2. Axiell belastning och yttryck

2.1	Axiell belastning

Sökt: σ_{AB} , σ_{BC}

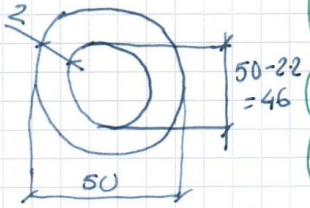
$\sigma = \frac{N}{A}$ s.25

$A_{BC} = \frac{\pi d^2}{4} = \frac{\pi \cdot 12^2}{4} = 113 mm^2$

$A_{AB} = \frac{(D^2 - (D-2t)^2) \cdot \pi}{4} =$

$= \frac{(50^2 - 46^2) \cdot \pi}{4} = 302 mm^2$

Arean finns på s.18 i KB. Hänvisar inte till detta i forts.
$A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$
Jag använder diameter då det nästan alltid är diametern som är det angivna måtten



$1 N/mm^2 = 1 MPa$

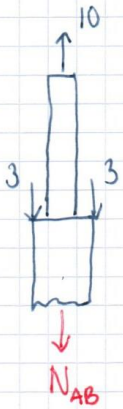
BC

$\downarrow N_{BC} - 10 = 0 \quad N_{BC} = 10 kN$

$\sigma_{BC} = \frac{N_{DC}}{A_{BC}} = \frac{10 \cdot 10^3}{113} = 88,5 N/mm^2$

$\sigma_{BC} = 88,5 MPa$

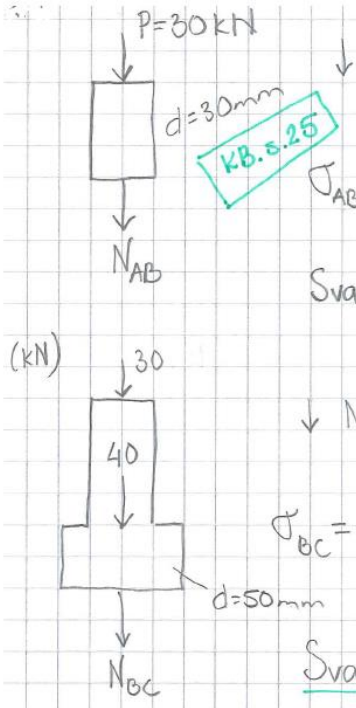
AB



$\downarrow N_{AB} + 2 \cdot 3 - 10 = 0 \quad N_{AB} = 4 kN$

$\sigma_{AB} = \frac{N_{AB}}{A_{AB}} = \frac{4 \cdot 10^3}{302} = 13,2 MPa$

2.2	Axiell belastning



$\downarrow N_{AB} + 30 = 0 \Rightarrow N_{AB} = -30\,kN$

$\sigma_{AB} = \frac{N_{AB}}{A_{AB}} = \frac{-30 \cdot 10^3 \cdot 4}{\pi \cdot 30^2} = -42{,}4\,N/mm^2 = -42{,}4\,MPa$

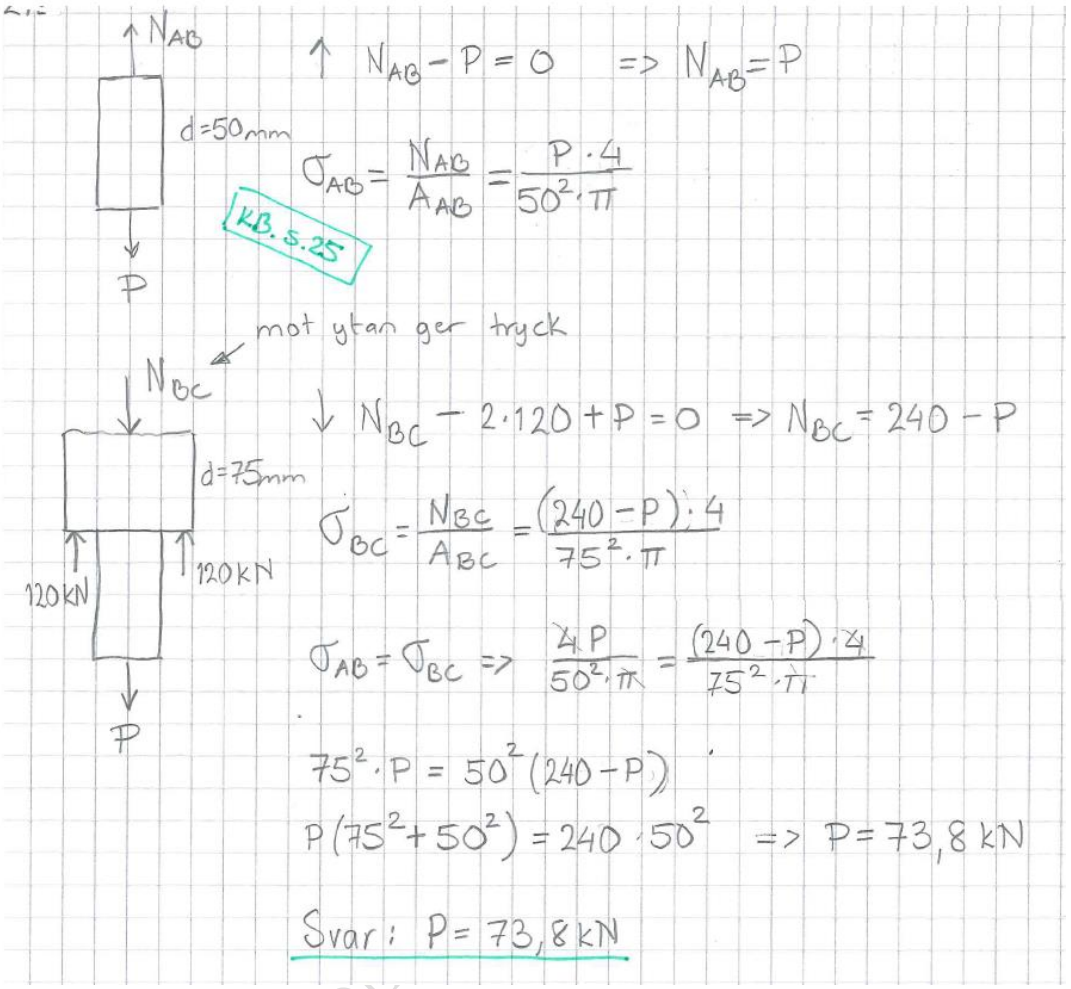
Svar: $\sigma_{AB} = -42{,}4\,MPa$ (tryck)

$\downarrow N_{BC} + 30 + 40 = 0 \Rightarrow N_{BC} = -70\,kN$

$\sigma_{BC} = \frac{N_{BC}}{A_{BC}} = \frac{-70 \cdot 10^3 \cdot 4}{50^2 \cdot \pi} = -35{,}7\,N/mm^2 = 35{,}7\,MPa$

Svar: $\sigma_{BC} = -35{,}7\,MPa$ (tryck)

2.3	Axiell belastning



$\uparrow N_{AB} - P = 0 \Rightarrow N_{AB} = P$

$\sigma_{AB} = \frac{N_{AB}}{A_{AB}} = \frac{P \cdot 4}{50^2 \cdot \pi}$

KB. s.25

$\downarrow N_{BC} - 2 \cdot 120 + P = 0 \Rightarrow N_{BC} = 240 - P$

$\sigma_{BC} = \frac{N_{BC}}{A_{BC}} = \frac{(240-P) \cdot 4}{75^2 \cdot \pi}$

$\sigma_{AB} = \sigma_{BC} \Rightarrow \frac{4P}{50^2 \cdot \pi} = \frac{(240-P) \cdot 4}{75^2 \cdot \pi}$

$75^2 \cdot P = 50^2 (240 - P)$

$P(75^2 + 50^2) = 240 \cdot 50^2 \Rightarrow P = 73{,}8 \text{ kN}$

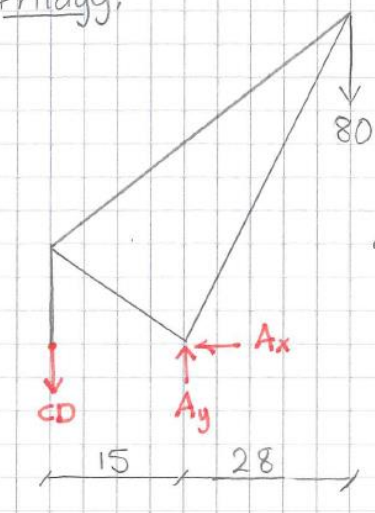
Svar: P = 73,8 kN

2.4	Axiell belastning

Sökt: σ_{CD}

CD är en tvåkraftskropp => endast kraft i dess riktning.

Frilägg:



$\curvearrowleft A: \quad 80 \cdot 10^3 \cdot 9{,}81 \cdot 28 - CD \cdot 15 = 0$

$CD = 1464960\ N$

$\sigma_{CD} = \frac{1464960}{50 \cdot 150} = 195\ MPa$

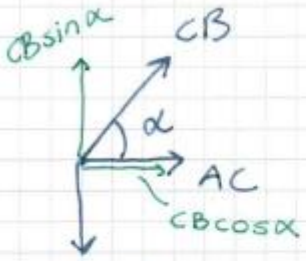
2.5	Axiell belastning

Axiell belastning

$\sigma = \frac{N}{A}$

Frilägg knutpunkt C

$\alpha = \arctan\left(\frac{3}{1,5}\right) = 63,4°$



$\uparrow \quad CB \sin 63,4° - 25 = 0$

$CB = 27,95 \text{ kN}$

$\rightarrow \quad AC + CB\cos 63,4 = 0$

$AC = -12,52 \text{ kN}$

a)

$A = 40 \cdot 4 = 160 \text{ mm}^2$

$\sigma_{CB} = \frac{CB}{A} = \frac{27,95 \cdot 10^3}{160} = \underline{175 \text{ MPa}}$ (drag)

$\sigma_{AC} = \frac{AC}{A} = \frac{-12,52 \cdot 10^3}{160} = \underline{-78 \text{ MPa}}$ (tryck)

b) $n = \frac{\text{Jämförandevärde}}{\text{Faktiskt värde}}$

S235 s.50 kB
$R_{eL} = 210 \text{ MPa}$
$R_m = 360 \text{ MPa}$

$n_S = \frac{R_{eL}}{\sigma_{CB}} = \frac{210}{175} = \underline{1,2}$ permanenta def.

$n_B = \frac{R_m}{\sigma_{CB}} = \frac{360}{175} = \underline{2,05}$ brott

2.6	Axiell belastning

Sökt: b

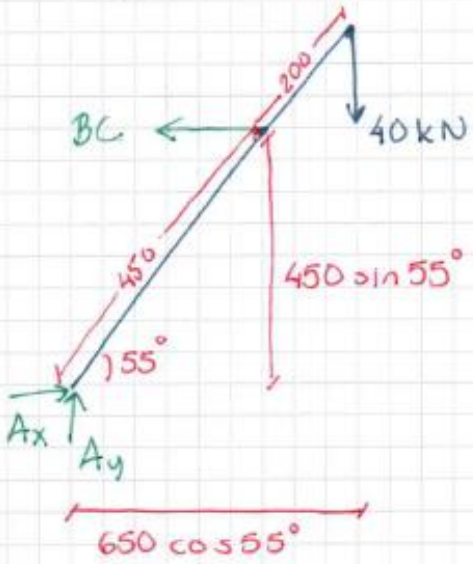
Givet:

$t = 6\,mm$

$\sigma_{till} = 150\,MPa$

Bestäm kraften i BC

Frilägg:



$\curvearrowleft A \quad BC \cdot 450 \sin 55^\circ - 40 \cdot 650 \cos 55^\circ = 0$

$BC = 40{,}46\,kN$

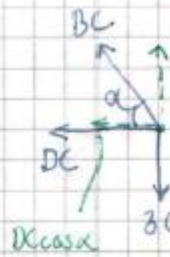
$\sigma_{till} = \frac{BC}{t \cdot b} \Rightarrow b = \frac{BC}{t \cdot \sigma_{till}} = \frac{40460}{6 \cdot 100} = 45\,mm$

2.7	Axiell belastning

Sökt: n_{AB} , d_{BE}

Krafter i AB & BC måste bestämmas.

Knutpunkt C

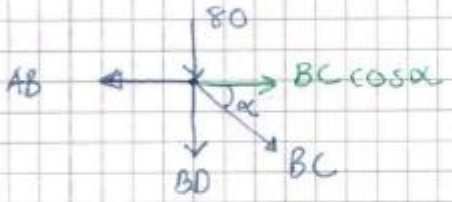


$\alpha = \arctan\left(\frac{900}{1300}\right) = 34,7°$

$\downarrow 80 - BC\sin\alpha = 0$

$BC = 52,70$ kN.

Knutpunkt B



$\leftarrow AB - BC\cos\alpha = 0$

$AB = 52,70 \cdot \cos 34,7° = 43,3$ kN

a) $n_{AB} = \frac{AB_{max}}{AB} = \frac{55}{43,3} = 1,27 = 1,2$ (avrunda nedåt)

b) jämnstarka $\Rightarrow \sigma_{AB} = \sigma_{BC}$

$\sigma_{AB} = \frac{43,3 \cdot 10^3 \cdot 4}{24^2 \cdot \pi} = \frac{52,7 \cdot 10^3 \cdot 4}{d^2 \cdot \pi}$

$d = \sqrt{\frac{52,7}{43,3} \cdot 24^2} = \sqrt{\frac{52,7}{43,3}} \cdot 24 = 26,5\text{ mm} = 27\text{ mm}$

2.8	Axiell belastning

Sökt: d om $n_s = 2$ och matrialet S275JR

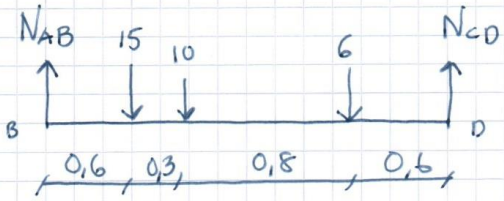
KB s.50 $\Rightarrow R_{eL} = 250$ MPa

$\sigma_{till} = \frac{R_{eL}}{n} = \frac{250}{2} = 125$ MPa

KB s.25

$\sigma = \frac{N}{A}$

Friläggning



$\circlearrowleft B$ $N_{CD}(0,6+0,3+0,8+0,6) - 6(0,6+0,3+0,8) \ldots$
$-10(0,3+0,6) - 15 \cdot 0,6 = 0$

$$N_{CD} = \frac{6 \cdot 1,7 + 10 \cdot 0,9 + 15 \cdot 0,6}{2,3} = 12,26 \text{ kN}$$

$\uparrow N_{AB} + N_{CD} - 15 - 10 - 6 = 0 \Rightarrow N_{AB} = 18,74$ kN

$N_{AB} > N_{CD} \Rightarrow F_{max} = N_{AB} = 18,74$ kN (dimensionerande kraft)

$$\sigma_{till} = \frac{F_{max}}{A} = \frac{F_{max} \cdot 4}{\pi d^2}$$

$$d = \sqrt{\frac{F_{max} \cdot 4}{\pi \cdot \sigma_{till}}} = \sqrt{\frac{18740 \cdot 4}{\pi \cdot 125}} = 13,8 \text{ mm}$$

Svar: Välj $d = 14$ mm

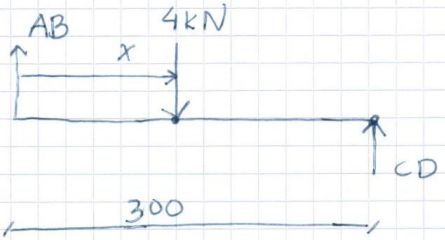
2.9	Axiell belastning

Sökt: x då dragspänningen stången (d) lika stor som tryckspänningen i fyrkantsstaven

$d = 18\,mm \Rightarrow A_{AB} = \frac{\pi \cdot 18^2}{4} = 255\,mm^2$

$a \times a = 25 \times 25\,mm \quad A_{CD} = 25^2 = 625\,mm^2$

Frilägging



$\uparrow \quad AB + CD - 4 = 0 \qquad CD = 4 - AB \qquad (1)$

$\circlearrowright_B \quad 4x - 300 \cdot CD = 0 \qquad (2)$

Spänningsvillkor

$\sigma_{AB} = |\sigma_{CD}|$

$\frac{AB}{A_{AB}} = \frac{CD}{A_{CD}} \Rightarrow CD = \frac{A_{CD}}{A_{AB}} \cdot AB = \frac{625}{255} AB = 2{,}45\,AB$

$(1) \Rightarrow 2{,}45\,AB = 4 - AB \qquad AB = \frac{4}{2{,}45 + 1} = 1{,}16\,kN.$

$CD = 2{,}45 \cdot 1{,}16 = 2{,}84\,kN$

$(2) \quad x = \frac{300\,CD}{4} = \frac{300 \cdot 2{,}84}{4} = \underline{213\,mm}$

$\sigma_{AB} = \frac{1160}{255} = 4{,}55\,MPa$

$\sigma_{CD} = \frac{2840}{625} = 4{,}55\,MPa \quad OK!$

2.10	Axiell belastning

Du kan även ta krafterna från uppgift 2.2. I denna lösning sätts tryck vid friläggning (NAB och NBC) då man ser att det blir tryck i dessa stänger.

Sökt: δ_{tot} , δ_B

$\delta = \frac{FL}{AE}$ KB s. 25

$\delta_{tot} = \delta_{AB} + \delta_{BC}$

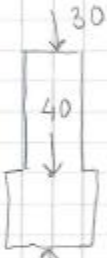
$E_{AB} = 200\,GPa = 200\,kN/mm^2$

Bestäm F_{AB} & F_{BC}

$E_{BC} = 105\,GPa = 105\,kN/mm^2$

$\uparrow F_{AB} - 30 = 0 \Rightarrow F_{AB} = 30\,kN$

$N_{AB} = F_{AB}$



$\uparrow F_{BC} - 30 - 40 = 0 \quad F_{BC} = 70\,kN$

$N_{BC} = F_{BC}$

$$\delta_{AB} = \frac{30 \cdot 10^3 \cdot 250 \cdot 4}{30^2 \cdot \pi \cdot 200 \cdot 10^3} = 0{,}0531\,mm$$

$$\delta_{BC} = \frac{70 \cdot 10^3 \cdot 300 \cdot 4}{50^2 \cdot \pi \cdot 105 \cdot 10^3} = 0{,}102\,mm$$

$\delta_{tot} = 0{,}0531 + 0{,}102 = 0{,}155\,mm$ (hoptryckning)

$\delta_B = \delta_{BC} = 0{,}102\,mm \downarrow$

Svar: $\delta_{tot} = 0{,}155\,mm$ (hoptrycks)

$\delta_B = 0{,}102\,mm \downarrow$

2.11	Axiell belastning

Sökt: P då $\delta_{tot} = 0$ samt δ_B för P.

Givet:

$E_{AB} = 200\ GPa = 200\ kN/mm^2$

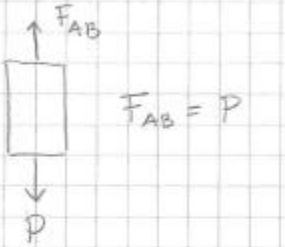
$E_{BC} = 105\ GPa = 105\ kN/mm^2$

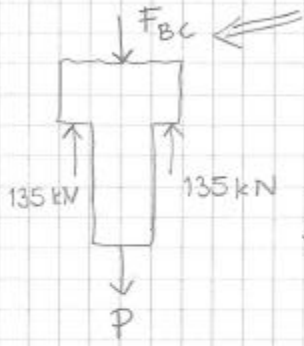
$\delta = \frac{FL}{AE}$ KB s.25

Villkor: $\delta_{AB} + \delta_{BC} = 0 \Rightarrow \delta_{AB} = -\delta_{BC}$

⇑ antar tryck i BC

$|\delta_{AB}| = |\delta_{BC}|$ (1)





↓ $F_{BC} + P - 2 \cdot 135 \cdot 10^3 = 0$

$F_{BC} = 270 \cdot 10^3 - P$

$$(1) \Rightarrow \frac{P \cdot 1000 \cdot 4}{50^2 \cdot \pi \cdot 200 \cdot 10^3} = \frac{(270 \cdot 10^3 - P) \cdot 750 \cdot 4}{75^2 \cdot \pi \cdot 105 \cdot 10^3}$$

$$\frac{1000\,P}{50^2 \cdot 200} = \frac{(270 \cdot 10^3 - P) \cdot 750}{75^2 \cdot 105}$$

$$\frac{P}{500} = \frac{(270 \cdot 10^3 - P)}{787{,}5}$$

$$\frac{P}{500} + \frac{P}{787{,}5} = \frac{270 \cdot 10^3}{787{,}5} \Rightarrow P = 104{,}8\ kN$$

$$\delta_B = \delta_{BC} = \frac{(270 - 104{,}8) \cdot 10^3 \cdot 750 \cdot 4}{75^2 \cdot \pi \cdot 105 \cdot 10^3} = 0{,}267\ mm \uparrow$$

Svar: $P = 104{,}8\ kN$

$\delta_B = 0{,}267\ mm \uparrow$

2.12	Axiell belastning

Du kan även ta krafterna från uppgift 2.1

Sökt: a) δ_{AC} b) δ_B

$E = 70\ GPa$

Bestäm krafterna AB & BC

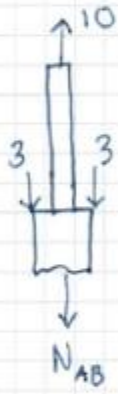
$\delta = \frac{FL}{AE}$

$\downarrow N_{BC} - 10 = 0$

$N_{BC} = 10\ kN$

Tvärsnitt

$A_{BC} = \frac{12^2 \cdot \pi}{4} = 113\ mm^2$



$\downarrow N_{AB} + 2 \cdot 3 - 10 = 0$

$N_{AB} = 10 - 6 = 4\ kN$

$A_{AB} = \frac{\pi}{4}(50^2 - 46^2) = 302\ mm^2$

alternativ för tunnväggigt rör

A = omkrets x tjocklek.

medel

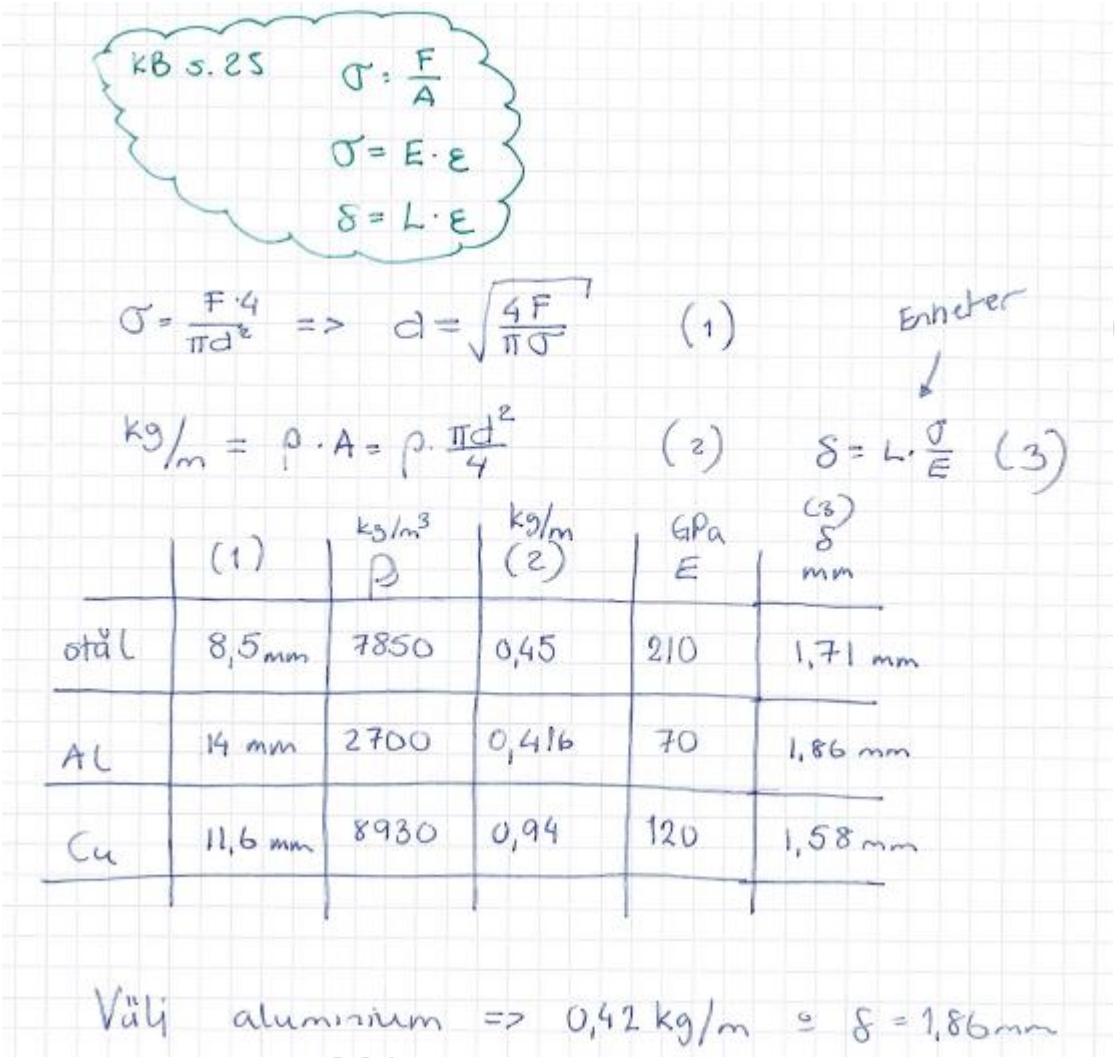
$A_{AB} = 48 \cdot \pi 2 = 302\ mm^2$

a) $\delta_{AC} = \delta_{AB} + \delta_{BC} = \frac{4000 \cdot 700}{302 \cdot 70 \cdot 10^3} + \frac{10\,000 \cdot 1500}{113 \cdot 70 \cdot 10^3} =$

$= 0{,}1325 + 1{,}8963 = 2{,}03\ mm$ (längre)

b) $\delta_B = \delta_{AB} = 0{,}13\ mm$ uppåt

2.13	Axiell belastning



	(1)	kg/m³ ρ	kg/m (2)	GPa E	(3) δ mm
stål	8,5 mm	7850	0,45	210	1,71 mm
Al	14 mm	2700	0,416	70	1,86 mm
Cu	11,6 mm	8930	0,94	120	1,58 mm

2.14	Axiell belastning

Sökt: d_{min}, σ

Givet: $L = 60\,m$

$\delta_{max} = 48\,mm$

$F = 6\,kN$

$E = 200\,GPa = 200 \cdot 10^9\,Pa = 200 \cdot 10^9\,N/m^2$

$\delta = \frac{FL}{AE}$ KB s.25

$A_{min} = \frac{FL}{\delta_{max} \cdot E} = \frac{\pi d^2}{4}$

$d = \sqrt{\frac{4FL}{\pi \delta_{max} E}} = \sqrt{\frac{4 \cdot 6000 \cdot 60}{\pi \cdot 0{,}048 \cdot 200 \cdot 10^9}} = 6{,}9 \cdot 10^{-3}\,m = 7\,mm$

$\sigma = \frac{F}{A} = \frac{6000 \cdot 4}{7^2 \cdot \pi} = 156\,MPa$

Svar: Välj $d = 7\,mm \Rightarrow \sigma = 156\,MPa$

2.15	Axiell belastning

Sökt: d samt ε i %

Givet: $F = 10\ N$

$E = 2{,}8\ GPa$

$\sigma_{tu} = 40\ MPa = 40\ N/mm^2$

$\sigma_{tu} = \frac{F \cdot 4}{\pi d^2}$

$d = \sqrt{\frac{10 \cdot 4}{40 \cdot \pi}} = 0{,}564\ mm$

Välj $d = 0{,}6\ mm \Rightarrow \sigma = \frac{10 \cdot 4}{\pi \cdot 0{,}6^2} = 35{,}4\ MPa$

Hooks lag KB. s.24

$\sigma = E \cdot \varepsilon \Rightarrow \varepsilon = \frac{35{,}4 \cdot 10^6}{2{,}8 \cdot 10^9} = 0{,}0126 \Rightarrow 1{,}26\ \%$

Svar: $d = 0{,}6\ mm$

$\varepsilon = 1{,}26\ \%$

2.16	Axiell belastning

Sökt: d_{min}

Givet: $F = 11\,N$

$E = 3{,}1\,GPa$

$\sigma_{till} = 40\,MPa$

$\varepsilon_{max} = 1\% = 0{,}01$

Kontrollera om töjning eller spänning är dimensionerande.

Hooks lag: $\sigma = E \cdot \varepsilon$

ε_{till} om $\sigma_{till} = 40\,MPa$ $\quad \varepsilon_{till} = \frac{\sigma_{till}}{E} = \frac{40 \cdot 10^6}{3{,}1 \cdot 10^9} = 0{,}0129 \Rightarrow 1{,}3\%$

Om spänningen är 40 MPa får vi töjningen 1,3% men endast $\varepsilon = 1\%$ är tillåten => töjningen är dimensionerande.

Bestäm spänning som ger 1% töjning.

$$\sigma_{dim} = E \cdot \varepsilon_{max} = \frac{F \cdot 4}{\pi d_{min}^2}$$

$$d_{min} = \sqrt{\frac{4F}{E\,\varepsilon_{max}\,\pi}} = \sqrt{\frac{4 \cdot 11}{3{,}1 \cdot 10^9 \cdot 0{,}01 \cdot \pi}} = 0{,}672\,mm$$

Svar: $d_{min} = 0{,}672\,mm$

2.17	Axiell belastning

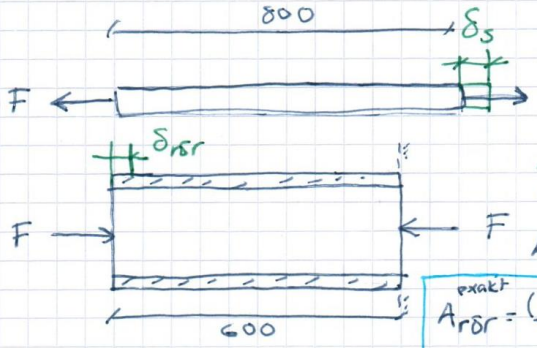
Sökt: C:s förflyttning

$\delta = \frac{FL}{AE}$ KB s. 25

$E_{al} = 70 \cdot 10^3 \ N/mm^2$

$E_{stål} = 210 \cdot 10^3 \ N/mm^2$

Frilägging



$A_s = \frac{d^2 \cdot \pi}{4} = \frac{20^2 \cdot \pi}{4} = 314 mm^2$

enkelt

$A_{rör} = d_y \pi t$ om $t << d_y$

$A_{rör} = 60 \cdot \pi \cdot 2 = 377 mm^2$

exakt

$A_{rör} = \frac{(60^2 - 56^2) \cdot \pi}{4} = 364 mm^2$

ca: 3% avvikelse

$\delta_{tot} = \delta_s + \delta_{rör}$

$\delta_{tot} = \frac{40 \cdot 10^3 \cdot 800}{314 \cdot 70 \cdot 10^3} + \frac{40 \cdot 10^3 \cdot 600}{377 \cdot 210 \cdot 10^3} = 1,456 + 0,303$

364 0,314

$\delta_{tot} = 1,76 \ mm$ åt höger.

1,77 A exakt

2.18	Axiell belastning

Sökt: σ_s, σ_r, δ_s, δ_r

Givet:

stigning $P = 1{,}5\,mm$

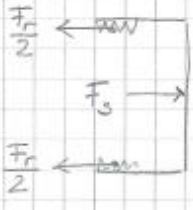
1/4 varv $\Rightarrow \delta_{tot} = \frac{1{,}5}{4}$

$E_{al} = 70\,kN/mm^2$ $E_m = 105\,kN/mm^2$

$\delta = \frac{FL}{AE}$ KB s.25

- Röret dras ut.
- Stången trycks ihop.
- Rör och stång påverkas av samma kraft |F|

Frilägg lock



$\rightarrow F_s = 2 \cdot \frac{F_r}{2} \Rightarrow F_s = F_r = F$

Kraften lika $\Rightarrow$ seriekopplade

$\delta_{tot} = |\delta_r| + |\delta_s|$

$$\frac{1{,}5}{4} = \frac{P \cdot 250 \cdot 4}{(36^2 - 28^2)\cdot\pi \cdot 70 \cdot 10^3} + \frac{P \cdot 250 \cdot 4}{25^2 \pi \cdot 105 \cdot 10^3}$$

$$\frac{1{,}5}{4} = \frac{4 \cdot 250 \cdot P}{\pi \cdot 10^3}\left(\frac{1}{(36^2 - 28^2)\cdot 70} + \frac{1}{25^2 \cdot 105}\right)$$

$0{,}3183$

$$0{,}375 = 1{,}373 \cdot 10^{-5} P$$

$$P = 27310\,N$$

$$\sigma_s = \frac{27310 \cdot 4}{25^2 \cdot \pi} = 55{,}6\,MPa$$

$$\delta_s = \frac{FL}{AE} = \sigma_s \frac{L}{E_m} = 55{,}6 \cdot \frac{250}{105 \cdot 10^3} = 0{,}132\,mm$$

$$\sigma_r = \frac{27310 \cdot 4}{(36^2 - 28^2)\cdot\pi} = 67{,}9\,MPa\ (tryck)$$

$$\delta_r = \sigma_r \frac{L}{E_A} = 67{,}9 \cdot \frac{250}{70 \cdot 10^3} = 0{,}243\,mm$$

Tanke hjälp

1. Anta rör $\Rightarrow$ stelt

2. Anta stång stel

3.

Delar δ_{tot} går åt att dra ut röret resten åt att trycka ihop stången.

$\delta_{tot} = |\delta_r| + |\delta_s|$

Svar: $\sigma_s = 55{,}6\,MPa$ (drag) $\delta_s = 0{,}132\,mm$

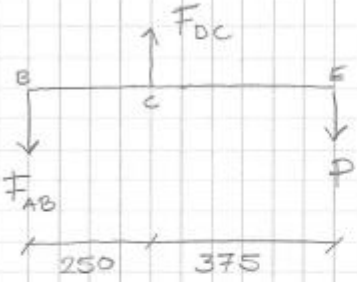
$\sigma_r = 67{,}9\,MPa$ (tryck) $\delta_r = 0{,}243\,mm$

2.19	Axiell belastning

Sökt: P om $\delta_{B,max} = 0{,}25$ mm

Givet: $E = 200$ GPa

Friläggning



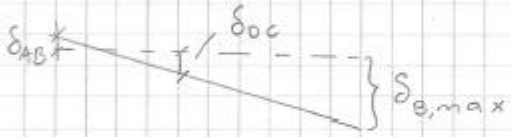
$\curvearrowleft C$ $\quad F_{AB} \cdot 250 - P \cdot 375 = 0$

$F_{AB} = 1{,}5\,P$

$\uparrow \quad F_{DC} - F_{AB} - P = 0$

$F_{DC} = P + 1{,}5\,P = 2{,}5P$

Geometri:



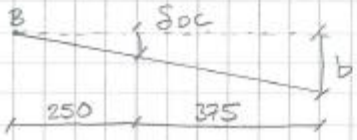
Geometriska samband genom superposition

Tänk en effekt i taget & summera

DC antas stel

$\frac{a}{375} = \frac{\delta_{AB}}{250} \Rightarrow a = 1{,}5\,\delta_{AB}$

AB antas stel



$\frac{b}{(250+375)} = \frac{\delta_{DC}}{250} \Rightarrow b = 2{,}5\,\delta_{DC}$

2.19 forts.	Axiell belastning

$\delta_{B,max} = a + b = 1{,}5\,\delta_{AB} + 2{,}5\,\delta_{CD}$

$L_{AB} = L_{CD} = 200\,mm$

$A_{AB} = A_{CD} = 6 \cdot 25 = 150\,mm^2$

$E = 200\,GPa = 200\,kN/mm^2$

$\delta = \frac{FL}{AE}$ KB s. 25

$$0{,}25 = 1{,}5\,\frac{1{,}5P \cdot 200}{150 \cdot 200 \cdot 10^3} + 2{,}5\,\frac{2{,}5P \cdot 200}{150 \cdot 200 \cdot 10^3}$$

$$0{,}25 = \frac{P}{150 \cdot 10^3}\left(1{,}5^2 + 2{,}5^2\right) \Rightarrow P = 4412\,N$$

Svar $P = 4412\,N$ om $\delta_{B,max} = 0{,}25\,mm$

2.20	Axiell belastning

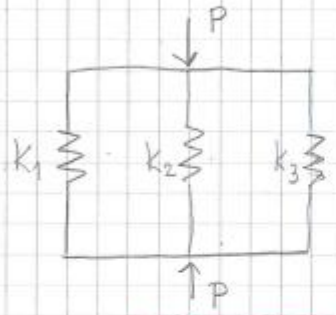
Sökt: σ_{AL} , σ_m

Givet:

$E_{al} = 70\ GPa = 70 \cdot kN/mm^2$

$E_m = 105\ GPa = 105\ kN/mm^2$

$\delta_{al} = \delta_m = \delta$ samma deformation för alla material delar. => analogi med parallell-kopplade fjädrar.



Deformation lika för alla fjädrar.
Kraft olika P fördelas över fjädrarna $P = P_1 + P_2 + P_3$

KB. s25

$\delta = \frac{FL}{AE} \Rightarrow F = \frac{\delta AE}{L}$

$A_{al} = A_m = A = 5 \cdot 30 = 150 mm^2$

$L_{al} = L_m = L = 250\ mm$

$$P = \frac{\delta A_{al} \cdot E_{al}}{L_{al}} \cdot 2 + \frac{\delta A_m E_m}{L_m} = \frac{\delta A}{L}\left(2 \cdot E_{al} + E_m\right)$$

$$30 \cdot 10^3 = \frac{\delta \cdot 150}{250}\left(2 \cdot 70 + 105\right) \cdot 10^3$$

$\delta = 0{,}204\ mm \Rightarrow F_{al} = \frac{0{,}204 \cdot 150 \cdot 70 \cdot 10^3}{250} = 8571\ N$

$F_m = 30000 - 2 \cdot 8571 = 12857\ N$

$\sigma_{al} = \frac{8571}{150} = 57{,}1\ MPa$ (tryck)

$\sigma_m = \frac{12857}{150} = 85{,}7\ MPa$ (tryck)

2.21	Axiell belastning

Sökt: δ, σ_{al}, σ_s

$\delta = \frac{FL}{AE}$ KB, s25

δ lika för kompositstången

Givet:

$E_{al} = 70\,GPa$

$E_s = 200\,GPa$

$L = 250\,mm$

$A_s = \frac{25^2 \cdot \pi}{4}$

$A_{al} = \frac{\pi}{4}(64^2 - 25^2)$

$P = F_s + F_{al}$

$P = \frac{\delta A_s E_s}{L} + \frac{\delta A_{al} E_{al}}{L}$

$P = \frac{\delta}{250}\left(\frac{\pi}{4} 25^2 \cdot 200 \cdot 10^3 + \frac{\pi}{4}(64^2 - 25^2)\, 70 \cdot 10^3\right)$

$180 \cdot 10^3 = \frac{\delta \cdot \pi \cdot 10^3}{4 \cdot 250}\left(25^2 \cdot 200 + (64^2 - 25^2) \cdot 70\right)$

$\delta = 0{,}156\,mm$

$\frac{F}{A} = \frac{\delta \cdot E}{L}$ $\left[\sigma = \underbrace{\varepsilon}_{\frac{\delta}{L}} \cdot E\right]$

$\sigma_s = \frac{0{,}156}{250} \cdot 200 \cdot 10^3 = 125\,MPa$ (tryck)

$\sigma_{al} = \frac{0{,}156}{250} \cdot 70 \cdot 10^3 = 43{,}7\,MPa$ (tryck)

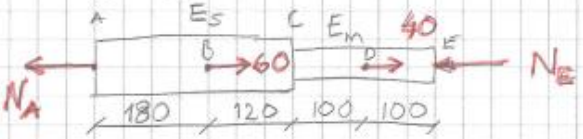
2.22	Axiell belastning

Sökt: N_A, N_E, δ_C

Givet: $E_s = 200\,GPa = 200\,kN/mm^2$

$E_m = 105\,GPa = 105\,kN/mm^2$

Frilägging

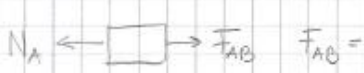


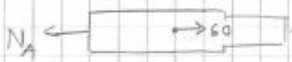
$\delta = \frac{FL}{AE}$ KB, s25

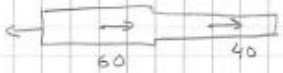
$\leftarrow \quad N_A - 60 - 40 + N_E = 0 \quad (1)$

2 obekanta 1 ekv => ej lösbart => fler villkor krävs

$\delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE} = 0$







$$\frac{N_A \cdot 180}{200 \cdot 20^2 \cdot \pi} + \frac{(N_A - 60) \cdot 120}{200 \cdot 20^2 \cdot \pi} + \frac{(N_A - 60) \cdot 100}{105 \cdot 15^2 \cdot \pi} + \frac{(N_A - 100) \cdot 100}{105 \cdot 15^2 \cdot \pi} = 0$$

$$\frac{N_A \cdot 180}{200 \cdot 20^2} + \frac{N_A \cdot 120}{200 \cdot 20^2} + \frac{N_A \cdot 100}{105 \cdot 15^2} + \frac{N_A \cdot 100}{105 \cdot 10^2} = \frac{60 \cdot 120}{200 \cdot 20^2} + \frac{60 \cdot 100}{105 \cdot 15^2} + \ldots + \frac{100 \cdot 100}{105 \cdot 15^2}$$

$$N_A \left(\frac{300}{200 \cdot 20^2} + \frac{200}{105 \cdot 15^2} \right) = \frac{60 \cdot 120}{200 \cdot 20^2} + \frac{16000}{105 \cdot 15^2}$$

$N_A = 62{,}79\,kN$ => (1) => $N_E = 100 - 62{,}79 = 37{,}21\,kN$

$\delta_C = \delta_{AB} + \delta_{BC}$

$$\delta_C = \frac{F_{AB} \cdot 180}{200 \cdot 10^3 \cdot 20^2 \cdot \pi} + \frac{F_{BC} \cdot 120}{200 \cdot 10^3 \cdot 20^2 \cdot \pi} = \frac{1 \cdot 10^3}{200 \cdot 10^3 \cdot 20^2 \cdot \pi} \left[62{,}79 \cdot 180 + 2{,}79 \cdot 120 \right]$$

$F_{AB} = N_A = 62{,}79\,kN \quad F_{BC} = N_A - 60 = 2{,}79\,kN$

$\delta_C = 0{,}0463\,mm$

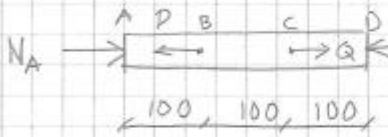
2.23	Axiell belastning

Sökt: N_A, N_D, δ_{BC}

Givet: $D_y = 32\,mm$ $E = 103\,GPa$

$D_i = 26\,mm$ $P = 190\,kN$ $Q = 160\,kN$

Friläggning



$\rightarrow N_A - N_D + 160 - 190 = 0$ (1)

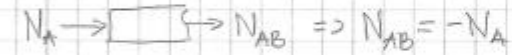
2 obekanta, 1 ekv

$\Downarrow$

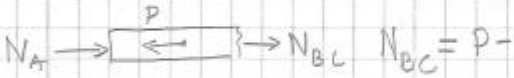
ej lösbart

$\delta_{AB} + \delta_{BC} + \delta_{CD} = 0$ (2)

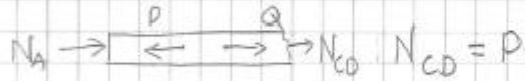
$\delta = \frac{FL}{AE}$ KB. s25



$\Rightarrow N_{AB} = -N_A$



$N_{BC} = P - N_A = 190 - N_A$



$N_{CD} = P - Q - N_A = 190 - 160 - N_A = 30 - N_A$

$$(2) \Rightarrow -\frac{N_A \cdot 100}{AE} + \frac{(190 - N_A) \cdot 100}{AE} + \frac{(30 - N_A) \cdot 100}{AE} = 0$$

$$-N_A + 190 - N_A + 30 - N_A = 0 \Rightarrow N_A = \frac{220}{3} = \underline{73{,}3\,kN}$$

$$(1) \Rightarrow N_D = N_A - 30 = 73{,}3 - 30 = \underline{43{,}3\,kN}$$

$$\delta_{BC} = \frac{N_{BC} L}{AE} = \frac{(190 - 73{,}3) \cdot 10^3 \cdot 100 \cdot 4}{(32^2 - 26^2) \cdot \pi \cdot 103 \cdot 10^3} = \underline{0{,}414\,mm}$$

2.24	Axiell belastning

Sökt: N_A, N_E, δ_C

Givet: $E_s = 200$ GPa $\quad \delta_E = 0{,}12$ mm

$E_m = 105$ GPa

Steg 1: Bestäm δ_{tot} om högra väggen inte finns.

Steg 2: Bestäm N_E som krävs för att $\delta_E = 0{,}12$ mm

dvs N_E trycker tillbaks $\delta_{tot} - \delta_E$

① $\delta_{tot} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE}$ $\quad \delta_{DE} = 0$ ty obelastad

Frilägging $\Rightarrow$ $N_{AB} = 100$ kN $\quad N_{BC} = 40$ kN

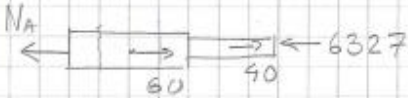
(eller insikt) $\quad N_{CD} = 40$ kN

$$\delta_{tot} = \frac{100\cdot10^3\cdot180\cdot4}{200\cdot10^3\cdot40^2\cdot\pi} + \frac{40\cdot10^3\cdot120\cdot4}{200\cdot10^3\cdot40^2\cdot\pi} + \frac{40\cdot10^3\cdot100\cdot4}{105\cdot10^3\cdot30^2\cdot\pi} = 0{,}1446$$

② $\delta_{åter} = 0{,}1446 - 0{,}12 = 0{,}0246$ mm

$$\frac{N_E\cdot(180+120)\cdot4}{200\cdot10^3\cdot40^2\cdot\pi} + \frac{N_E(100+100)\cdot4}{105\cdot10^3\cdot30^2\cdot\pi} = 0{,}0246$$

$$\frac{4\cdot N_E}{10^3\cdot\pi}\left[\frac{300}{200\cdot40^2} + \frac{200}{105\cdot30^2}\right] = 0{,}0246 \Rightarrow N_E = 6327\text{ N}$$



$N_A = 60 + 40 - 6{,}327 = 93{,}7$ kN

$$\delta_C = \delta_{AB} + \delta_{BC} = \frac{93{,}7\cdot10^3\cdot180\cdot4}{200\cdot10^3\cdot40^2\cdot\pi} + \frac{33{,}67\cdot10^3\cdot120\cdot4}{200\cdot10^3\cdot40^2\cdot\pi} = 0{,}083\text{ mm}$$

$N_{AB} = N_A = 93{,}7$ kN $\quad N_{BC} = 93{,}7 - 60 = 33{,}67$ kN

2.25	Axiell belastning

Sökt: Spänningen i stängerna

$\delta = \frac{FL}{AE}$ $\sigma = \frac{F}{A}$ KB s. 25

Friläggning

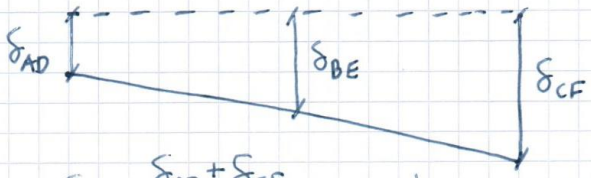
3 obekanta AD, BE, CF

2 ekv ↑ ↻ => statiskt obest.

$\uparrow \quad AD + BE + CF - 50 - 80 = 0 \qquad (1)$

$\circlearrowright D \quad 50 \cdot 1 - BE \cdot 2 + 80 \cdot 3 - CF \cdot 4 = 0 \qquad (2)$

Balken D-F är stel



$\delta_{BE} = \frac{\delta_{AD} + \delta_{CF}}{2}$ ty symmetri

$\frac{BE \cdot L}{AE} = \frac{1}{2}\left(\frac{AD \cdot L}{AE} + \frac{CF \cdot L}{AE}\right)$

$BE = \frac{1}{2}(AD + CF)$

2.25 fortsättning	Axiell belastning

(1) $AD + CF + BE = 130$

(2) $2BE + 4CF = 290$

(3) $2BE = AD + CF$

(1) o (3) => $2BE + BE = 130 \Rightarrow BE = 43{,}3\ kN$

(2) => $2 \cdot 43{,}3 + 4CF = 290$ => $CF = 50{,}8\ kN$.

(1) => $AD = 130 - 43{,}3 - 50{,}8 = 35{,}9\ kN$

$A = \frac{\pi \cdot 18^2}{4} = 255\ mm^2$

$\sigma_{AD} = \frac{35{,}9 \cdot 10^3}{255} = 141\ MPa$

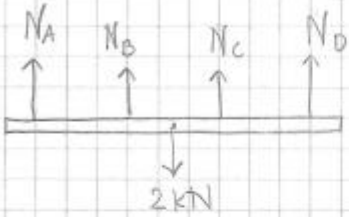
$\sigma_{BE} = \frac{43{,}3 \cdot 10^3}{255} = 170\ MPa$

$\sigma_{CF} = \frac{50{,}8 \cdot 10^3}{255} = 199\ MPa$

2.26	Axiell belastning

Sökt: N_A, N_B, N_C, N_D

$\delta_A, \delta_B, \delta_C, \delta_D$



Symmetri

$N_A = N_D \quad \delta_A = \delta_D$

$N_B = N_C \quad \delta_B = \delta_C$

$\uparrow \; 2N_A + 2N_B = 2$

$N_A + N_B = 1 \Rightarrow N_B = 1 - N_A$

Då stången är stel och krafterna ($N_A \dots N_D$) är symmetriska kring lasten kommer

$\delta_A = \delta_B = \delta_C = \delta_D$

$$\delta_A = \delta_B \Rightarrow \frac{N_A(125+150)\cdot 4}{70\cdot 10^3 \cdot 2{,}5^2 \cdot \pi} = \frac{(1-N_A)\cdot 150 \cdot 4}{200\cdot 10^3 \cdot 2^2 \cdot \pi}$$

$$\frac{N_A \cdot 275}{70 \cdot 2{,}5^2} = \frac{(1-N_A)\cdot 150}{200 \cdot 2^2}$$

$$N_A\left(\frac{275}{70\cdot 2{,}5^2} + \frac{150}{200\cdot 4}\right) = \frac{150}{200\cdot 4}$$

$N_A = 0{,}230 \text{ kN} = 230 \text{ N} = N_D$

$N_B = 1 - 0{,}230 = 0{,}770 \text{ kN} = 770 \text{ N} = N_C$

$$\delta_A = \frac{230(125+150)\cdot 4}{70\cdot 10^3 \cdot 2{,}5^2 \cdot \pi} = 0{,}184 \text{ mm} = \delta_B$$

Kontrollera att $\delta_B = 0{,}184$ mm

$$\delta_B = \frac{770 \cdot 150 \cdot 4}{200\cdot 10^3 \cdot 2^2 \cdot \pi} = 0{,}184 \text{ mm}$$ OK!!!

2.27	Axiell belastning

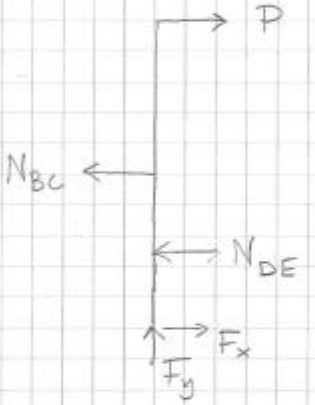
Sökt: N_{BC}, N_{DE}, δ_A

Givet: $P = 2{,}5\ kN$

tvärsnitt $12 \times 6\ mm$ $\quad E = 200\ GPa$

BC & DE är tvåkraftskroppar (länkar) tar endast upp drag / tryck

Friläggning:



4 obekanta & 3 ekvationer ↑ ← ↻

=> ej lösbart

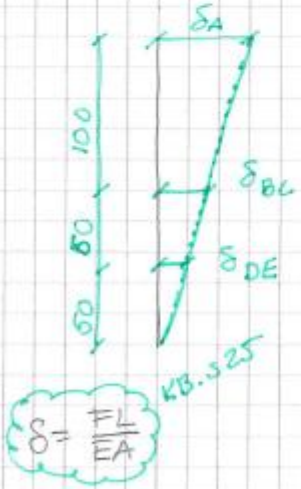
$\circlearrowright F$ $\quad P \cdot 200 - N_{BC} \cdot 100 - N_{DE} \cdot 50 = 0 \quad (1)$

(2) → (1)

$200P = 100 \cdot 2{,}5 N_{DE} + N_{DE} \cdot 50$

$$N_{DE} = \frac{200 \cdot 2{,}5}{250 + 50} = 1{,}67\ kN$$

Geometri:



KB. s 25

$\delta = \frac{FL}{EA}$

Triangellikformighet =>

$$\frac{\delta_{BC}}{100} = \frac{\delta_{DE}}{50} \Rightarrow \delta_{BC} = 2 \cdot \delta_{DE}$$

$$\frac{N_{BC} \cdot 100}{EA} = 2 \cdot \frac{N_{DE} \cdot 125}{EA} \Rightarrow N_{BC} = 2{,}5\, N_{DE}$$

(2) => $N_{BC} = 2{,}5 \cdot 1{,}67 = 4{,}17\ kN$

Triangellikformighet =>

$$\frac{\delta_A}{200} = \frac{\delta_{BC}}{100} \Rightarrow \delta_A = 2 \cdot \delta_{BC}$$

$$\delta_A = 2 \cdot \frac{4{,}17 \cdot 10^3 \cdot 100}{200 \cdot 10^3 \cdot 12 \cdot 6} = 0{,}0579\ mm$$

2.28	Yttryck

a) $p_c = \frac{F}{A_{trä}} = \frac{50 \cdot 10^3}{120 \cdot 120} = 3{,}5\ MPa$

b) $A_c = a^2$

$p_{tju} = 150\ kPa = 0{,}15\ MPa$

$p_{tju} = \frac{F}{A_c} \Rightarrow a = \sqrt{\frac{F}{p_{tju}}} = \sqrt{\frac{50 \cdot 10^3}{0{,}15}} = 577\ mm$

$a = 580\ mm$

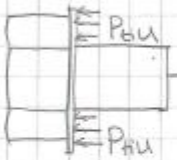
2.29	Yttryck

Sökt: σ_{tiu} i bulten

Bestäm kraften, F, i bulten som ger p_{tiu}.

Givet:

$p_{tiu} = 5\ MPa$



$$p_{tiu} = \frac{F}{A_p} = \frac{F \cdot 4}{(30^2 - 15^2)\cdot \pi} = 5$$

$$F = \frac{5\pi(30^2 - 15^2)}{4} = 2651\ N$$

$$\sigma_{tiu} = \frac{F}{A_{bult}} = \frac{2651 \cdot 4}{12^2 \cdot \pi} = 23{,}4\ N/mm^2 = 23{,}4\ MPa$$

Svar: $\sigma_{tiu} = 23{,}4\ MPa$