Teknologi A
Mekanik
Lösningar

I detta dokument finns lösningsförslag till samtliga uppgifter i boken ”Teknologi A – Mekanik”.

På YouTube kanalen, ”Lösningar Teknologi A Mekanik”, finns inspelade lösningsförslag till många av uppgifterna. Det finns även en spellista med föreläsningar som är anpassad till boken (dock ej kombinerade lastfall och knäckning).

Jag använder mig att Karl Björks ”Formler och Tabeller för Mekanisk konstruktion” när jag löser uppgifterna.

Om du inte har en formelsamling i maskinkonstruktion så rekommenderar jag starkt Karl Björks formelsamling. Denna finns att beställa på bjorksforlag.se

En komplett översikt av kanalen och materialet finns på edume.nu

/Madeleine

YouTube SE https://www.youtube.com/channel/UCZWty6uAUlkab9XyHQIAu9Q

Madeleine Hermann

EduME – Education and Mechanical Engineering

2.1 $\rho = 2{,}65 \cdot 10^3 \text{ kg/m}^3$

$V = 1{,}21 \cdot 2{,}03 \cdot 0{,}545 = 1{,}339 \text{ m}^3$

$m = \rho \cdot V = 2{,}65 \cdot 10^3 \cdot 1{,}339 = 3550 \text{ kg}$

2.2 $L = 547 \text{ mm}$

$d = 40{,}0 \text{ mm}$

$\rho = 7{,}80 \cdot 10^3 \text{ kg/m}^3$

$G = mg = V \cdot \rho \cdot g = 0{,}547 \cdot \frac{0{,}04^2 \cdot \pi}{4} \cdot 7{,}8 \cdot 10^3 \cdot 9{,}81 = 52{,}6 \text{ N}$

2,3 $d_y = 40 \text{mm}$ $d_i = 35 \text{mm}$

a) $m = V \cdot \rho = 0{,}547 \cdot \left(\frac{0{,}04^2 \cdot \pi}{4} - \frac{0{,}035^2 \cdot \pi}{4} \right) \cdot 7{,}8 \cdot 10^3 =$

$= 0{,}547 \frac{(0{,}04^2 - 0{,}035^2)\pi}{4} \cdot 7{,}8 \cdot 10^3 = 1{,}257 \text{ kg}$

b) $\%\text{ viktförändring} = \frac{m_{rör}}{m_{stång}} = \frac{A_{rör} \cdot L \cdot \rho}{A_{stång} \cdot L \cdot \rho} = \frac{A_{rör}}{A_{stång}}$

$\%\text{ viktförändring} = \frac{(40^2 - 35^2) \cdot \pi \cdot 4}{40^2 \cdot \pi \cdot 4} = 0{,}2344$

Röret har 23,4% av stångens massa

Minskningen blir 76,6 %

2,4 IPE 180 =

$L = 5{,}50\,m \qquad m = 18{,}8\,kg/m$

$G = m \cdot L \cdot g = 18{,}8 \cdot 5{,}50 \cdot 9{,}81 = 1\,kN$

2.5 a) $1{,}51 \cdot 10^5\,mm = 1{,}51 \cdot 10^2\,m$

b) $2{,}00 \cdot 10^7\,mm^2 = 2{,}00 \cdot 10^7 \cdot (10^{-3})^2 = 20\,m^2$

c) $3{,}10\,m^3 = 3{,}10 \cdot 100^3 = 3{,}10 \cdot 10^6\,cm^3$

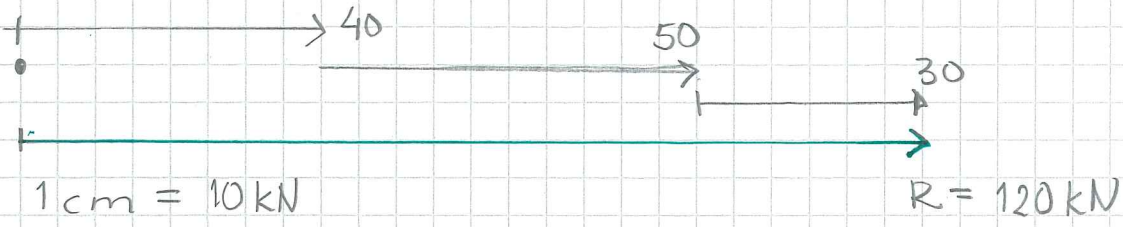
d) $1{,}00\,N/mm^2 = 0{,}001\,kN/mm^2 = 0{,}001 \frac{1}{0{,}001^2} = 1000\,kN/m^2$

2.6 a) $1{,}00 \cdot 10^6\,mm = 1{,}00\,km$

b) $2{,}05 \cdot 10^{11}\,W = 205\,GW$

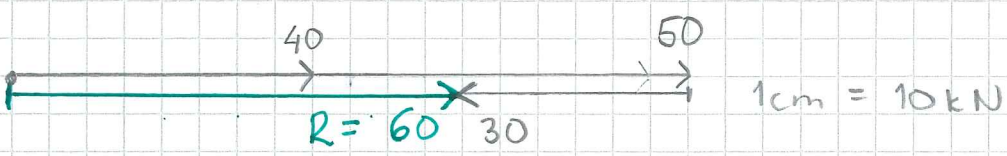
c) $2{,}05 \cdot 10^{-4}\,s = 205\,\mu s = 0{,}205\,ms$

3.1 a)



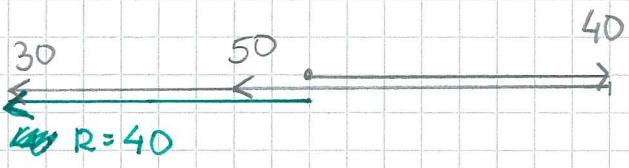
$\rightarrow R_x = 40+50+30 = 120$ kN.

b



$\rightarrow R_x = 40+50-30 = 60$ kN

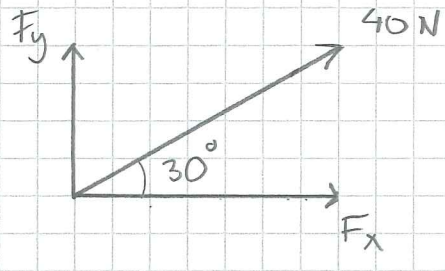
c



$\rightarrow R_x = 40-50-30 = -40$ kN

3.2

a)



1 cm = 10 N

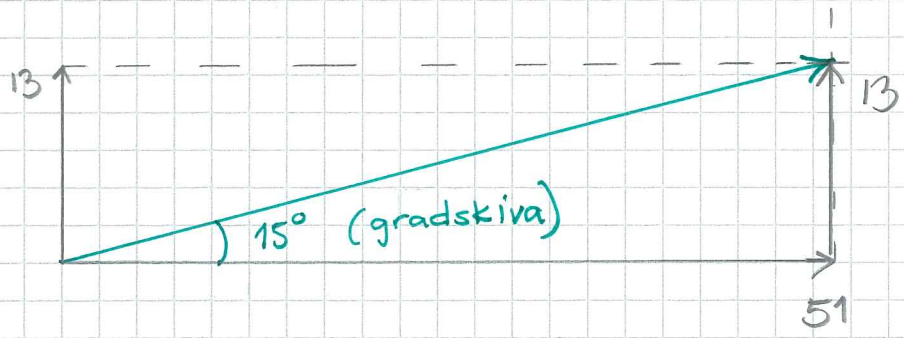
Mätning $\Rightarrow F_x = 35$ N

$F_y = 20$ N

b) $F_x = 40 \cos 30^\circ = 34{,}6$ N

$F_y = 40 \sin 30^\circ = 20$ N

3.3 a) 2 cm = 10 N

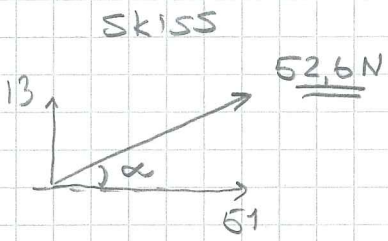


$R = 52{,}5$ N (10,5 cm)

b) $\rightarrow R_x = 51$ N

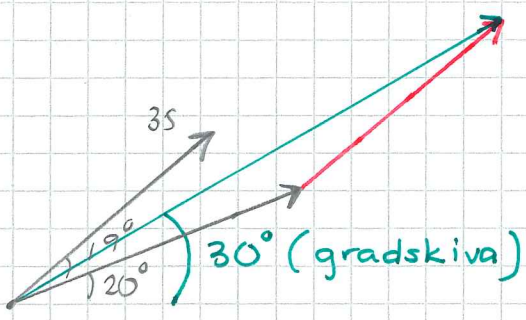
$\uparrow R_y = 13$ N

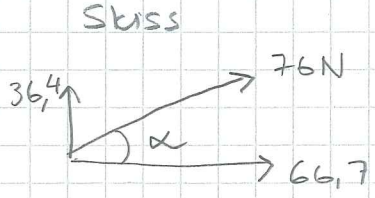
$R = \sqrt{51^2 + 13^2} = 52{,}6$ N



$\tan\alpha = \frac{13}{51} \Rightarrow \alpha = \arctan\frac{13}{51} = 14{,}3°$

3,4 a)





b $\rightarrow R_x = 42\cos 20° + 35\cos(20° + 19°) = 66{,}7$ kN

$\uparrow R_y = 42\sin 20° + 35\sin(20° + 19°) = 36{,}4$ kN

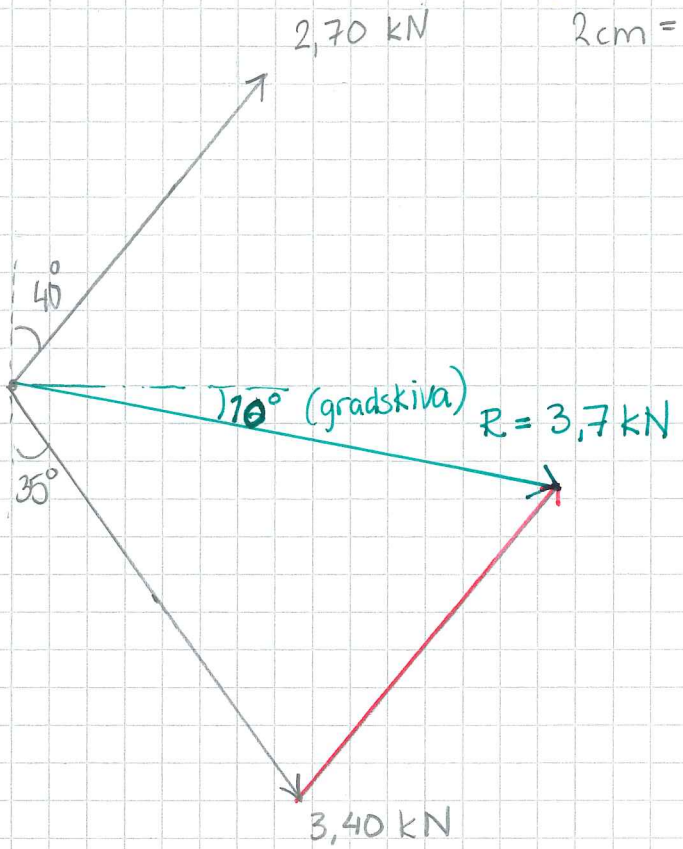
$R = \sqrt{66{,}7^2 + 36{,}4^2} = 76$ kN

$\alpha = \arctan\left(\frac{36{,}4}{66{,}7}\right) = 28{,}6°$

3.5

• Flytta längs verkningslinje för gemensam angreppspunkt.

a)



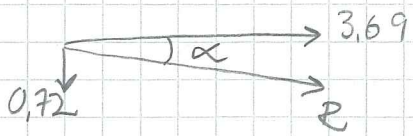
b) $\rightarrow R_x = 2{,}7 \sin 40^\circ + 3{,}4 \sin 35^\circ = 3{,}69 \text{ kN}$

$\uparrow R_y = 2{,}7 \cos 40^\circ - 3{,}4 \cos 35^\circ = -0{,}72 \text{ kN}$

$R = \sqrt{3{,}69^2 + 0{,}72^2} = \underline{\underline{3{,}76 \text{ kN}}}$

$\alpha = \arctan\left(\frac{0{,}72}{3{,}69}\right) = \underline{\underline{11^\circ}}$

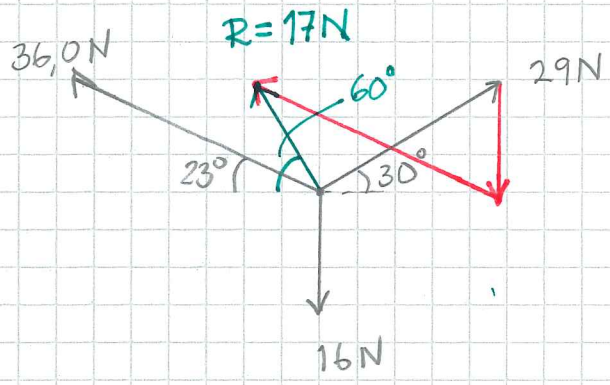
Skiss



3.6

1cm = 10N

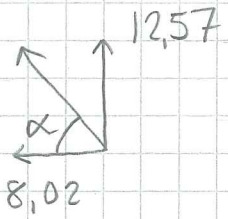
Huvud svans (eller kraftpolygon)



$\overrightarrow{R_x} = 29\cos 30^\circ - 36\cos 23^\circ = -8{,}02\ N$

$\uparrow R_y = 29\sin 30^\circ + 36\sin 23^\circ - 16 = 12{,}57\ N$

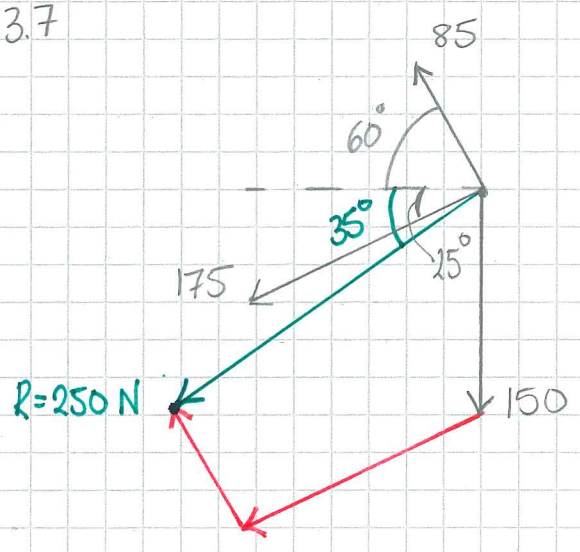
Skiss



$R = \sqrt{8{,}02^2 + 12{,}57^2} = \underline{\underline{14{,}9\ N}}$

$\alpha = \arctan\frac{12{,}57}{8{,}02} = \underline{\underline{57{,}5^\circ}}$

3.7

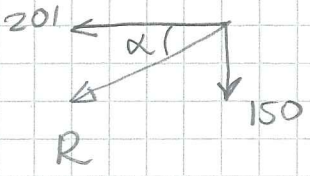


1 cm = 50 N

$\uparrow R_y = 85 \sin 60^\circ - 175 \sin 25^\circ - 150 = -150 \text{ N}$

$\rightarrow R_x = -85 \cos 60^\circ - 175 \cos 25^\circ = -201$

Skiss



$R = \sqrt{150^2 + 201^2} = 251 \text{ N}$

$\alpha = \arctan\left(\frac{150}{201}\right) = 36{,}7^\circ$

3,8 a Huvud-svansmetoden

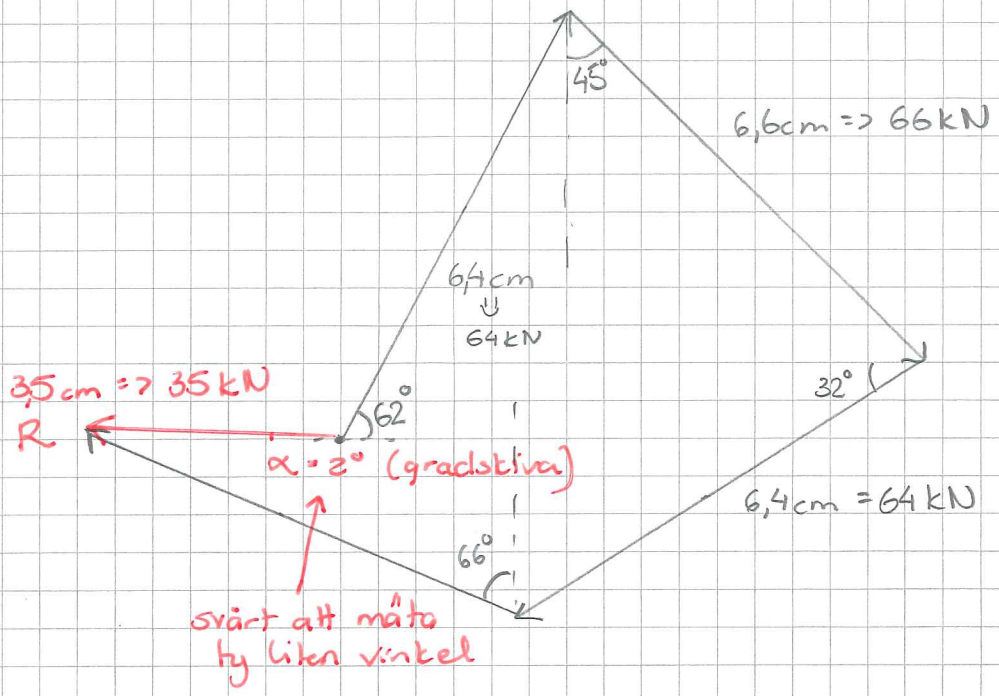
Börja med en av krafterna (valfri kraft)

Jag väljer 64 kN (62°)

Skalan jag väljer 1 cm = 10 kN

Rita med rätt riktning (gradskiva)

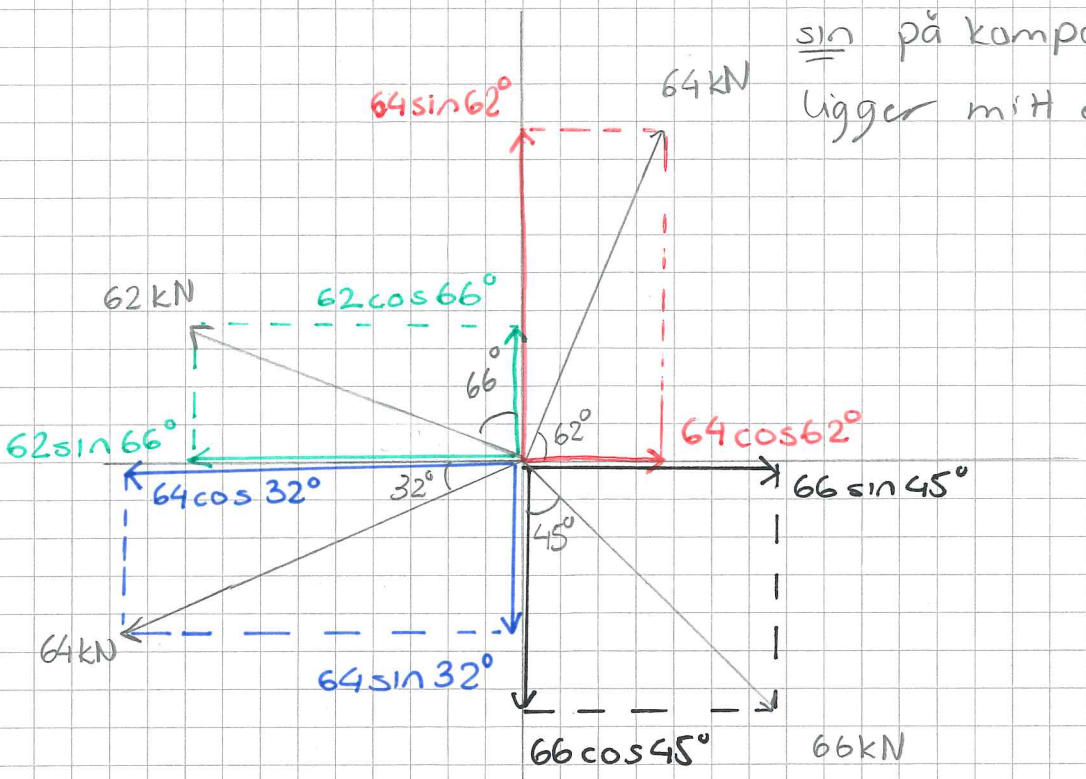
Fortsätt med 66 kN, 64 kN (66°+56°-90°=32°), 62 kN (66°)



3:8 b

cos på komposant som ligger nära vinkeln.

sin på komposant som ligger mitt emot.



Summera alla komposanter i x-led

pilen anger min valda positiva riktning

$$\rightarrow R_x = 64\cos 62^\circ + 66\sin 45^\circ - 62\sin 66^\circ - 64\cos 32^\circ$$

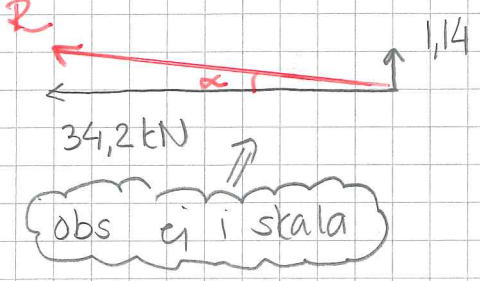
$$R_x = -34{,}2 \text{ kN}.$$

positivt uppåt

$$\uparrow R_y = 64\sin 62^\circ + 62\cos 66^\circ - 64\sin 32^\circ - 66\cos 45^\circ$$

$$R_y = 1{,}14 \text{ kN}$$

Rita R_x o R_y



$$R = \sqrt{34{,}2^2 + 1{,}14^2} = 34{,}2 \text{ kN}$$

ty liten R_y

$$\tan\alpha = \frac{1{,}14}{34{,}2}$$

$$\alpha = \arctan\left(\frac{1{,}14}{34{,}2}\right) = 1{,}9^\circ$$

3.8 a) Lös grafiskt med huvud/svans-metoden (kraftpolygon)

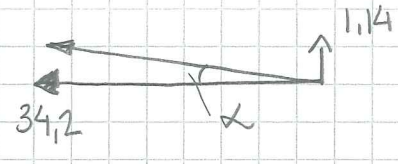
b) $\rightarrow R_x = 64\cos 62^\circ - 62\sin 66^\circ - 64\cos 32^\circ + 66\sin 45^\circ$

$R_x = -34{,}2\ \text{kN}$

$\uparrow R_y = 64\sin 62^\circ + 62\cos 66^\circ - 64\sin 32^\circ - 66\cos 45^\circ$

$R_y = 1{,}14\ \text{kN}$

Skiss

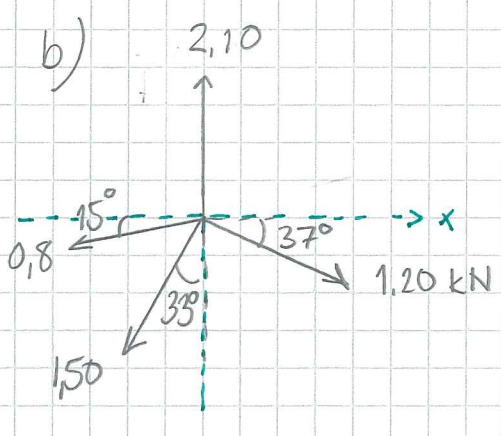


$R = \sqrt{34{,}2^2 + 1{,}14^2} = 34{,}2\ \text{kN}$

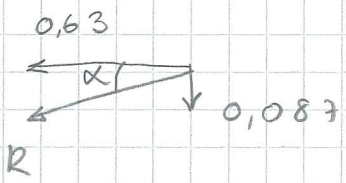
$\alpha = \arctan\left(\frac{1{,}14}{34{,}2}\right) = 1{,}9^\circ$

3.9 a) se ovan

b)



Skiss



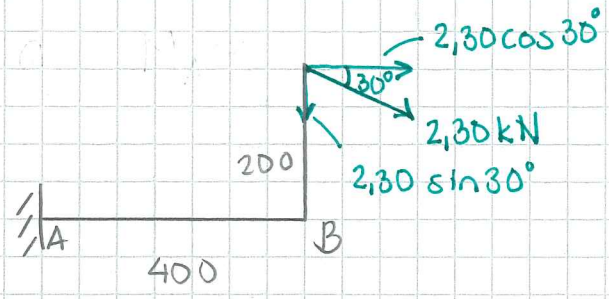
$\alpha = \arctan\left(\frac{0{,}087}{0{,}63}\right) = 7{,}9^\circ$

$\uparrow R_y = 2{,}10 - 0{,}8\sin 15^\circ - 1{,}5\cos 33^\circ - 1{,}20\sin 37^\circ = -0{,}087$

$\rightarrow R_x = 1{,}20\cos 37^\circ - 0{,}8\cos 15^\circ - 1{,}50\sin 33^\circ = -0{,}63$

$R = \sqrt{0{,}087^2 + 0{,}63^2} = 0{,}63\ \text{kN}$

3.10



Anta: mm

a) $\curvearrowleft M_A = 2{,}30 \sin 30^\circ \cdot 0{,}4 + 2{,}30 \cos 30^\circ \cdot 0{,}2$

$\curvearrowleft M_A = 0{,}858 \text{ kNm} = 858 \text{ Nm}$ medurs

b) $\curvearrowright M_B = 0{,}2 \cdot 2{,}30 \cos 30^\circ = 0{,}398 \text{ kNm} = 398 \text{ Nm}$ medurs

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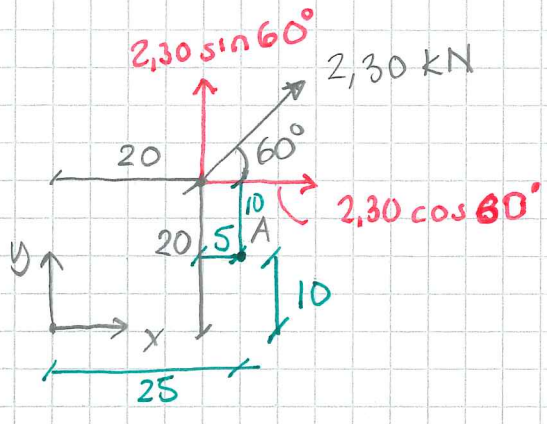
3.11

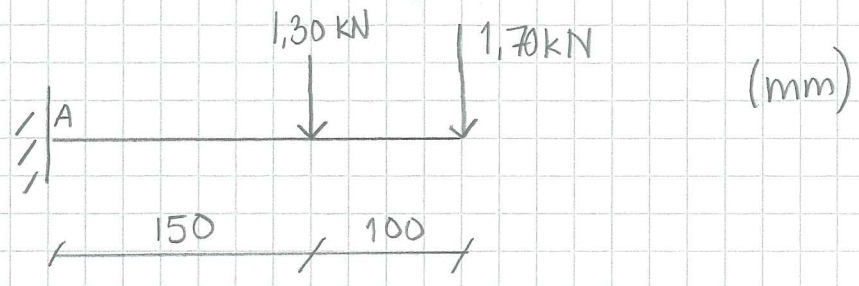


Anta: mm

a) $\curvearrowleft M_0 = 0{,}02 \cdot 2{,}30 \sin 60^\circ - 0{,}02 \cdot 2{,}30 \cos 60^\circ = 16{,}8 \text{ Nm}$ moturs

b) $\curvearrowright M_A = 0{,}01 \cdot 2{,}30 \cos 60^\circ + 0{,}005 \cdot 2{,}30 \sin 60^\circ = 21{,}5 \text{ Nm}$ medurs

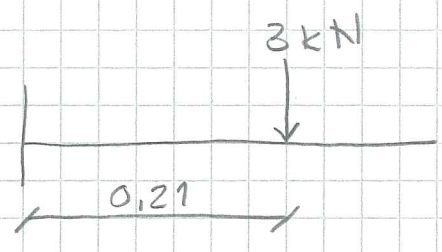
3.12



a) $\curvearrowleft M_A = 1{,}30 \cdot 0{,}150 + 1{,}70 \cdot 0{,}250 = 0{,}62\ \text{kNm}$

$\curvearrowright M_A = 620\ \text{Nm}$

b) $R = 1{,}30 + 1{,}70 = 3\ \text{kN}$



$R \cdot r = M_A$

$r = \frac{M_A}{R} = \frac{620}{3000} = 0{,}21\ \text{m}$

3.13

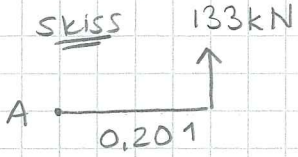
$\uparrow R_y = 39 + 44 + 50 = 133\ \text{kN}$

$\curvearrowleft M_A = 44 \cdot 0{,}203 + 50(0{,}154 + 0{,}203) = 26{,}8\ \text{kNm}$

(39 kN ger inget moment m.a.p A då hävarmen är noll)

$M_A = r \cdot R_y \qquad r = \frac{26{,}8}{133} = 0{,}201$

skiss



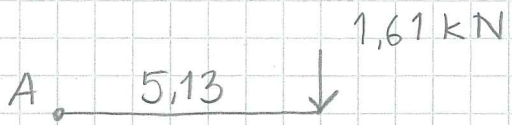
3.14

$\downarrow R_y = R = 2{,}54 + 1{,}17 - 2{,}10 = 1{,}61\ kN$

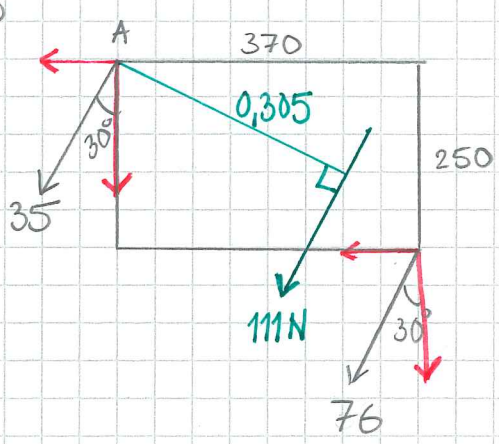
$\circlearrowleft M_A = 1{,}17 \cdot 1{,}7 - 2{,}1(1{,}7+2{,}03) + 2{,}54(1{,}7+2{,}03+1{,}82)$

$\circlearrowleft M_A = 8{,}253\ kNm$

$M_A = r \cdot R \qquad r = \frac{8{,}253}{1{,}61} = 5{,}13\ m$



3,15



Båda krafterna har samma riktning =>
att resultanten kommer ha lika riktning (30°)

$R = 35 + 76 = 111\ N$

Om inte detta inses kan man räkna R_x & R_y

$\leftarrow R_x = 35 \sin 30^\circ + 76 \sin 30^\circ = 55{,}5\ N$

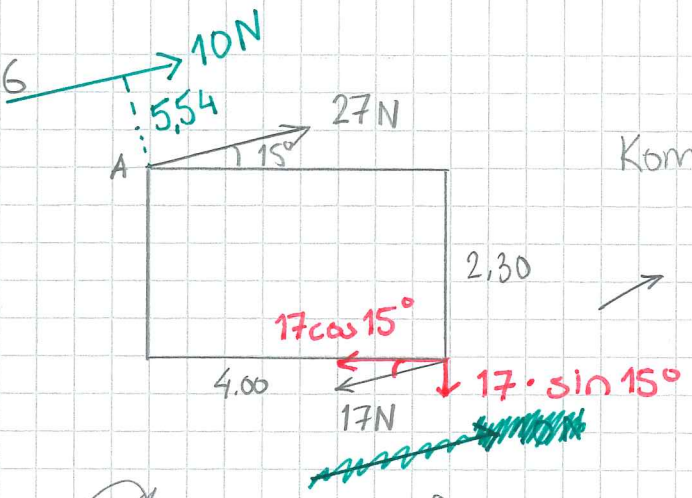
$\downarrow R_y = 35 \cos 30^\circ + 76 \cos 30^\circ = 96{,}13\ N$

$R = \sqrt{55{,}5^2 + 96{,}13^2} = 111\ N$ som ovan Ü

$\circlearrowleft M_A = 76 \sin 30^\circ \cdot 0{,}250 + 76 \cos 30^\circ \cdot 0{,}370 = 33{,}85\ Nm$

$r = \frac{33{,}85}{111} = 0{,}305\ m$

3,16



Kommentar se uppg. 3.15

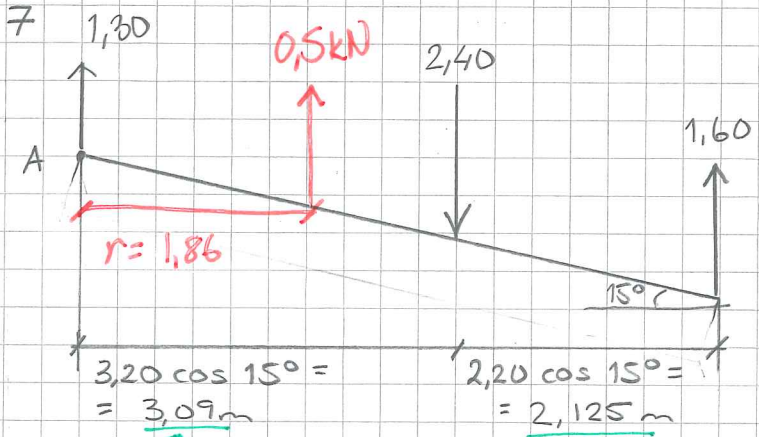
$\rightarrow R = 27 - 17 = 10\ N$

$\circlearrowleft M_A = 17 \sin 15^\circ \cdot 4{,}0 + 17 \cos 15^\circ \cdot 2{,}3 = 55{,}37\ Nm$

$r = \frac{55{,}37}{10} = 5{,}54 m$

3,17

kN, m



$\uparrow R_y = R = 1{,}3 - 2{,}4 + 1{,}6 = 0{,}5\ kN$

· Bestäm hävarmar till A, vinkelräta avstånd

$\circlearrowleft M_A = 2{,}40 \cdot 3{,}09 - 1{,}6(3{,}09 + 2{,}125) = -0{,}93\ kNm$

$\circlearrowright M_A = 0{,}93\ kNm$ (moturs)

$r = \frac{M_A}{R} = \frac{0{,}93}{0{,}5} = 1{,}86 m$

3.18

a) $M_A = 1,30 \cdot 7,35 = 9,56\ kNm$

b) $M_B = 1,30 \cdot 7,35 = 9,56\ kNm$

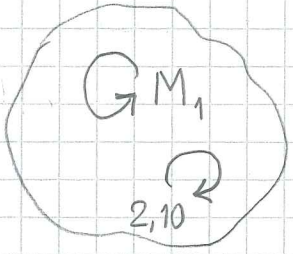
c) $M_C = 1,30 \cdot 4,50 + 1,3\,(7,35 - 4,50)$

$M_C = \cancel{1,30 \cdot 4,50} + 1,3 \cdot 7,35 - \cancel{1,3 \cdot 4,50} = 1,3 \cdot 7,35$

$M_C = 9,56\ kNm$

3.19

Ett kraftpar är ett rent moment och in platsberoende på stela kroppar



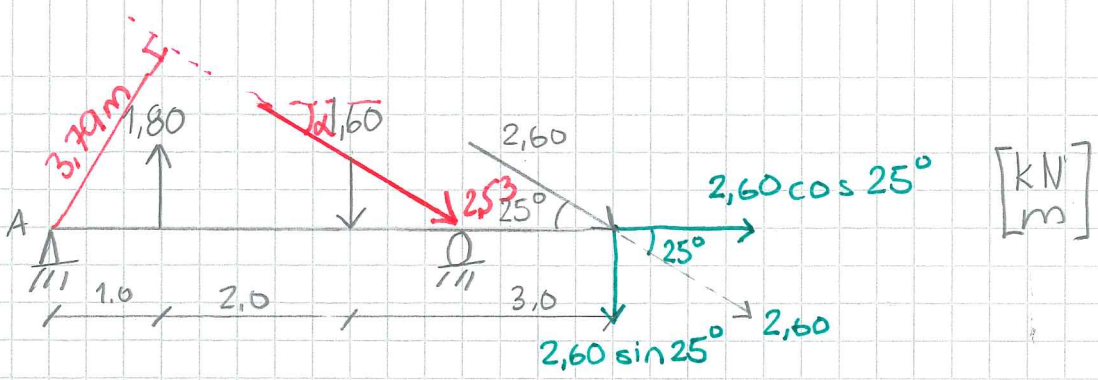
$M_1 = 10 \cdot 0,5 = 5\ kNm$

$M_{tot} = 5 - 2,10 = 2,90\ kNm$

3.20

$M_A = 3,50 \cdot 2,70 - 9,30 = 0,15\ kNm$

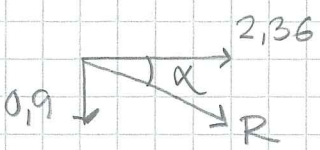
3.21



$\rightarrow R_x = 2{,}60 \cos 25^\circ = 2{,}36 \text{ kN}$

$\uparrow R_y = 1{,}8 - 1{,}6 - 2{,}6 \sin 25^\circ = -0{,}90 \text{ kN}$

Skiss



$R = \sqrt{0{,}9^2 + 2{,}36^2} = 2{,}53 \text{ kN}$

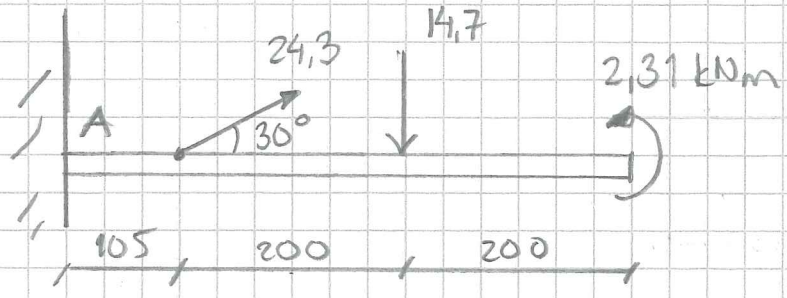
$\alpha = \arctan\left(\frac{0{,}9}{2{,}36}\right) = 20{,}9^\circ$

$\circlearrowright M_A = 1{,}6 \cdot 3{,}0 - 1{,}8 \cdot 1{,}0 + 6{,}0 \cdot 2{,}6 \sin 25^\circ = 9{,}59 \text{ kNm}$

$r = \frac{M_A}{R} = \frac{9{,}59}{2{,}53} = 3{,}79 \text{ m}$

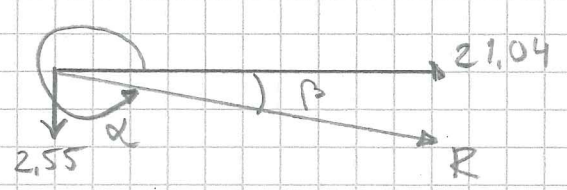
3.22 Sökt: Resultanten.

kN, cm



$\rightarrow F_{Rx} = 24{,}3 \cos 30^\circ = 21{,}04 \text{ kN}$

$\uparrow F_{Ry} = 24{,}3 \sin 30^\circ - 14{,}7 = -2{,}55 \text{ kN}$



$R = \sqrt{21{,}04^2 + 2{,}55^2} = \underline{\underline{21{,}19 \text{ kN}}}$

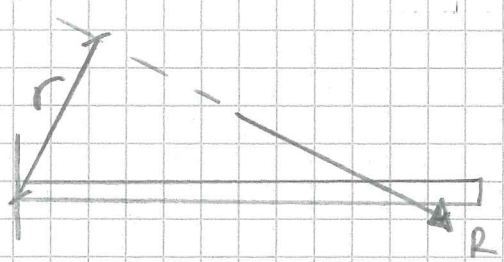
$\beta = \arctan\left(\frac{2{,}55}{21{,}04}\right) = \underline{\underline{6{,}91^\circ}}$

"boken" $\alpha = 360 - 6{,}91 = 353^\circ$

R ska ge samma vridande förmåga som de två krafterna + momentet

$\curvearrowleft M_A = 2{,}31 - 14{,}7 \cdot 3{,}05 + 1{,}05 \cdot 24{,}3 \sin 30^\circ =$

$= -29{,}8 \text{ kNm} \Rightarrow \curvearrowright M_A = 29{,}8 \text{ kNm}$

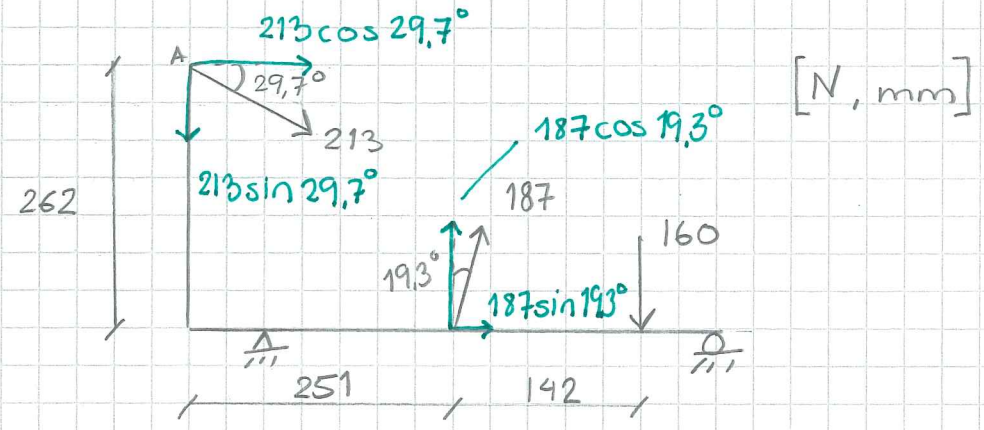


$\curvearrowright M_A = r \cdot R$

$r = \frac{29{,}8}{21{,}2} = \underline{\underline{1{,}4 \text{ m}}}$

$R_y x = M_A \Rightarrow x = \frac{M_A}{R_y} = \frac{29{,}8}{2{,}55}$

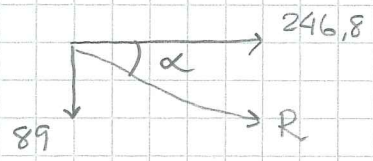
3.23



$\downarrow R_y = 213\sin 29{,}7^\circ - 187\cos 19{,}3^\circ + 160 = 89{,}04\ \text{N}$

$\rightarrow R_x = 213\cos 29{,}7^\circ + 187\sin 19{,}3^\circ = 246{,}8\ \text{N}$

Stöss

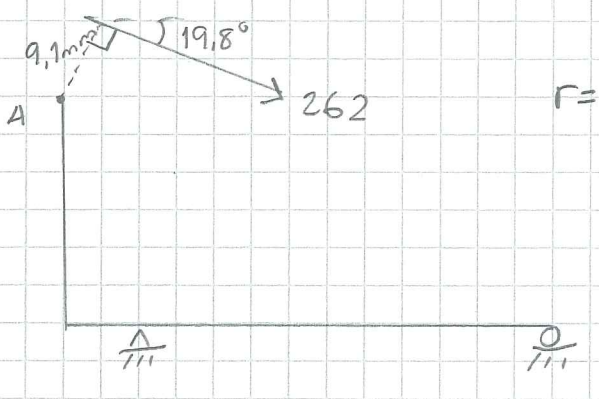


$R = \sqrt{246{,}8^2 + 89^2} = 262\ \text{N}$

$\alpha = \arctan\left(\frac{89}{246{,}8}\right) = 19{,}8^\circ$

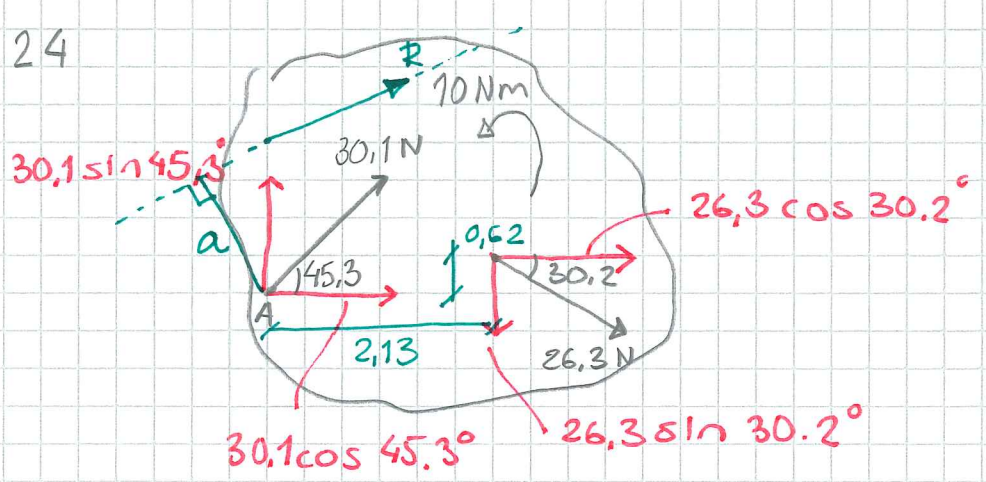
$\circlearrowleft M_A = 160(0{,}251 + 0{,}142) - 187\cos 19{,}3^\circ \cdot 0{,}251 - \ldots$

$\ldots 187\sin 19{,}3^\circ \cdot 0{,}262 = 2{,}39\ \text{Nm}$



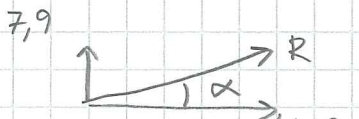
$r = \frac{M_A}{R} = \frac{2{,}39}{262} = 9{,}1\ \text{mm}$

3.24



$\uparrow \quad R_y = 30{,}1 \sin 45{,}3^\circ - 26{,}3 \sin 30{,}2^\circ = 7{,}9\,\text{N}$

$\rightarrow \quad R_x = 26{,}3 \cos 30{,}2^\circ + 30{,}1 \cos 45{,}3^\circ = 44{,}3\,\text{N}$



$R = \sqrt{7{,}9^2 + 44{,}3^2} = 45\,\text{N}$

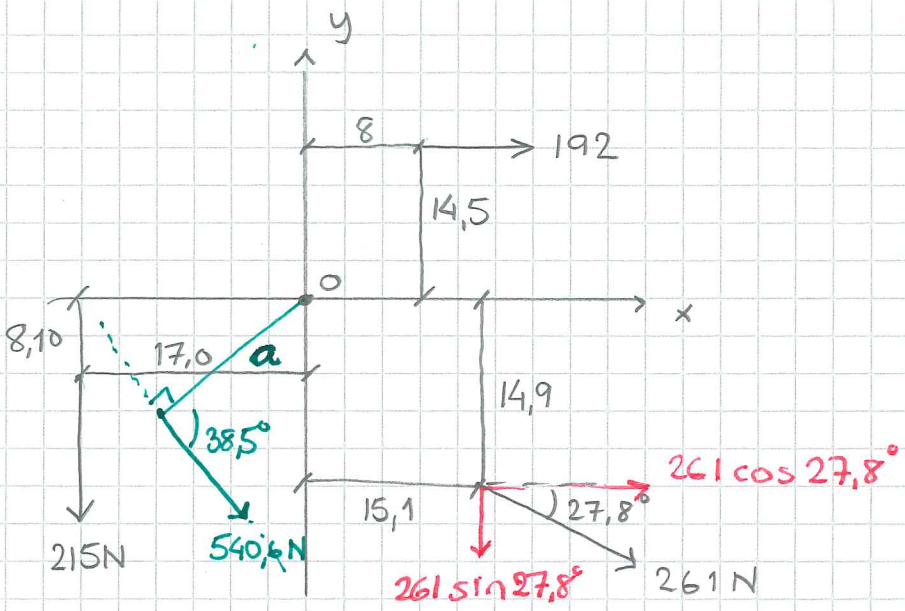
$\alpha = \arctan\left(\frac{7{,}9}{44{,}3}\right) = 10{,}1^\circ$

$M_A = 0{,}62 \cdot 26{,}3 \cos 30{,}2^\circ + 2{,}13 \cdot \overbrace{26{,}3}^{\text{sin}} \sin 30{,}2^\circ - 10 = 33\,\text{Nm}$

$R \cdot a = M_A \qquad a = \frac{33}{45} = 0{,}733\,\text{m}$

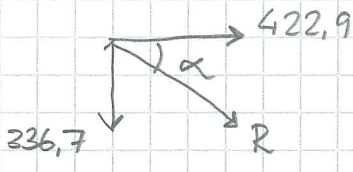
a enligt figur (vinkelrät avstånd från A till kraftens verkningslinje, placera R så att det blir medurs i förhållande till A.

3.25



$$\rightarrow \quad R_x = 192 + 261 \cos 27{,}8^\circ = 422{,}9\ \text{N}$$

$$\downarrow \quad R_y = 215 + 261 \sin 27{,}8^\circ = 336{,}7\ \text{N}$$



$$R = \sqrt{422{,}9^2 + 336{,}7^2} = 540{,}6\ \text{N}$$

$$\alpha = \arctan\left(\frac{336{,}7}{422{,}9}\right) = 38{,}5^\circ$$

$$\circlearrowright M_O = 192 \cdot 14{,}5 - 215 \cdot 17 + 15{,}1 \cdot 261 \sin 27{,}8^\circ \ldots$$

$$\ldots - 14{,}9 \cdot 261 \cos 27{,}8^\circ = -2470\ \text{Nmm}$$

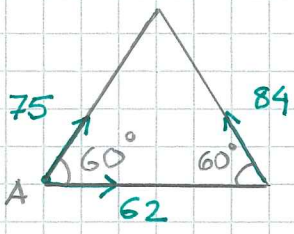
$$\circlearrowleft M_O = 2470\ \text{Nmm}$$

$$a = \frac{2470}{540{,}6} = 4{,}57\ \text{mm}$$

3.26

3x65 cm

(N, cm)

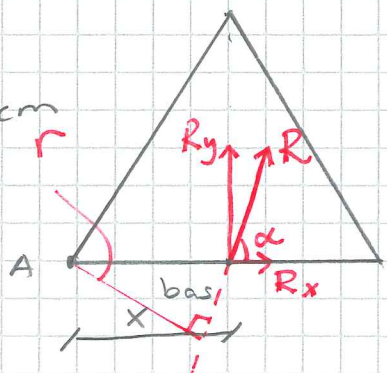


$\uparrow R_y = 75 \sin 60^\circ + 84 \sin 60^\circ = 138{,}7\ N$

$\rightarrow R_x = 75 \cos 60^\circ - 84 \cos 60^\circ + 62 = 57{,}5\ N$

$\circlearrowleft_A \quad M_A = 84 \sin 60^\circ \cdot 65 = 4728\ Ncm$

$r = \frac{4728}{150} = 31{,}5\ cm$



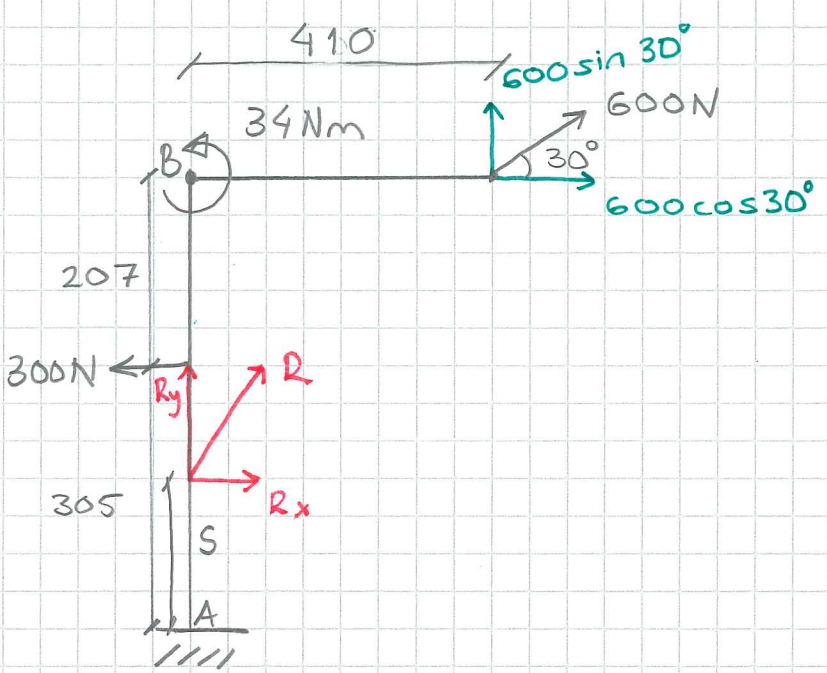
$\alpha = \arctan \frac{138{,}7}{57{,}5} = 67{,}5^\circ$

$R = \sqrt{138{,}7^2 + 57{,}5^2} = 150\ N$

b) Då R står basen ger endast Ry moment i A

$R_y \cdot x = M_A \Rightarrow x = \frac{M_A}{R_y} = \frac{4728}{138{,}7} = 34\ cm$

3,27



$\uparrow R_y = 600 \sin 30^\circ = 300\,N$

$\rightarrow R_x = 600 \cos 30^\circ - 300 = 220\,N$

$R = \sqrt{300^2 + 220^2} = 372\,N$

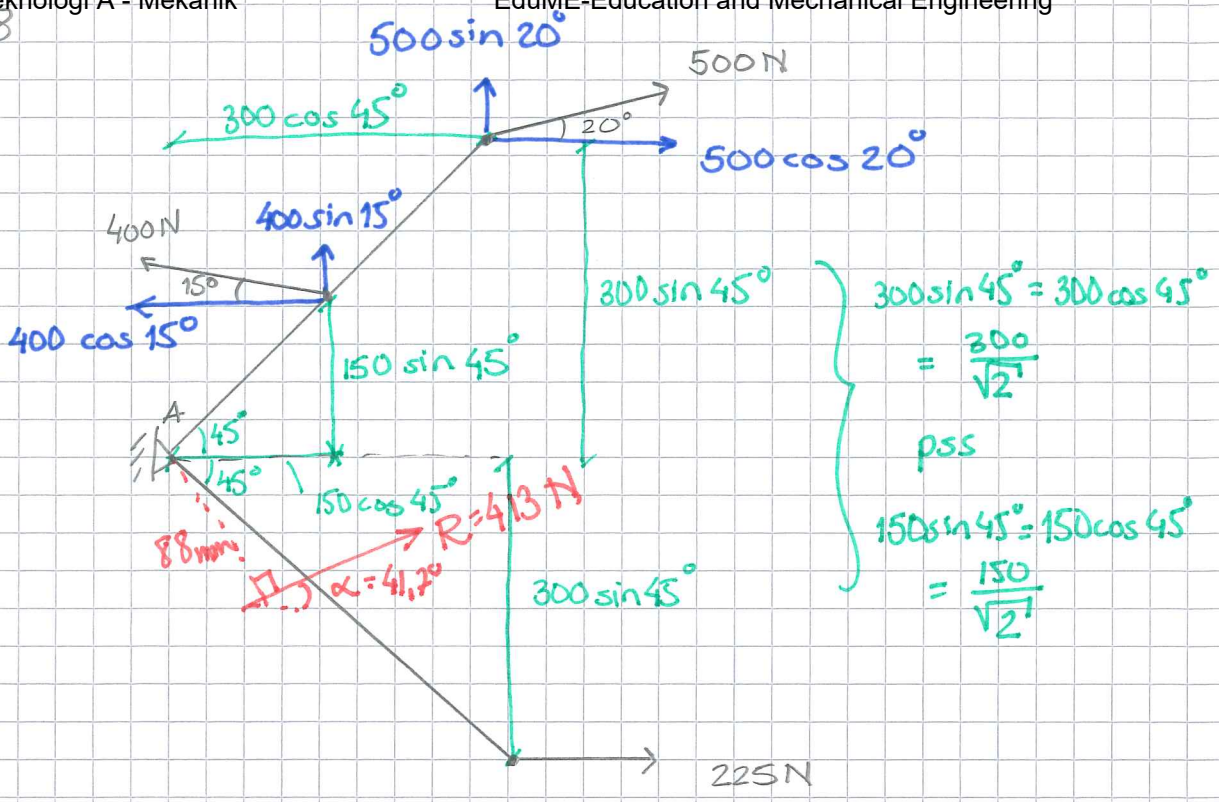
$\alpha = \arctan\left(\frac{300}{220}\right) = 53{,}7^\circ$

$M_A = 0{,}512 \cdot 600 \cos 30^\circ - 0{,}41 \cdot 600 \sin 30^\circ - 34 - 300 \cdot 0{,}305$

$M_A = 17{,}5\,Nm$

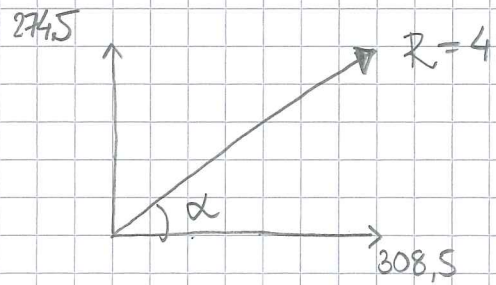
$S = \frac{M_A}{R_x} = \frac{17{,}5}{220} = \underline{\underline{0{,}08\,m}}$

3.28



$\uparrow \quad R_y = 500 \sin 20° + 400 \sin 15° = 274{,}5 \text{ N}$

$\rightarrow \quad R_x = 225 + 500 \cos 20° - 400 \cos 15° = 308{,}5 \text{ N}$



$R = \sqrt{274{,}5^2 + 308{,}5^2} = 413 \text{ N}$

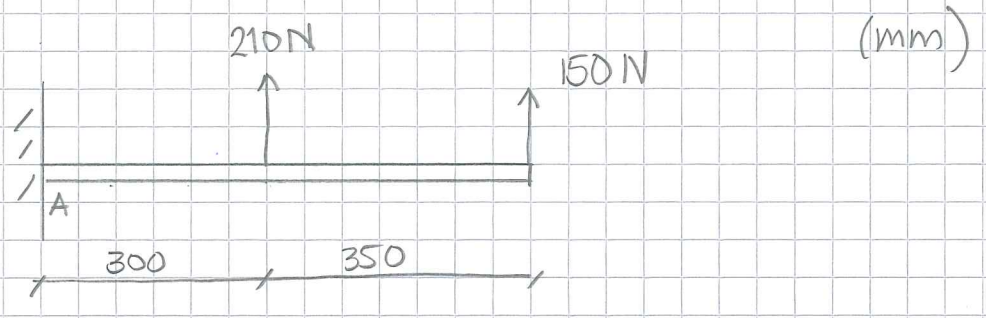
$\alpha = \arctan \frac{274{,}5}{308{,}5} = 41{,}7°$

$\circlearrowleft A \quad 225 \cdot \frac{300}{\sqrt{2}} + 400 \cos 15° \cdot \frac{150}{\sqrt{2}} + 400 \sin 15° \cdot \frac{150}{\sqrt{2}} + 500 \sin 20° \cdot \frac{300}{\sqrt{2}} \ldots$

$\ldots - 500 \cos 20° \cdot \frac{300}{\sqrt{2}} = M_A \quad \Rightarrow \quad \circlearrowleft M_A = 36298 \text{ Nmm}$

$a = \frac{36298}{413} = 88 \text{ mm}$

3.29

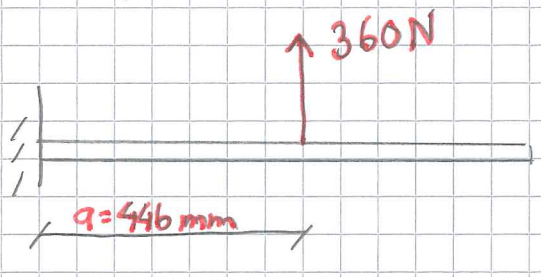


$\uparrow \quad R = 210 + 150 = 360\,N$

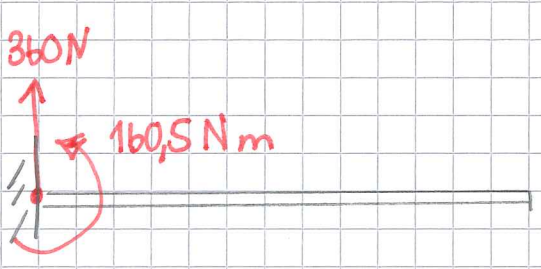
$M_A = 210 \cdot 300 + 150 \cdot 650 = 160500\,Nmm$

$a = \frac{M_A}{R} = \frac{160500}{360} = 446\,mm$

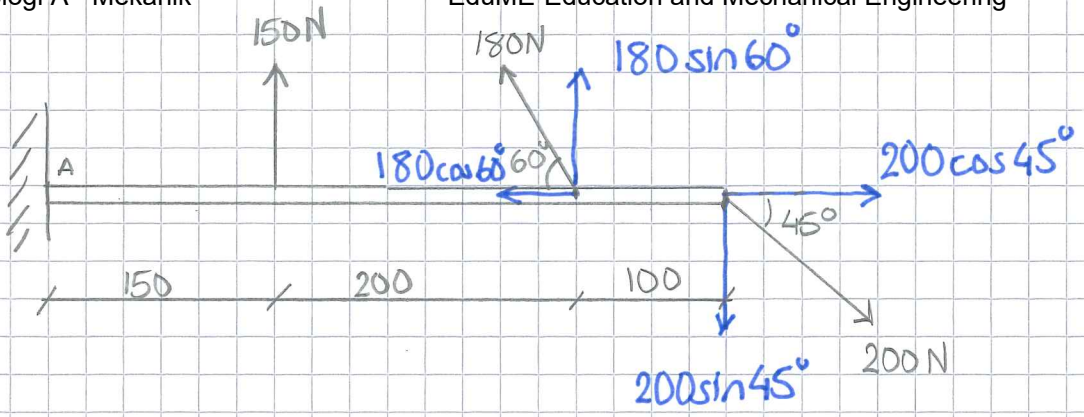
a)



b)

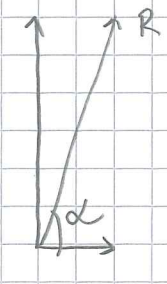


3:30



$$\uparrow \quad R_y = 150 + 180\sin 60^\circ - 200\sin 45^\circ = 164{,}5\ \text{N}$$

$$\rightarrow \quad R_x = 200\cos 45^\circ - 180\cos 60^\circ = 51{,}4\ \text{N}$$



$$R = \sqrt{51{,}4^2 + 164{,}5^2} = 172\ \text{N}$$

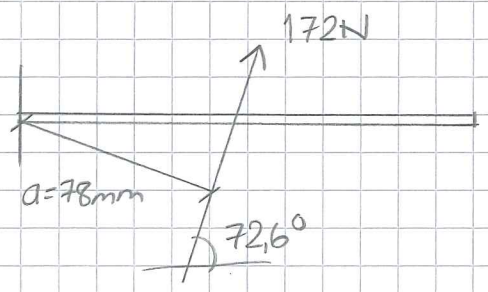
$$\alpha = \arctan\left(\frac{164{,}5}{51{,}4}\right) = 72{,}6^\circ$$

$$M_A = 150 \cdot 150 + 180\sin 60^\circ \cdot (150+200) - 200\sin 45^\circ (150+200+100)$$

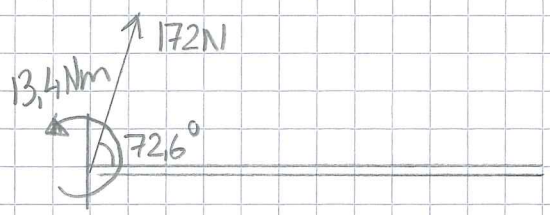
$$M_A = 13420\ \text{Nmm}$$

$$a = \frac{M_A}{R} = \frac{13420}{172} = 78\ \text{mm}$$

a)

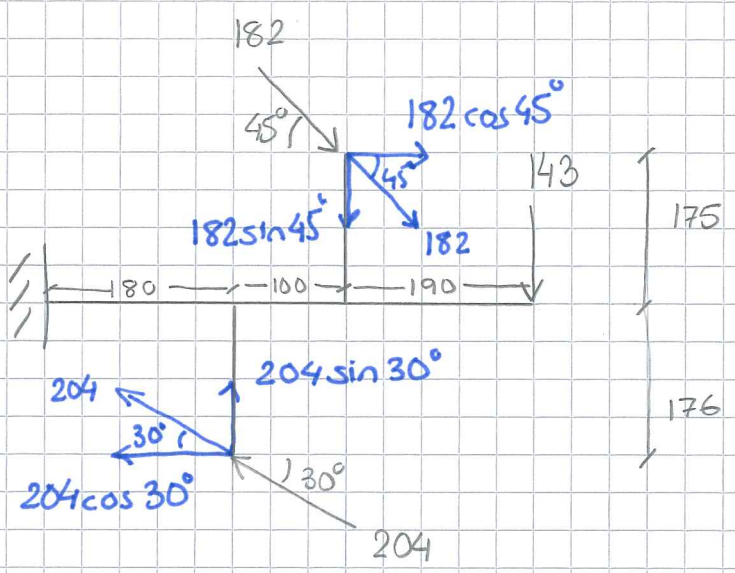


b



3.31

(N, mm)

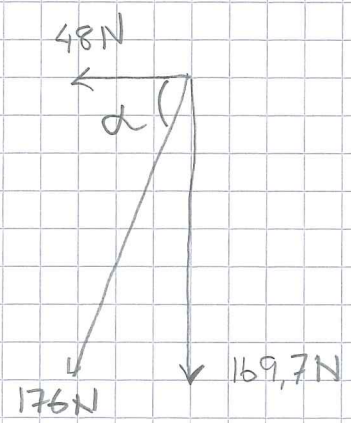


$$\uparrow R_y = 204\sin 30^\circ - 143 - 182\sin 45^\circ = -169{,}7\ \text{N}$$

$$\rightarrow R_x = 182\cos 45^\circ - 204\cos 30^\circ = -48{,}0\ \text{N}$$

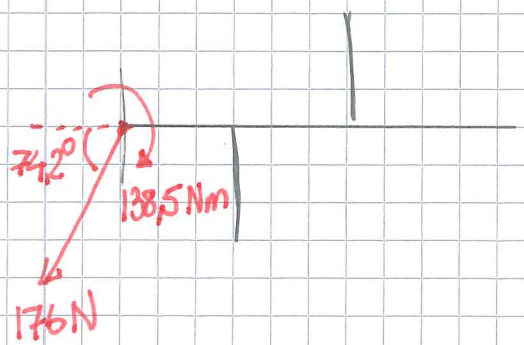
$$R = \sqrt{169{,}7^2 + 48^2} = 176\ \text{N}$$

$$\alpha = \arctan\left(\frac{169{,}7}{48}\right) = 74{,}2^\circ$$

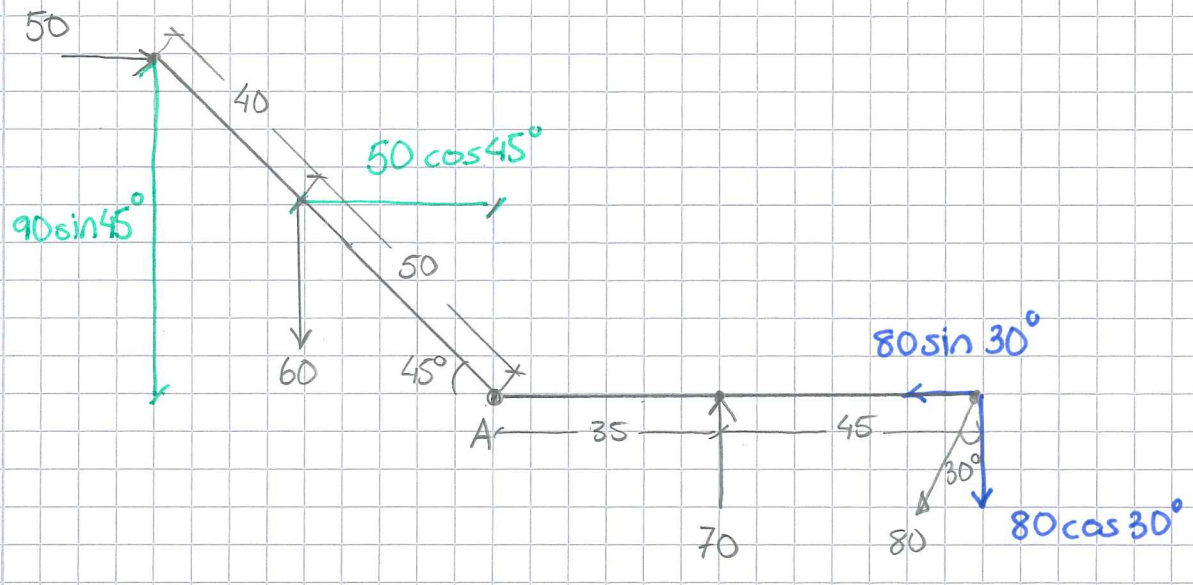


$$\circlearrowleft M_A = 143(180+100+190) + 182\sin 45^\circ(180+100) + 182\cos 45 \cdot 175 \ldots$$

$$+ 204\cos 30^\circ \cdot 176 - 204\sin 30^\circ \cdot 180 = 138499\ \text{Nmm}$$



3.32

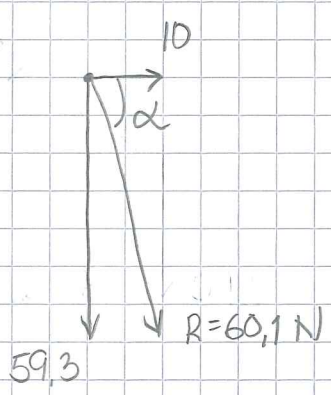


$$\uparrow R_y = 70 - 60 - 80\cos 30^\circ = -59{,}3\ \text{N}$$

$$\rightarrow R_x = 50 - 80\sin 30^\circ = 10$$

$$R = \sqrt{10^2 + 59{,}3^2} = 60{,}1\ \text{N}$$

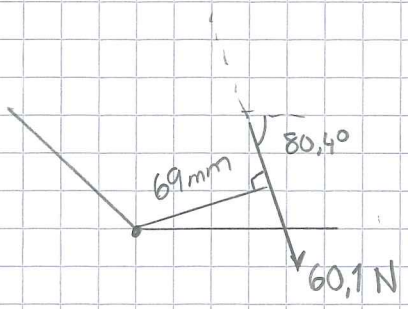
$$\alpha = \arctan\left(\frac{59{,}3}{10}\right) = 80{,}4^\circ$$



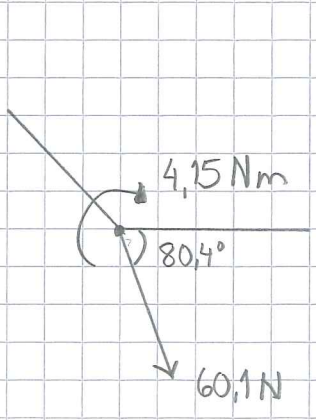
$$M_A = 50 \cdot 90\sin 45^\circ - 60 \cdot 50\cos 45^\circ - 70 \cdot 35 + 80\cos 30^\circ \cdot 80$$

$$M_A = 4153\ \text{Nmm} \qquad a = \frac{M_A}{R} = \frac{4153}{60{,}1} = 69\ \text{mm}$$

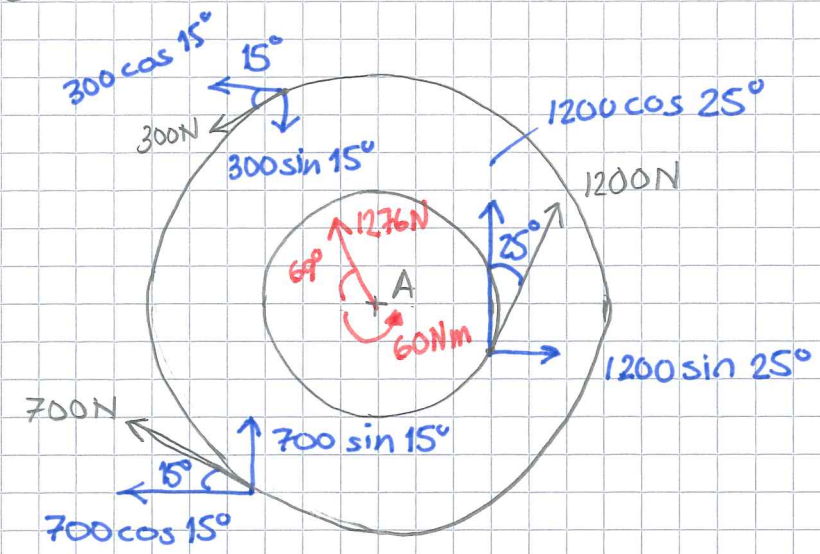
a)



b)



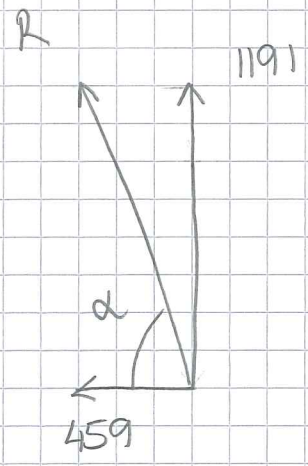
3.55



$$\rightarrow R_x = 1200\sin 25^\circ - 700\cos 15^\circ - 300\cos 15^\circ = -459\ \text{N}$$

$$\uparrow R_y = 1200\cos 25^\circ + 700\sin 15^\circ - 300\sin 15^\circ = 1191\ \text{N}$$

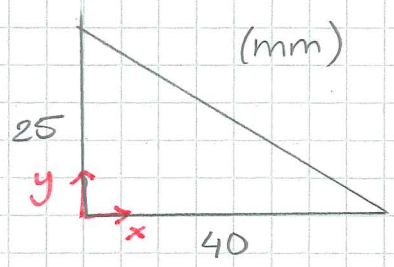
$$\circlearrowleft M_A = 300\cdot 300 + 1200\cdot 150 - 700\cdot 300 = 60000\ \text{Nmm}$$



$$R = \sqrt{1191^2 + 459^2} = 1276\ \text{N}$$

$$\alpha = \arctan\left(\frac{1191}{459}\right) = 69^\circ$$

4.1



Formelsamling => $t_p = \frac{h}{3}$

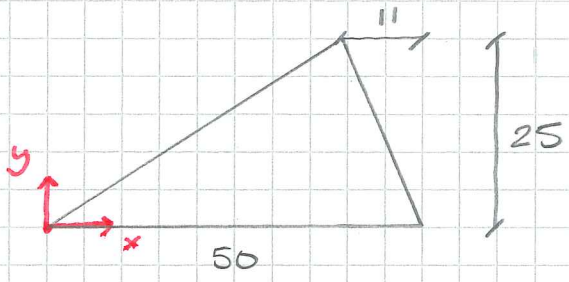
Mitt råd: Använd alltid $\frac{h}{3}$ för att beräkna t_p inte $x_{tp} = \frac{A+C+D}{3}$ (Malmendahl)

$\frac{h}{3}$ gäller från basen.

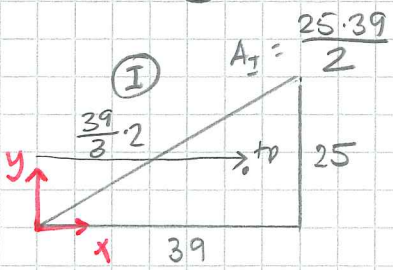
$$y_{tp} = \frac{25}{3} = 8{,}33 \text{ mm}$$

$$x_{tp} = \frac{40}{3} = 13{,}3 \text{ mm}$$

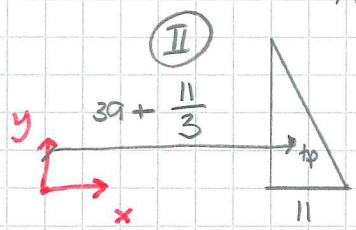
4.2



$$y_{tp} = \frac{25}{3} = 8{,}33 \text{ mm}$$



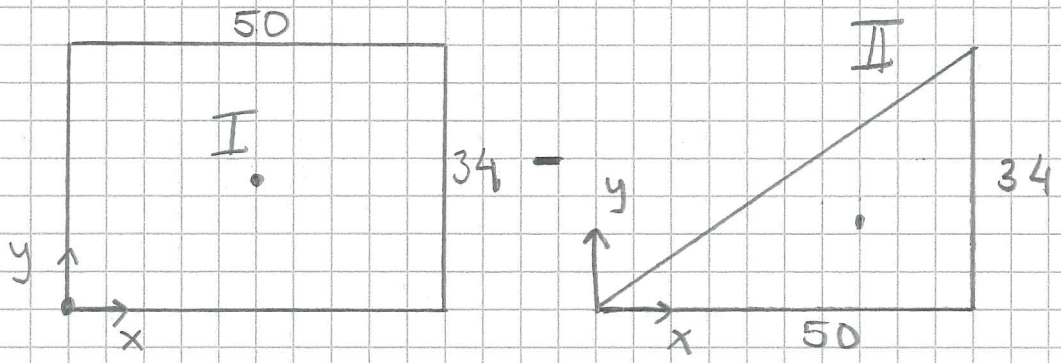
+

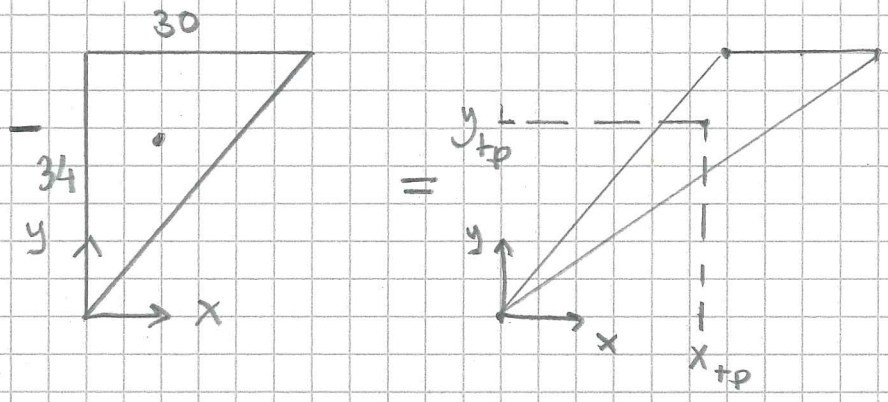


	A	x_{tp}	$A x_{tp}$
I	487,5	26	12675
II	137,5	42,7	5867
	625	\	18542

$$x_{tp} = \frac{18542}{625} = 29{,}7 \text{ mm}$$

4.3 Sökt: x_{tp}, y_{tp}





n	A_n	x_{tp}^n	y_{tp}^n	$A_n \cdot x_{tp}^n$	$A_n \cdot y_{tp}^n$
I	1700	25	17	42500	28 900
II	-850	$\frac{2 \cdot 50}{3} = 33{,}3$	$\frac{34}{3} = 11{,}3$	-28333	-9633
III	-510	$\frac{30}{3} = 10$	$\frac{2 \cdot 34}{3} = 22{,}7$	-5100	-11560
Σ	340			9067	7707

$$x_{tp} = \frac{\Sigma A_n x_{tp}^n}{A_{tot}} = \frac{9067}{340} = \underline{\underline{26{,}7 \text{ mm}}}$$

$$y_{tp} = \frac{7707}{340} = \underline{\underline{22{,}7 \text{ mm}}}$$

4.4

$$x_{tp} = \frac{2R \sin\alpha}{3\alpha}$$

Formelbladet säger α i rad

α i nämnaren måste alltid vara i radianer.

sin α => om α i rad måste miniräknare stå på radianer.

om α i grader kan miniräknaren vara på grader (enklast)

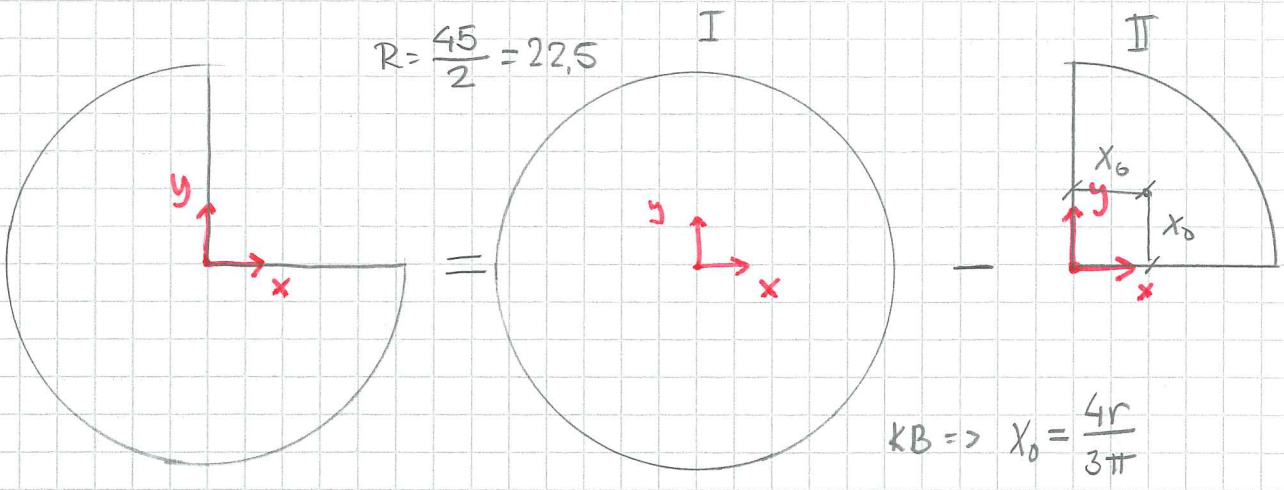
Mitt råd: radianer i nämnaren
grader för sinα för miniräknare står på grader.

$\alpha = 13°$ $\qquad \alpha_{rad} = \frac{13}{360} \cdot 2\pi = 13 \cdot \frac{\pi}{180}$

$$x_{tp} = \frac{2 \cdot 30 \cdot \sin 13°}{3 \cdot 13 \cdot \frac{\pi}{180}} = 19{,}8 \text{ mm}$$

4,5

$R = \frac{45}{2} = 22,5$



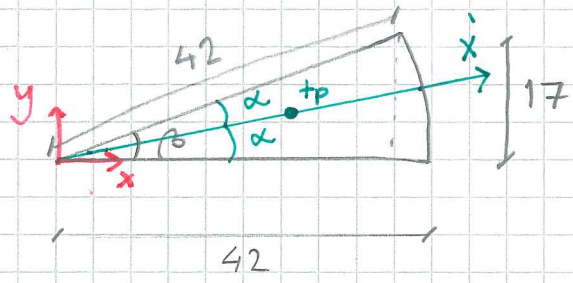
KB => $x_D = \frac{4r}{3\pi}$

$x_D = \frac{4 \cdot 22,5}{3\pi} = 9,55$

	A	x_{tp}	y_{tp}	Ax_{tp}	Ay_{tp}
I	$22,5^2\pi = 1590$	0	0	0	0
II	$-\frac{1590}{4} = -397,5$	9,55	9,55	-3796	-3796
	1193			-3796	-3796

$$x_{tp} = \frac{-3796}{1193} = \underline{\underline{-3,18}} = y_{tp}$$

4,6



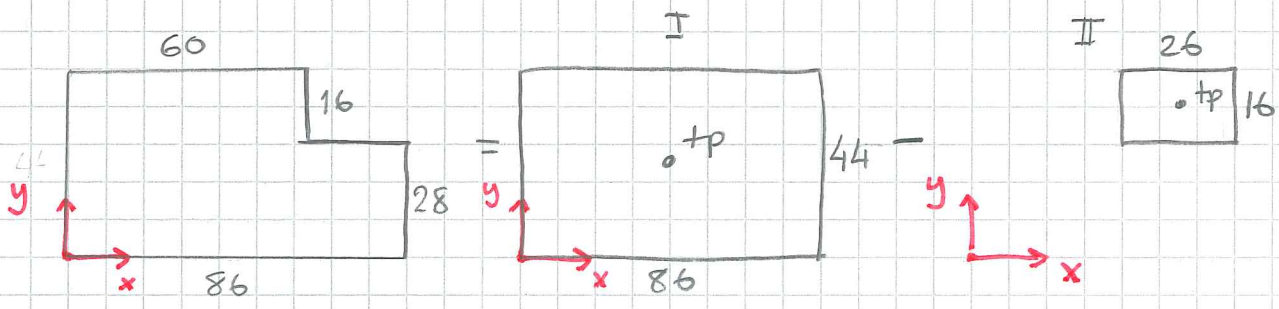
$$\beta = \arcsin\left(\frac{17}{42}\right) = 23{,}88°$$

$$x'_{tp} = \frac{2 \cdot 42 \cdot \sin 11{,}9°}{3 \cdot 11{,}9 \cdot \frac{\pi}{180}} = 27{,}8\ mm$$

$$x_{tp} = 27{,}8 \cdot \cos 11{,}9° = \underline{27{,}2\ mm}$$

$$y_{tp} = 27{,}8 \sin 11{,}9° = \underline{5{,}73\ mm}$$

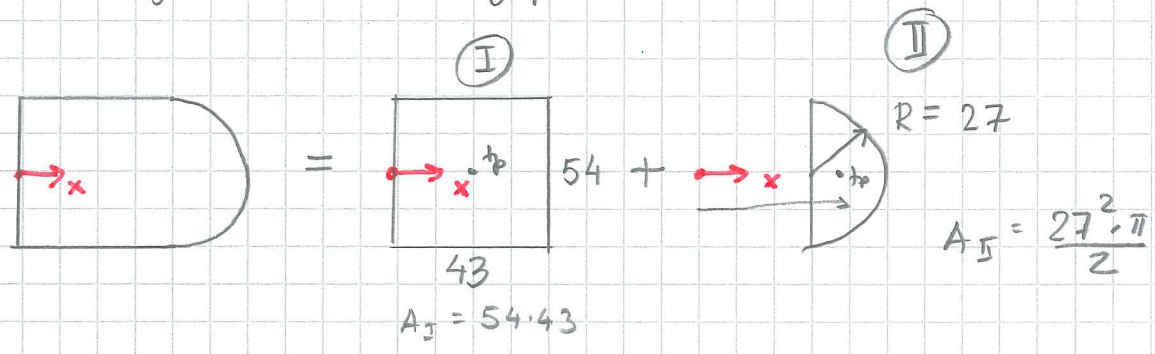
4,7



	A	x_{tp}	y_{tp}	Ax_{tp}	Ay_{tp}
I	$44 \cdot 86 = 3784$	$\frac{86}{2} = 43$	$\frac{44}{2} = 22$	162712	83248
II	$-26 \cdot 16 = -416$	$60 + \frac{26}{2} = 73$	$28 + \frac{16}{2} = 36$	−30368	−14976
	3368			132 344	68272

$$x_{tp} = \frac{132\,344}{3368} = \underline{39{,}3} \qquad y_{tp} = \frac{68272}{3368} = \underline{20{,}3}$$

4.8 Symmetri $\Rightarrow$ $y_{tp} = 0$



$A_I = 54 \cdot 43$

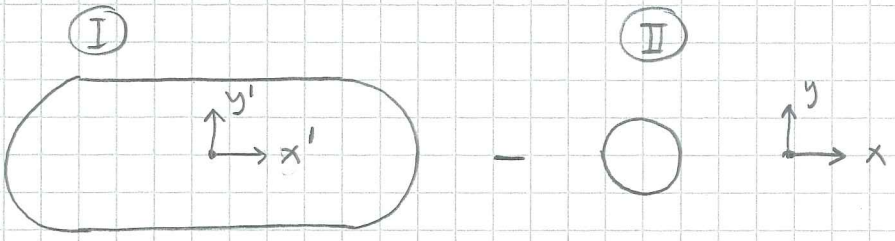
$A_{II} = \frac{27^2 \cdot \pi}{2}$

	A	x_{tp}	$A x_{tp}$
I	2322	21,5	49923
II	1145	$43 + \frac{4 \cdot 27}{3\pi} = 54,5$	62402
Σ	3467		112325

$$x_{tp} = \frac{112325}{3467} = \underline{\underline{32.4}} \text{ mm}$$

4.9 Tänk till INNAN! Leta symmetrier o kända tyngdpunkter

Här: flytta koordinatsystemet



$A_I = 19 \cdot 26 + 95^2\pi = 777$

$x'_{tp} = 0$

$A_{II} = 5^2\pi = 78{,}5$

$x'_{tp} = -\left(\frac{45}{2} - 95\right) = -13$

	A	x'_{tp}	Ax'_{tp}
I	777	0	0
II	−78,5	−13	1020,5
	698,5		1020,5

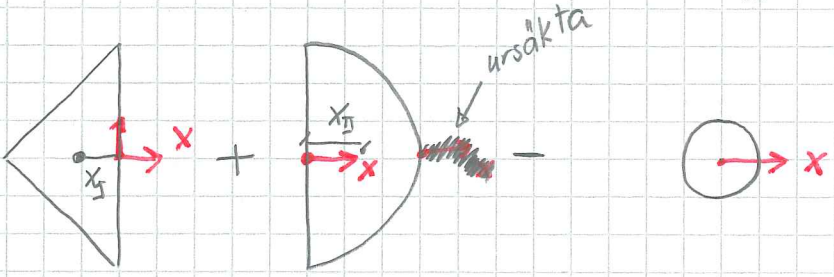
$$x'_{tp} = \frac{1020{,}5}{698{,}5} = 1{,}46$$

Koordinatsystem enl. uppg. $\Delta 13$ mm

$$x_{tp} = 13 + 1{,}46 = \underline{\underline{14{,}5\text{ mm}}}$$

4.10 Symmetri => $y_{tp} = 0$

I II III



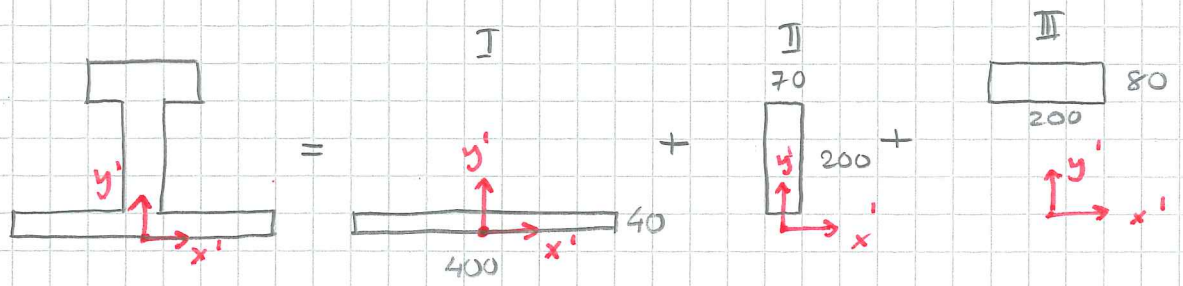
$A_I = \frac{17 \cdot 18}{2} = 306$ $A_{II} = \frac{18^2 \cdot \pi}{2} = 509$ $A_{III} = \frac{11^2 \cdot \pi}{4} = 95$

$x_I = -\frac{17}{3} = 5{,}67$ $x_{II} = \frac{4 \cdot 18}{3\pi} = 7{,}64$ $x_{III} = 0$

	A	x_{tp}	Ax_{tp}
I	306	-5,67	-1735
II	509	7,64	3888
III	-95	0	0
Σ	720	\	2153

$x_{tp} = \frac{2153}{720} = \underline{\underline{2{,}99}}$

4.11 Flytta koordinatsystemet



Symmetri => $x'_{tp} = 0$ eller $x_{tp} = 200\,mm$

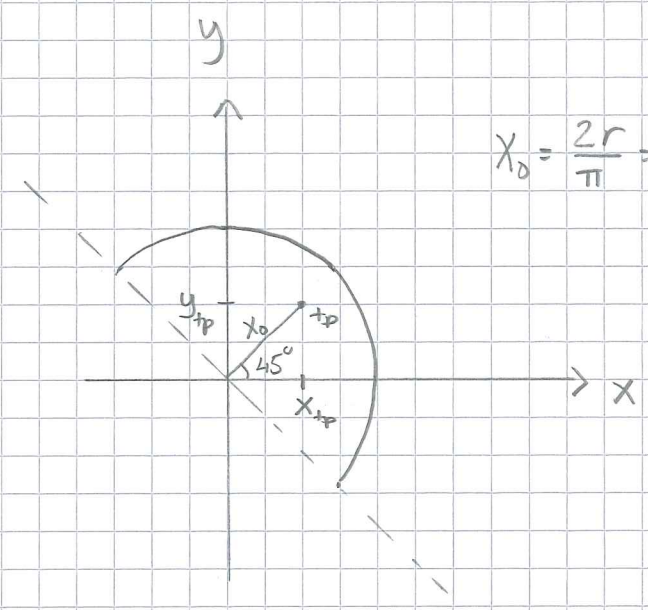
$A_I = 400 \cdot 40$ $A_{II} = 200 \cdot 70$ $A_{III} = 200 \cdot 80$

$y_I = 20$ $y_{II} = 100+40$ $y_{III} = 240+40$

	A	y_{tp}	Ay_{tp}
I	16000	20	320 000
II	14000	140	1960 000
III	16000	280	4480 000
	46000	/	6760000

$$y_{tp} = \frac{6760\,000}{46\,000} = \underline{\underline{147\,mm}}$$

4.12

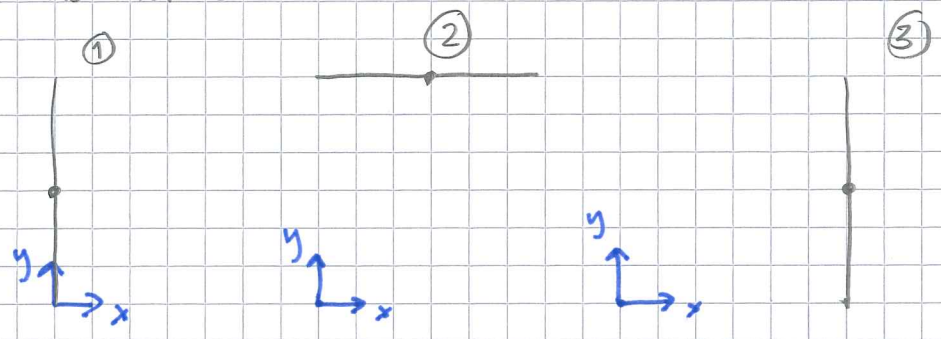


$$x_0 = \frac{2r}{\pi} = \frac{2 \cdot 15}{\pi} = 9{,}55$$

$$x_{tp} = 9{,}55 \cos 45^\circ = \underline{6{,}75}$$

$$y_{tp} = x_{tp} = \underline{6{,}75}$$

4.13 Area ersätts med längd då tvärsnitt är konstant.

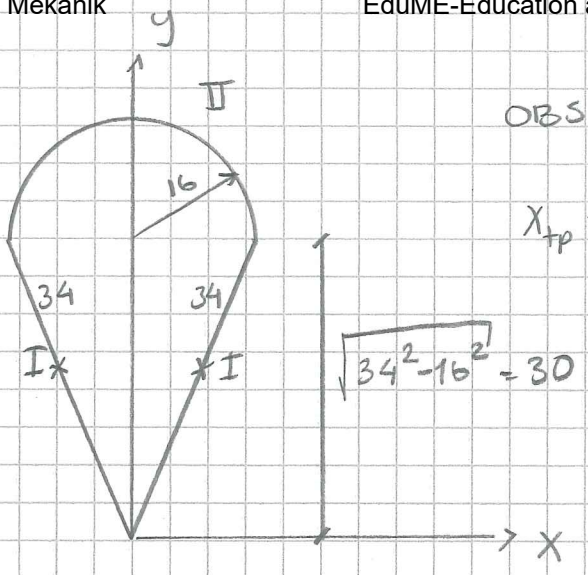


$L = 30$	$L = 30$	$L = 30$
$x^1_{tp} = 0$	$x^2_{tp} = 15$	$x^3_{tp} = 30$
$y^1_{tp} = 15$	$y^2_{tp} = 30$	$y^3_{tp} = 15$

$$x_{tp} = \frac{\Sigma L \cdot x_{tp}}{L_{tot}} = \frac{30 \cdot 0 + 30 \cdot 15 + 30 \cdot 30}{3 \cdot 30} = \underline{15}$$ "inses"

$$y_{tp} = \frac{30 \cdot 15 + 30 \cdot 30 + 30 \cdot 15}{3 \cdot 30} = \underline{20}$$

4,14



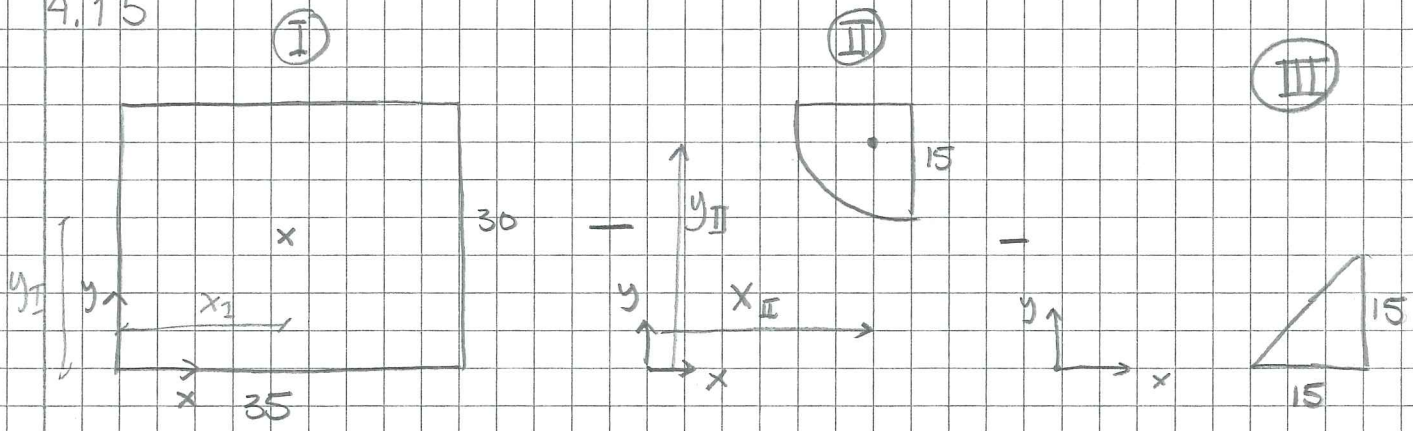
OBS! tråd

$x_{tp}=0$ ty symmetri

	L	y_{tp}	$L\cdot y_{tp}$
I	$2\cdot 34$	$\frac{\sqrt{34^2-16^2}}{2}=15$	1020
II	$\frac{32\cdot\pi}{2}=16\pi$	$30+\frac{2\cdot 16}{\pi}=40,18$	2020
Σ	118,27		3039,7

$$y_{tp}=\frac{3039.7}{118,27}=\underline{\underline{25,7}} \qquad \underline{\underline{x_{tp}=0}}$$

4.15



(I) $A_I = 35 \cdot 30 = 1050$

$x_I = \frac{35}{2} = 17,5$

$y_I = \frac{30}{2} = 15$

(II) $A_{II} = \frac{15^2 \cdot \pi}{4} = 176,7$

$x_{II} = 35 - x_0 = 35 - \frac{4 \cdot 15}{3\pi} = 28,6$ ($x_0 = 6,37$)

$y_{II} = 30 - x_0 = 30 - 6,37 = 23,6$

(III) $A_{III} = \frac{15 \cdot 15}{2} = 112,5$

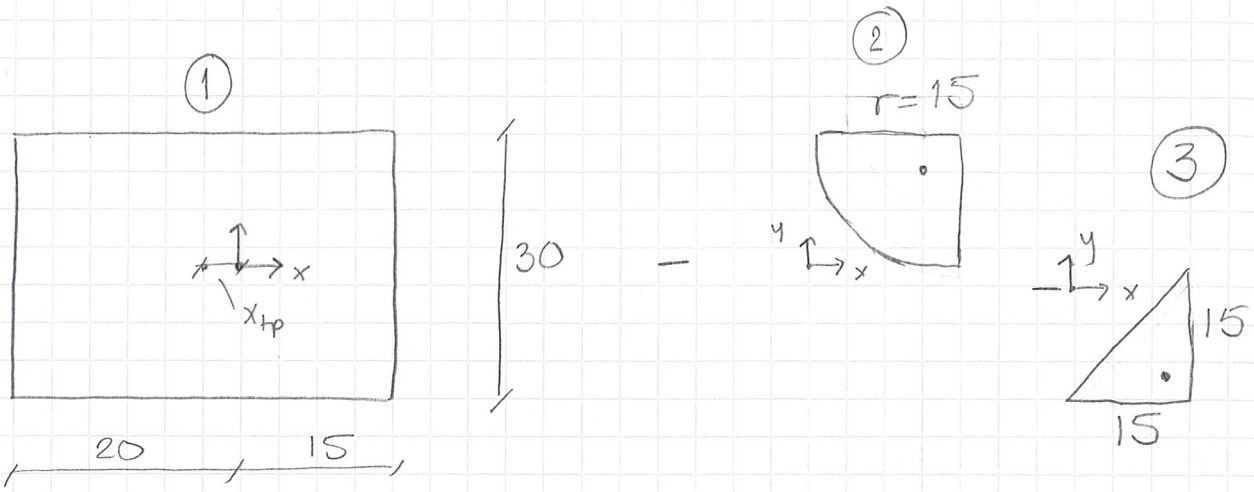
$x_{III} = 35 - \frac{h}{3} = 35 - 5 = 30$

$y_{III} = \frac{15}{3} = 5$

$$x_{tp} = \frac{1050 \cdot 17,5 - 177 \cdot 28,6 - 112,5 \cdot 30}{1050 - 177 - 112,5} = 13,07 \text{ mm}$$

$$y_{tp} = \frac{1050 \cdot 15 - 177 \cdot 23,6 - 112,5 \cdot 5}{1050 - 177 - 112,5} = 14,48 \text{ mm}$$

4.15 Bokens koordinatsystem

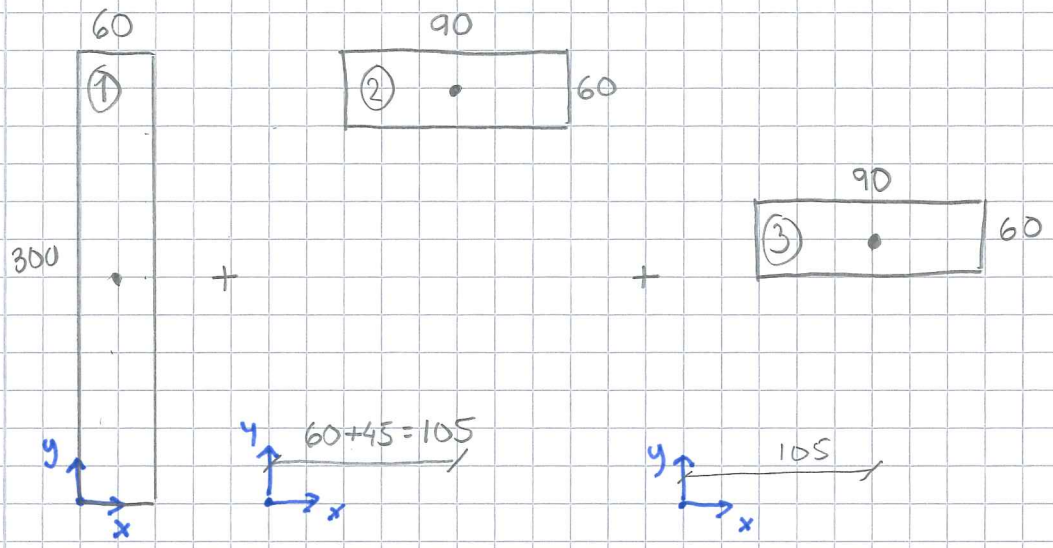


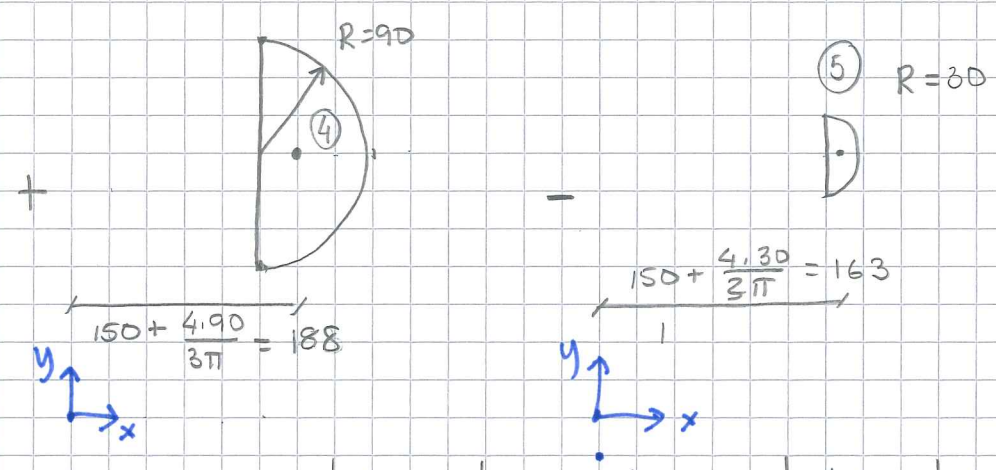
	A	x_{tp}	y_{tp}	$A x_{tp}$	$A y_{tp}$
1	$30 \cdot 35 = 1050$	$-2,5$	0	-2625	0
2	$-15^2\pi/4 = -177$	$15 - \frac{4 \cdot 15}{3\pi} = 8,63$	8,63	-1527	-1527
3	$-15^2/2 = -112,5$	10	-10	-1125	1125
	760,5			-5277	-402

$$x_{tp} = \frac{-5277}{760,5} = -6,9$$

$$y_{tp} = \frac{-402}{760,5} = 0,53$$

4.16 Många lösningsstrategier är möjliga.
Försök att minska antal delytor





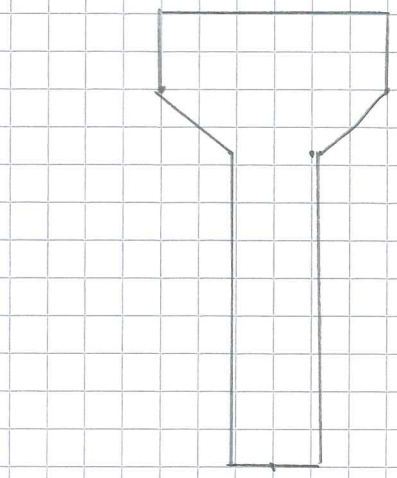
	A_n	x_{tp}	y_{tp}	$A_n x_{tp}$	$A_n y_{tp}$
1	60·300 = 18000	30	150	540000	2700000
2	90·60 = 5400	105	270	567000	1458000
3	90·60 = 5400	105	150	567000	810000
4	$\frac{90^2\cdot\pi}{2}$ = 12723	188	210	2391924	2671830
5	$\frac{-30^2\cdot\pi}{2}$ = -1414	163	210	-230482	-296940
Σ	40109	\	\	3835442	7342890

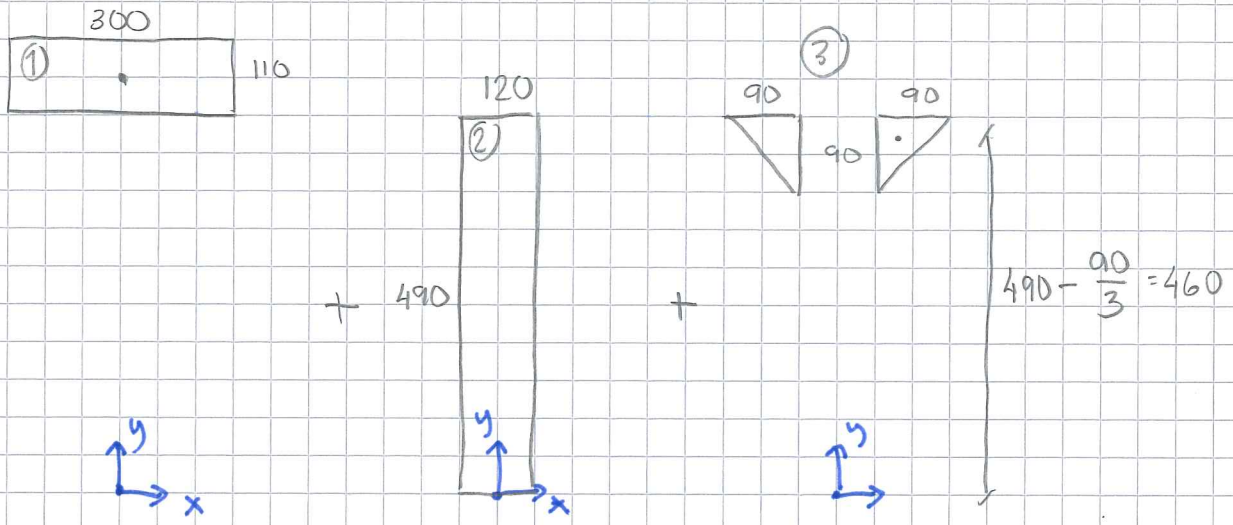
$$x_{tp} = \frac{3835442}{40109} = 95{,}6$$

$$y_{tp} = \frac{7342890}{40109} = 183$$

4.17

$x_{tp} = 0$ (symmetri)





	A_n	y_{tp}	$A_n y_{tp}$
①	$300 \cdot 110 = 33000$	545	17985 000
②	$490 \cdot 120 = 58800$	245	14406000
③	$\frac{90^2}{2} \cdot 2 = 8100$	460	3726000
	99900		36117 000

$$y_{tp} = \frac{36117000}{99900} = \underline{362\ mm}$$