = 19{,}32 \text{ kN}$

$\curvearrowleft_A \quad M_A - 1 \cdot 20 \sin 15^\circ = 0 \Rightarrow M_A = 5{,}18 \text{ kNm}$

IPE 160 $\quad A = 20{,}1 \text{ cm}^2 = 20{,}1 \cdot 10^{-4} \text{ m}^2$

$W_x = 109 \text{ cm}^3 = 109 \cdot 10^{-6} \text{ m}^3$

$$\sigma_d = \frac{19{,}32 \cdot 10^3}{20{,}1 \cdot 10^{-4}} = 9{,}61 \text{ MPa}$$

$$\sigma_b = \pm \frac{5{,}18 \cdot 10^3}{109 \cdot 10^{-6}} = \pm 47{,}52 \text{ MPa}$$

$\sigma_{drag}^{max} = 9{,}61 + 47{,}52 = 57{,}13 \text{ MPa}$ ovansida

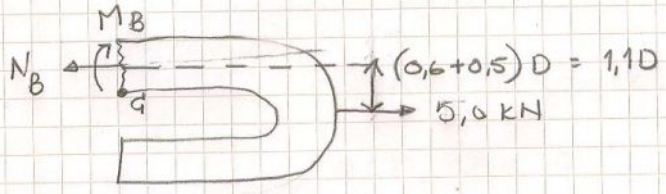
$\sigma_{tryck}^{max} = 9{,}61 - 47{,}52 = -37{,}91 \text{ MPa}$ undersida

8,3 Sökt: D då länken håller om även A-A brister

Givet: $F = 5{,}0\ kN$ $\sigma_s = 300\ MPa$ $n = 3$

$\sigma_{tillåt} = \frac{300}{3} = 100\ MPa$

Frilägging - efter A-A brustit



$\leftarrow \quad N_B = 5{,}0\ kN$

$\circlearrowleft B \quad M_B - 5{,}0 \cdot 1{,}1D = 0 \quad M_B = 5{,}5D\ kNm$

$\sigma_d = \frac{5 \cdot 10^3 \cdot 4}{\pi D^2} \qquad \sigma_b = \frac{5{,}5D \cdot 10^3 \cdot 32}{\pi D^3} = \frac{176 \cdot 10^3}{\pi D^2}$

Maxspänning i G (drag + drag)

$$\frac{5 \cdot 10^3 \cdot 4}{\pi D^2} + \frac{176 \cdot 10^3}{\pi D^2} = 100$$

$$\frac{196 \cdot 10^3}{\pi D^2} = 100 \Rightarrow D = \sqrt{\frac{196 \cdot 10^3}{100\pi}} = 25\ mm$$

Svar: $D = 25\ mm$

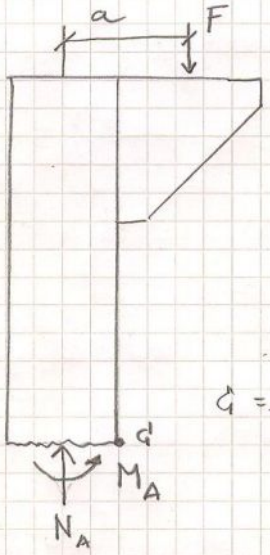
8.4 Sökt: F_{max}

Givet: $a = 200\,mm$

$\sigma_{hu} = 80\,MPa$

IPE 200 => $A = 28{,}5\,cm^2 = 28{,}5 \cdot 10^{-4}\,m^2$

$W_x = 194\,cm^3 = 194 \cdot 10^{-6}\,m^3$



$\uparrow N_A = F$ (tryck)

$\circlearrowleft_A \; M_A - F \cdot a = 0$

$M_A = F \cdot a = 0{,}2\,F$

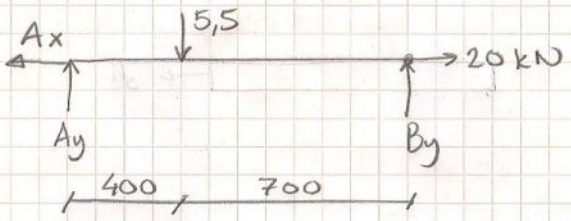
G => tryck + tryck => σ_{max}
(N_A, M_A)

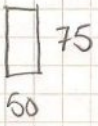
$$\sigma_d = \frac{F}{28{,}5 \cdot 10^{-4}} \qquad \sigma_b = \frac{0{,}2\,F}{194 \cdot 10^{-6}}$$

$$80 \cdot 10^6 = F\left(\frac{1}{28{,}5 \cdot 10^{-4}} + \frac{0{,}2}{194 \cdot 10^{-6}}\right)$$

$$F = 57{,}9\,kN$$

8.5 Sökt: σ_{max}

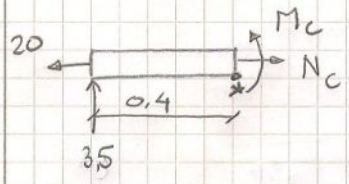




$\curvearrowright A$ $\quad 5,5 \cdot 400 - By \cdot 1100 = 0 \quad \Rightarrow \quad By = 2\ kN$

$\uparrow \quad Ay + By - 5,5 = 0 \quad \Rightarrow \quad Ay = 3,5\ kN$

$\rightarrow \quad 20 - Ax = 0 \quad \Rightarrow \quad Ax = 20\ kN$



$N_c = 20\ kN$

$\curvearrowright C \quad M_c - 3,5 \cdot 0,4 = 0$

$M_c = 1,4\ kNm$

$*$ => maxspänning drag N_c + drag M_c

$$\sigma_{max} = \frac{20 \cdot 10^3}{50 \cdot 75} + \frac{1,4 \cdot 10^{6\ Nmm} \cdot 6}{50 \cdot 75^2} = 5,33 + 29,9 = 35,2\ MPa$$

Svar: 35,2 MPa

8,6 Sökt: σ_e Givet: axel $d = 20\,mm$

Axel påverkas av böjning och vridning samtidigt.

$$\tau_v = \frac{M_v}{W_v} = \frac{44 \cdot 16}{\pi \cdot 0{,}02^3} = 28\,MPa$$

$M_v = 400 \cdot 0{,}110 = 44\,Nm$

$W_v = \frac{\pi d^3}{16}$

$$\sigma_b = \frac{M_b}{W_b} = \frac{400 \cdot 0{,}045 \cdot 32}{\pi \cdot 0{,}02^3} = 22{,}4\,MPa$$

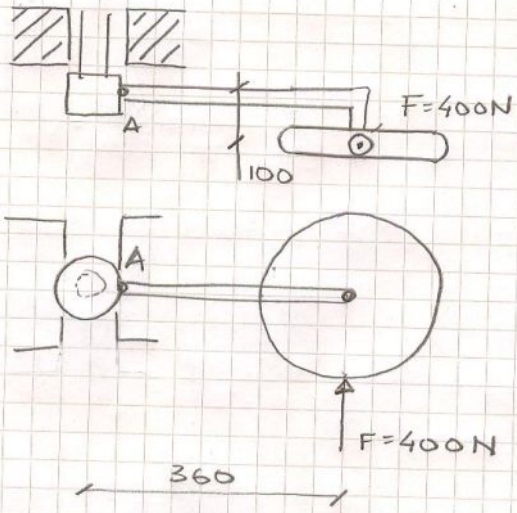
$M_b = 400 \cdot 0{,}045 =$

$W_b = \frac{\pi d^3}{32}$

$\sigma_e = \sqrt{\sigma^2 + 3\tau^2}$

$$\sigma_e = \sqrt{22{,}9^2 + 3 \cdot 28^2} = 53{,}6\,MPa$$

8.7 Sökt: d Givet: $\sigma_{tu} = 80\ MPa$



A påverkas av vridning + böjning

$$\tau_v = \frac{M_v}{W_v} = \frac{400 \cdot 0{,}1 \cdot 16}{\pi d^3} = \frac{203{,}72}{d^3}$$

$$\sigma_b = \frac{M_b}{W_b} = \frac{400 \cdot 0{,}36 \cdot 32}{\pi d^3} = \frac{1466{,}8}{d^3}$$

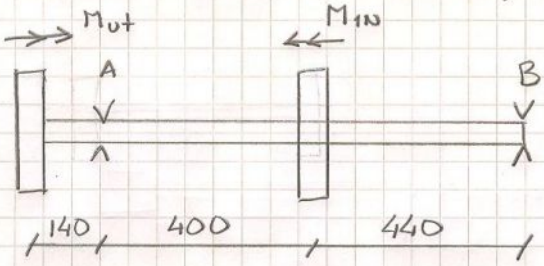
3. $\sigma_e^2 = \sigma_b^2 + 3\tau^2 = \sigma_{tu}^2$

$$\frac{1466{,}8^2}{d^6} + \frac{203{,}72^2}{d^6} = (80 \cdot 10^6)^2$$

$$1466{,}8^2 + 203{,}72^2 = (80 \cdot 10^6)^2 \cdot d^6$$

$d = 0{,}0264\ mm$ Väljer: $d = 27\ mm$

8.8 Sökt: säkerhetsfaktor, n.

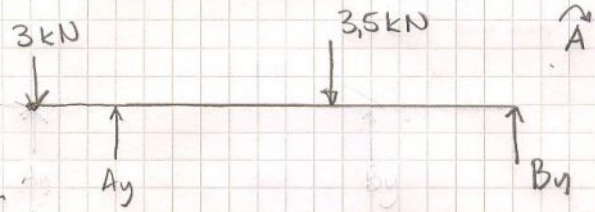


$M_V = 9{,}55 \cdot \frac{P}{n}$

$d = 50\,mm$

$$M_{IN} = M_{UT} = 9{,}55 \cdot \frac{15 \cdot 10^3}{120} = 1194\ Nm$$

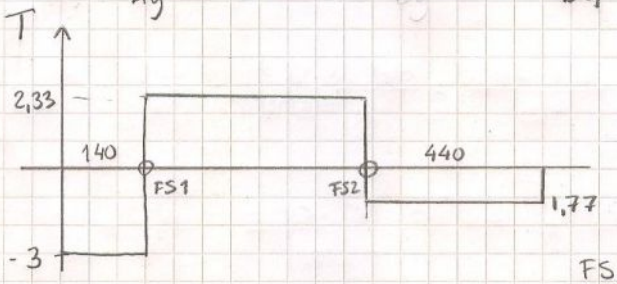
$$\sigma_V = \frac{M_V}{W_V} = \frac{1194 \cdot 16}{\pi \cdot 0{,}050^3} = 48{,}6\ MPa$$



$\curvearrowright A$ $3{,}5 \cdot 400 - B_y \cdot 840 - 3 \cdot 140 = 0$

$B_y = 1{,}17\ kN$

$A_y = 5{,}33\ kN$



FS1 => $M_1 = 0{,}14 \cdot 3 = 420\ Nm$

FS2 => $M_2 = 0{,}44 \cdot 1{,}77 = 779\ Nm$

$$\sigma_b = \frac{M_2}{W_b} = \frac{779 \cdot 32}{\pi \cdot 0{,}050^3} = 63{,}46\ MPa$$

$$\sigma_e = \sqrt{63{,}46^2 + 3 \cdot 48{,}6^2} = 105\ MPa$$

$$n = \frac{280}{105} = 2{,}66$$

9:1	KNÄCKNING

$$\lambda = \frac{l_k}{i_{min}} \qquad l_k = l = 500 \text{ mm} \quad (\text{Euler 2})$$

$$i_{min} = \frac{D}{A} \quad (TS) \qquad i_{min} = \frac{18,0}{4} = 4,50 \text{ mm}$$

$$\lambda = \frac{500}{4,50} = 111$$

$\lambda > \lambda_0$ (105) ger elastisk knäckning

SVAR: Elastisk knäckning föreligger.

9:2	KNÄCKNING

Beräkning av slankhetstal:

$$\lambda = \frac{h_k}{i_{min}} \qquad h_k = 2 \cdot 70 = 140 \text{ mm} \quad \text{(Euler 1)}$$

$$i_{min} = \frac{B}{\sqrt{12}} \qquad i_{min} = \frac{3}{\sqrt{12}} = 0{,}87$$

$$\lambda = \frac{140}{0{,}87} = 162$$

$$\lambda_0 \approx \pi\sqrt{\frac{E}{0{,}85\, R_{el}}} \qquad \lambda_0 = \pi\sqrt{\frac{2{,}1 \cdot 10^5}{0{,}85 \cdot 230}} = 103$$

$\lambda\,(162) > \lambda_0\,(103)$ ger elastisk knäckning.

Beräkning av tillåten kraft:

$$\sigma_k = \frac{\pi^2 E}{\lambda^2} \qquad \sigma_k = \frac{\pi^2 \cdot 2{,}1 \cdot 10^5}{162^2} = 79{,}3 \text{ MPa}$$

$$F_k = \sigma_k \cdot A \qquad F_k = 196 \cdot 20 \cdot 3 = 4{,}76 \cdot 10^3 \text{ N}$$

$$F_{till} = \frac{F_k}{n_k} \qquad F_{till} = \frac{4{,}76 \cdot 10^3}{6} = 7{,}93 \cdot 10^2 \text{ N} = 793 \text{ N}$$

SVAR: $F_{till} = 793$ N

9:3	KNÄCKNING

Beräkning av slankhetstal:

$$\lambda = \frac{L_k}{i_{min}}$$

$L_k = L = 75$ mm (Euler 2)

$$i_{min} = \frac{H}{\sqrt{12}} \qquad i_{min} = \frac{3{,}0}{\sqrt{12}} = 0{,}866$$

$$\lambda = \frac{75}{0{,}866} = 86{,}6$$

$\lambda < \lambda_0 (105)$ ger oelastisk knäckning och Tetmajers samband (TS)

$\sigma_k = 310 - 1{,}14 \cdot 86{,}6 = 211$ MPa $< R_c$ (220 MPa)

$F_k = \sigma_k \cdot A \qquad F_k = 2{,}11 \cdot 3{,}0^2 = 1{,}90 \cdot 10^3$ N $= 1{,}90$ kN

SVAR: $F_k = 1{,}90$ kN

9:4	KNÄCKNING

Antag elastisk knäckning ($\lambda > \lambda_0$)

$$F_k = \frac{\pi^2 E \cdot I_{min}}{l_k^2}$$

$F_k = n_k \cdot F_{till}$ $\quad F_k = 4 \cdot 90 \cdot 10^3 = 3{,}6 \cdot 10^5$ N

$l_k = l = 1200$ mm (Euler 2)

$$3{,}6 \cdot 10^5 = \frac{\pi^2 \cdot 2{,}1 \cdot 10^5 \cdot I_{erf}}{1200^2} \qquad I_{erf} = 2{,}50 \cdot 10^5 \text{ mm}^4$$

$$I = \frac{\pi D^4}{64} \qquad 2{,}50 \cdot 10^5 = \frac{\pi \cdot D_{erf}^4}{64} \qquad D_{erf} = 47{,}5 \text{ mm}$$

$D = 48$ mm (preferensserie 2)

Kontroll av slankhetstal:

$$\lambda = \frac{l_k}{i_{min}} \qquad i_{min} = \frac{D}{4} \qquad i_{min} = \frac{48}{4} = 12 \text{ mm}$$

$$\lambda = \frac{1200}{12} = 100 \qquad \lambda > \lambda_0 \ (90) \qquad \text{OK!}$$

SVAR: $D = 48$ mm ($D_{erf} = 47{,}5$ mm)

9:5	KNÄCKNING

Antag elastisk knäckning

$$F_k = \frac{\pi^2 E I_{min}}{h_k^2} \qquad F_k = 4 \cdot 75 \cdot 10^3 = 3{,}0 \cdot 10^5 \text{ N}$$

$$h_k = L = 4 \cdot 10^3 \text{ mm}$$

$$3{,}0 \cdot 10^5 = \frac{\pi^2 \cdot 2{,}1 \cdot 10^5 \cdot I_{erf}}{(4 \cdot 10^3)^2} \qquad I_{erf} = 2{,}32 \cdot 10^6 \text{ mm}^4$$

per stång $\frac{2{,}32 \cdot 10^6}{2} = 1{,}16 \cdot 10^6 \text{ mm}^4$

ger IPE 100 ($I_x = 1{,}71 \cdot 10^6 \text{ mm}^4$) (TS)

Kontroll av slankhetstal:

$$\lambda = \frac{h_k}{i_{min}} \qquad i_{min} = 40{,}7 \text{ mm (TS)}$$

$$\lambda = \frac{4{,}0 \cdot 10^3}{40{,}7} = 98 < \lambda_0 (105) \text{ ger oelastisk knäckning}$$

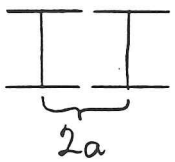
Prova med IPE 100:

$$\sigma_k = 310 - 1{,}14\,\lambda \qquad \sigma_k = 310 - 1{,}14 \cdot 98 = 198 \text{ MPa}$$

$$F_k = \sigma_k \cdot A \qquad A = 1030 \text{ mm}^2$$

$$F_k = 198 \cdot 2 \cdot 1030 = 4{,}08 \cdot 10^5 \text{ N} = 408 \text{ kN}$$

$$F_{till} = \frac{F_k}{n_k} \qquad F_{till} = \frac{408}{4} = 102 \text{ kN} > F\,(75 \text{ kN}) \text{ Ok!}$$

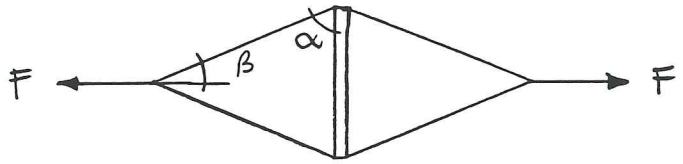


$$I_x = I_y \qquad 2 \cdot 171 = 2\,(15{,}9 + 10{,}3\,a^2)$$

$$a = 3{,}88 \text{ cm} \qquad 2a = 7{,}76 \text{ cm}$$

SVAR: Lämplig profil är IPE 100
Centrumavstånd mellan balkliven 7,76 cm

9:6 KNÄCKNING



$\tan\beta = \frac{1,0}{2,5}$ $\quad \beta = 21,8°$ $\quad \alpha = 90 - 21,8° = 68,2°$

$\lambda = \frac{h_k}{i_{min}}$ $\quad h_k = L = 2000$ mm (Euler 2)

$i_{min} = \frac{D}{4}$ $\quad i_{min} = \frac{50}{4} = 12,5$ mm

$\lambda = \frac{2000}{12,5} = 160 > \lambda_0\ (105)$

Alt I med utnyttjande av diagram

$\lambda = 160$ ger $\sigma_k = 82$ MPa (TS)

$\sigma_{ktill} = \frac{\sigma_k}{h_k}$ $\quad \sigma_{ktill} = \frac{82}{3} = 27,3$ MPa

$F_{till} = \sigma_{ktill} \cdot A$ $\quad F_{till} = 27,3 \cdot \pi \cdot 25^2 = 5,3 \cdot 10^4 N = 53$ kN

Alt II med Eulers elevation

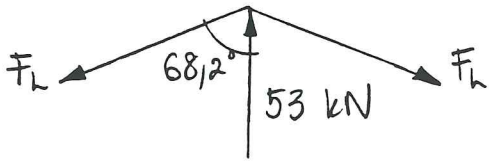
$\sigma_k = \frac{\pi^2 \cdot E}{\lambda^2}$ $\quad \sigma_k = \frac{\pi^2 \cdot 2,1 \cdot 10^5}{160^2} = 81$ MPa

$\sigma_{ktill} = \frac{\sigma_k}{h_k}$ $\quad \sigma_{ktill} = \frac{81}{3} = 27$ MPa

$F_{till} = \sigma_{ktill} \cdot A$ $\quad F_{till} = 27 \cdot \pi \cdot 25^2 = 5,3 \cdot 10^4 N = 53$ kN

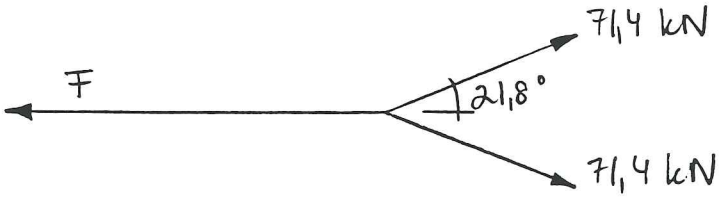
9:6	KNÄCKNING

FORTS.



$\uparrow: \quad -2 \cdot F_L \cdot \cos 68{,}2^\circ + 53 = 0$

$F_L = 71{,}4 \text{ kN}$



$\rightarrow: \quad -F + 2 \cdot 71{,}4 \cdot \cos 21{,}8^\circ = 0$

$F = 133 \text{ kN}$

SVAR: $F = 133$ kN

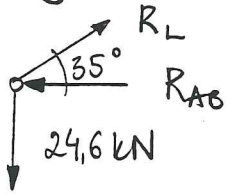
9:7 KNÄCKNING

$G = 5{,}0 \cdot 9{,}82 = 49{,}1$ kN

Av symmetriska skäl fördelas lasten lika på linorna vid A och B.

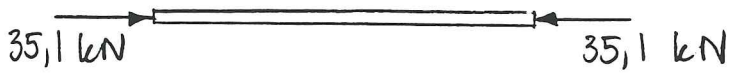
$F_{LA} = F_{LB} = \frac{49{,}1}{2} = 24{,}6$ kN

Beräkning av stångkraften vid A:



$\uparrow: R_L \cdot \sin 35^\circ - 24{,}6 = 0 \quad R_L = 42{,}8$ kN

$\rightarrow: 42{,}8 \cdot \cos 35^\circ - R_{AB} = 0 \quad R_{AB} = 35{,}1$ kN



Beräkning av U-profil:

Antag elastisk knäckning

$F_k = \frac{\pi^2 \cdot E \cdot I_{min}}{L_k^2} \qquad F_k = n_k \cdot F_{till} \qquad F_k = 3 \cdot 35{,}1 = 105$ kN

$L_k = L = 3000$ mm (Euler 2)

$105 \cdot 10^3 = \frac{\pi^2 \cdot 2{,}1 \cdot 10^5 \cdot I_{min}}{3000^2} \qquad I_{min} = 4{,}57 \cdot 10^5 \text{ mm}^3 = 45{,}7 \text{ cm}^4$

ger U-140 ($I_y = 62{,}7$ cm^4)

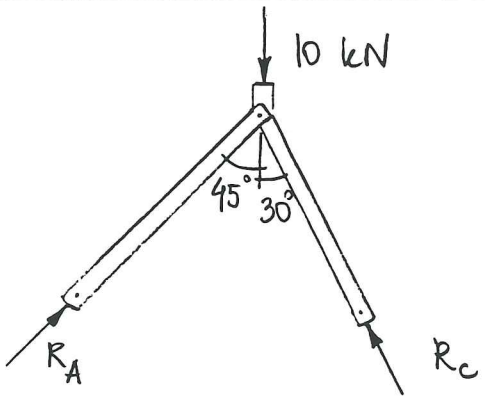
Kontroll av slankhetstal:

$\lambda = \frac{L_k}{i_{min}} \qquad i_{min} = 17{,}5$ mm (TS)

$\lambda = \frac{3000}{17{,}5} = 171 > \lambda_0\,(100)$ ger elastisk knäckning

SVAR: U140

9:8	KNÄCKNING



Beräkning av stångkrafter:

$\rightarrow$: $R_A \cdot \sin 45° - R_C \cdot \sin 30° = 0$ $\quad R_A = 0{,}7071\ R_C$

$\uparrow$: $0{,}7071\ R_C \cdot \cos 45° - 10 + R_C \cdot \cos 30° = 0$

$R_C = 7{,}32$ kN $\quad R_A = 0{,}7071 \cdot 7{,}32 = 5{,}18$ kN

Stången AB:

$l = \frac{500}{\cos 45°} = 70{,}7$ mm $= l_k$ (Euler 2)

Antag elastisk knäckning $F_k = \frac{\pi^2 \cdot E \cdot I_{min}}{l_k^2}$

$F_k = n_k \cdot F$ $\quad F_k = 3{,}0 \cdot 5180 = 1{,}55 \cdot 10^4$ N

$1{,}55 \cdot 10^4 = \frac{\pi^2 \cdot 2{,}1 \cdot 10^5 \cdot I_{min}}{707^2}$ $\quad I_{min} = 3748$ mm^4

$I = \frac{\pi D^4}{64}$ $\quad 3748 = \frac{\pi \cdot D_{erf}^4}{64}$ $\quad D_{erf} = 16{,}6$ mm

Välj $D = 18$ mm (preferensserie 2)

Kontroll av slankhetstal:

$\lambda = \frac{l_k}{i_{min}}$ $\quad i_{min} = \frac{D}{4}$ $\quad i_{min} = \frac{18}{4} = 4{,}5$ mm

$\lambda = \frac{707}{4{,}5} = 157 > \lambda_0\ (105)$ ger elastisk knäckning OK!

9:8	KNÄCKNING

FORTS.

Stången CB:

$h = \frac{300}{\cos 30°} = 346 \text{ mm} = h_k$ (Euler 2)

Antag elastisk knäckning $F_k = \frac{\pi^2 \cdot E \cdot I_{min}}{h_k^2}$

$F_k = n_k \cdot F \qquad F_k = 3{,}0 \cdot 7320 = 2{,}20 \cdot 10^4 \text{ N}$

$2{,}20 \cdot 10^4 = \frac{\pi^2 \cdot 2{,}1 \cdot 10^5 \cdot I_{min}}{346^2} \qquad I_{min} = 1{,}27 \cdot 10^3 \text{ mm}^4$

$I = \frac{\pi D^4}{64} \qquad 1{,}27 \cdot 10^3 = \frac{\pi D_{erf}^4}{64} \qquad D_{erf} = 12{,}7 \text{ mm}$

Välj $D = 14$ mm (preferensserie 2)

Kontroll av slankhetstal:

$\lambda = \frac{L_k}{i_{min}} \qquad i_{min} = \frac{D}{4} \qquad i_{min} = \frac{14}{4} = 3{,}5 \text{ mm}$

$\lambda = \frac{346}{3{,}5} = 99 < \lambda_0 (105)$ ger oelastisk knäckning

Prova med $D = 14$ mm

(TS) $\sigma_k = 310 - 1{,}14\,\lambda \qquad \sigma_k = 310 - 1{,}14 \cdot 99 = 197 \text{ MPa}$

$F_k = \sigma_k \cdot A \qquad F_k = 197 \cdot \pi \cdot 7^2 = 3{,}03 \cdot 10^4 \text{ N}$

$F_k > F_{kerf} (2{,}20 \cdot 10^4 \text{ N})$

Prova med $D = 12$ mm $\qquad i_{min} = \frac{12}{4} = 3{,}0 \text{ mm}$

$\lambda = \frac{346}{3} = 115 > \lambda_0 (105)$ ger elastisk knäckning

$D = 14$ mm

SVAR: $D_{AB} = 18$ mm $\qquad D_{CB} = 14$ mm