

Teknologi A

Mekanik

Lösningar

I detta dokument finns lösningsförslag till samtliga uppgifter i boken ”Teknologi A – Mekanik”.

På YouTube kanalen, ”Lösningar Teknologi A Mekanik”, finns inspelade lösningsförslag till många av uppgifterna. Det finns även en spellista med föreläsningar som är anpassad till boken (dock ej kombinerade lastfall och knäckning).

Jag använder mig att Karl Björks ”Formler och Tabeller för Mekanisk konstruktion” när jag löser uppgifterna.

Om du inte har en formelsamling i maskinkonstruktion så rekommenderar jag starkt Karl Björks formelsamling. Denna finns att beställa på bjorksforlag.se

En komplett översikt av kanalen och materialet finns på edume.ru

/Madeleine



<https://www.youtube.com/channel/UCZWty6uAUlkab9XyHQAu9Q>



Madeleine Hermann

EduME – Education and Mechanical Engineering

$$2.1 \quad \rho = 2,65 \cdot 10^3 \text{ kg/m}^3$$

$$V = 1,21 \cdot 2,03 \cdot 0,545 = 1,339 \text{ m}^3$$

$$m = \rho \cdot V = 2,65 \cdot 10^3 \cdot 1,339 = 3550 \text{ kg}$$

$$2.2 \quad L = 547 \text{ mm}$$

$$d = 40,0 \text{ mm}$$

$$\rho = 7,80 \cdot 10^3 \text{ kg/m}^3$$

$$G = mg = V \cdot \rho \cdot g = 0,547 \cdot \frac{0,04^2 \cdot \pi}{4} \cdot 7,8 \cdot 10^3 \cdot 9,81 = 52,6 \text{ N}$$

$$2.3 \quad d_y = 40 \text{ mm} \quad d_i = 35 \text{ mm}$$

$$a) \quad m = V \cdot \rho = 0,547 \cdot \left(\frac{0,04^2 \cdot \pi}{4} - \frac{0,035^2 \cdot \pi}{4} \right) \cdot 7,8 \cdot 10^3 =$$

$$= 0,547 \cdot \frac{(0,04^2 - 0,035^2) \pi}{4} \cdot 7,8 \cdot 10^3 = 1,257 \text{ kg}$$

$$b) \quad \% \text{ viktförändring} = \frac{m_{\text{rör}}}{m_{\text{stäng}}} = \frac{A_{\text{rör}} \cdot L \cdot \rho}{A_{\text{stäng}} \cdot L \cdot \rho} = \frac{A_{\text{rör}}}{A_{\text{stäng}}}$$

$$\% \text{ viktförändring} = \frac{(40^2 - 35^2) \cdot \pi \cdot 4}{40^2 \cdot \pi \cdot 4} = 0,2344$$

röret har 23,4% av stängens massa

Minskningen blir 76,6%

2,4 IPE 180 =

$$L = 5,50 \text{ m} \quad m = 18,8 \text{ kg/m}$$

$$G = m \cdot L \cdot g = 18,8 \cdot 5,50 \cdot 9,81 = 1 \text{ kN}$$

2.5 a) $1,51 \cdot 10^5 \text{ mm} = 1,51 \cdot 10^2 \text{ m}$

b) $2,00 \cdot 10^7 \text{ mm}^2 = 2,00 \cdot 10^7 \cdot (10^{-3})^2 = 20 \text{ m}^2$

c) $3,10 \text{ m}^3 = 3,10 \cdot 100^3 = 3,10 \cdot 10^6 \text{ cm}^3$

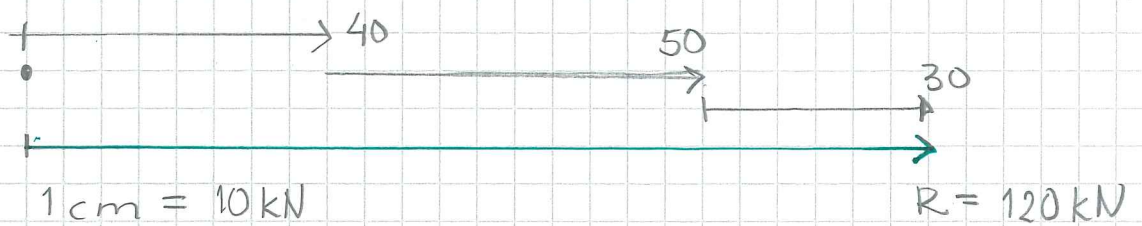
d) $1,00 \text{ N/mm}^2 = 0,001 \text{ kN/mm}^2 = 0,001 \frac{1}{0,001^2} = 1000 \text{ kN/m}^2$

2.6 a) $1,00 \cdot 10^6 \text{ mm} = 1,00 \text{ km}$

b) $2,05 \cdot 10^8 \text{ W} = 205 \text{ GW}$

c) $2,05 \cdot 10^{-4} \text{ s} = 205 \mu\text{s} = 0,205 \text{ ms}$

3.1 a)



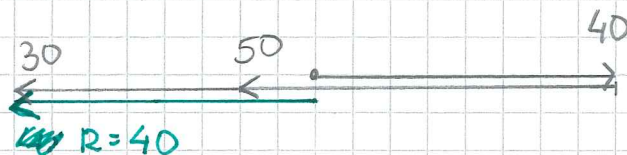
$$\rightarrow R_x = 40 + 50 + 30 = 120 \text{ kN}$$

b)



$$\rightarrow R_x = 40 + 50 - 30 = 60 \text{ kN}$$

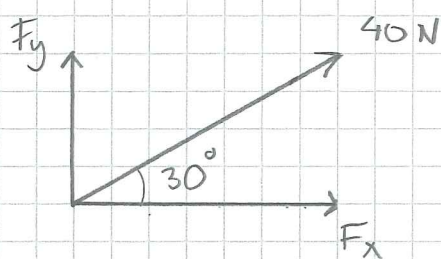
c)



$$\rightarrow R_x = 40 - 50 - 30 = -40 \text{ kN}$$

3.2

a)

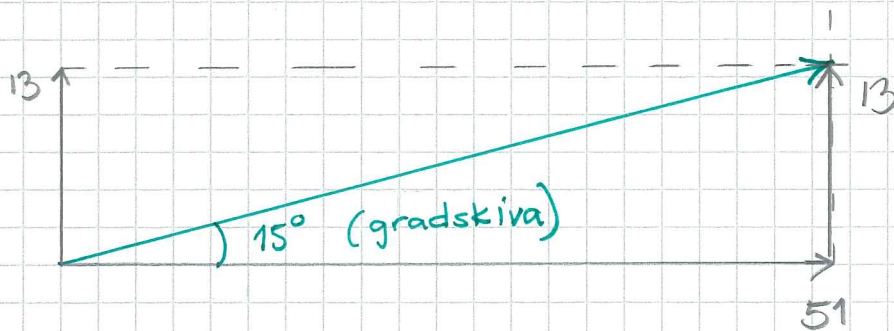


$1 \text{ cm} = 10 \text{ N}$

Mätning $\Rightarrow F_x = 35 \text{ N}$
 $F_y = 20 \text{ N}$

b) $F_x = 40 \cos 30^\circ = 34,6 \text{ N}$
 $F_y = 40 \sin 30^\circ = 20 \text{ N}$

3.3 a) $2 \text{ cm} = 10 \text{ N}$

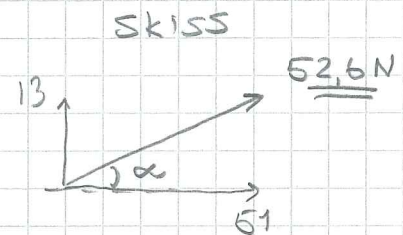


$R = 52,5 \text{ N} \quad (10,5 \text{ cm})$

b) $\rightarrow R_x = 51 \text{ N}$

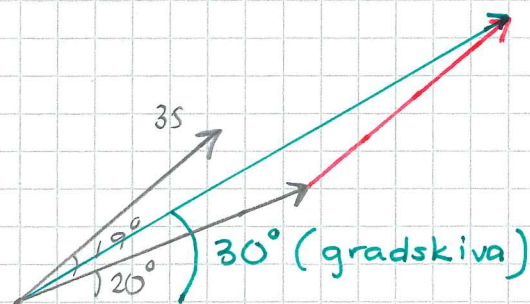
$\uparrow R_y = 13 \text{ N}$

$R = \sqrt{51^2 + 13^2} = 52,6 \text{ N}$

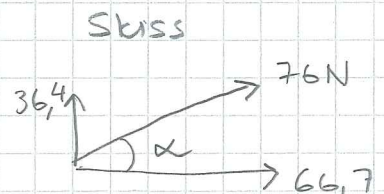


$\tan \alpha = \frac{13}{51} \Rightarrow \alpha = \arctan \frac{13}{51} = \underline{\underline{14,3^\circ}}$

3.4 a)



$R = 75 \text{ kN} \quad (7,5 \text{ cm})$



b) $\rightarrow R_x = 42 \cos 20^\circ + 35 \cos (20^\circ + 19^\circ) = 66,7 \text{ kN}$

$\uparrow R_y = 42 \sin 20^\circ + 35 \sin (20^\circ + 19^\circ) = 36,4 \text{ kN}$

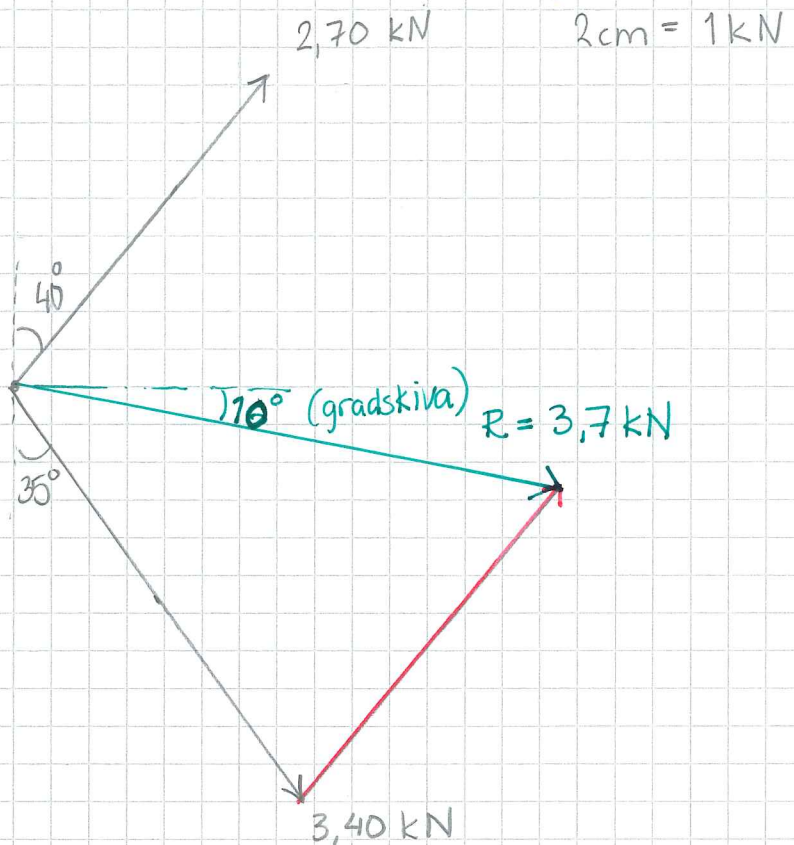
$R = \sqrt{66,7^2 + 36,4^2} = 76 \text{ kN}$

$\alpha = \arctan \left(\frac{36,4}{66,7} \right) = \underline{\underline{28,6^\circ}}$

3.5

- Flytta längs verkningslinje för gemensam angreppspunkt.

a)



b)

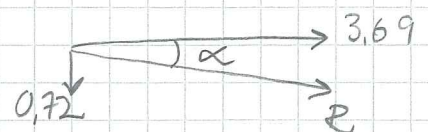
$$\vec{R}_x = 2,7 \sin 40^\circ + 3,4 \sin 35^\circ = 3,69 \text{ kN}$$

$$\uparrow R_y = 2,7 \cos 40^\circ - 3,4 \cos 35^\circ = -0,72 \text{ kN}$$

$$R = \sqrt{3,69^2 + 0,72^2} = \underline{\underline{3,76 \text{ kN}}}$$

$$\alpha = \arctan\left(\frac{0,72}{3,69}\right) = \underline{\underline{11^\circ}}$$

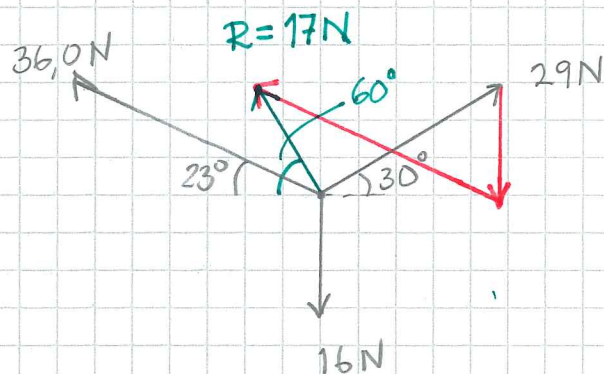
SKISS



3.6

$$1\text{cm} = 10\text{N}$$

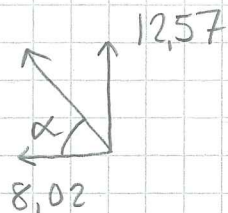
Huvud svans (eller kraftpolygon)



$$\vec{R}_x = 29 \cos 30^\circ - 36 \cos 23^\circ = -8,02 \text{ N}$$

$$\uparrow R_y = 29 \sin 30^\circ + 36 \sin 23^\circ - 16 = 12,57 \text{ N}$$

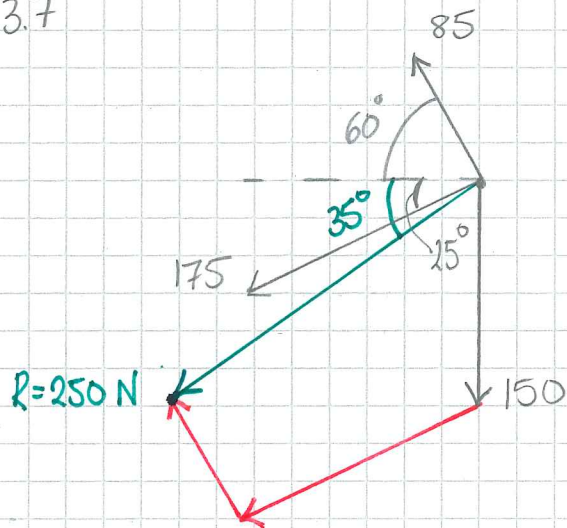
SKISS



$$R = \sqrt{8,02^2 + 12,57^2} = \underline{\underline{14,9 \text{ N}}}$$

$$\alpha = \arctan \frac{12,57}{8,02} = \underline{\underline{57,5^\circ}}$$

3.7

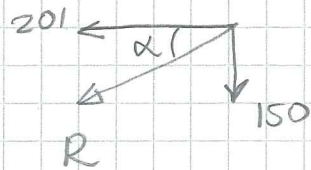


$$1 \text{ cm} = 50 \text{ N}$$

$$\uparrow R_y = 85 \sin 60^\circ - 175 \sin 25^\circ - 150 = -150 \text{ N}$$

$$\rightarrow R_x = -85 \cos 60^\circ - 175 \cos 25^\circ = -201$$

Skiss



$$R = \sqrt{150^2 + 201^2} = \underline{\underline{251 \text{ N}}}$$

$$\alpha = \arctan\left(\frac{150}{201}\right) = \underline{\underline{36,7^\circ}}$$

3,8a Huvud-svansmetoden

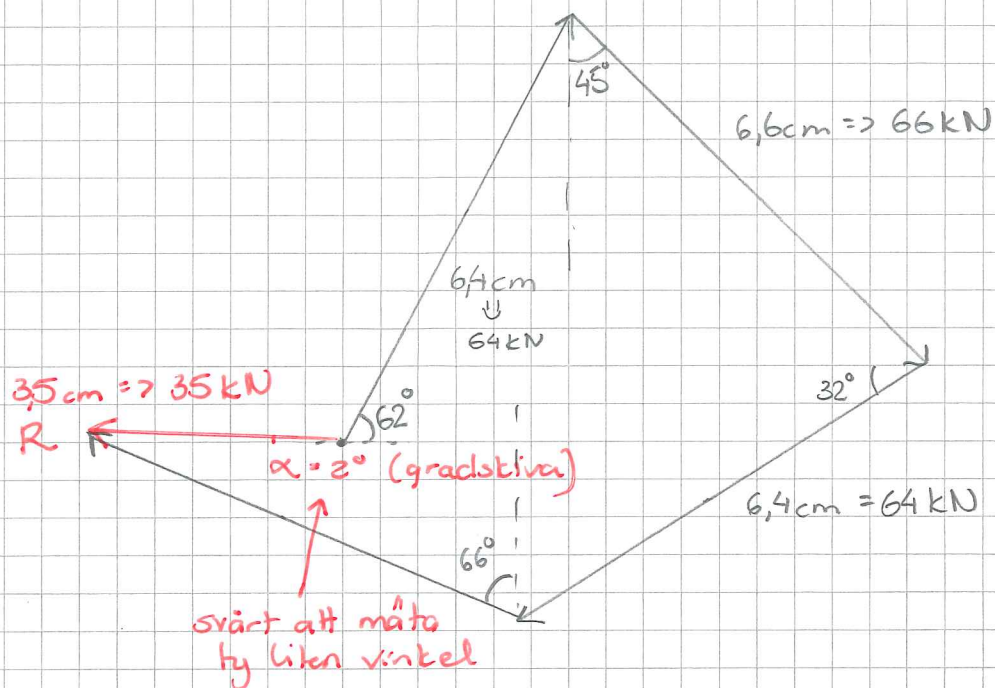
Börja med en av krafterna (valfri kraft)

Jag väljer 64 kN (62°)

Skalan jag väljer 1 cm = 10 kN

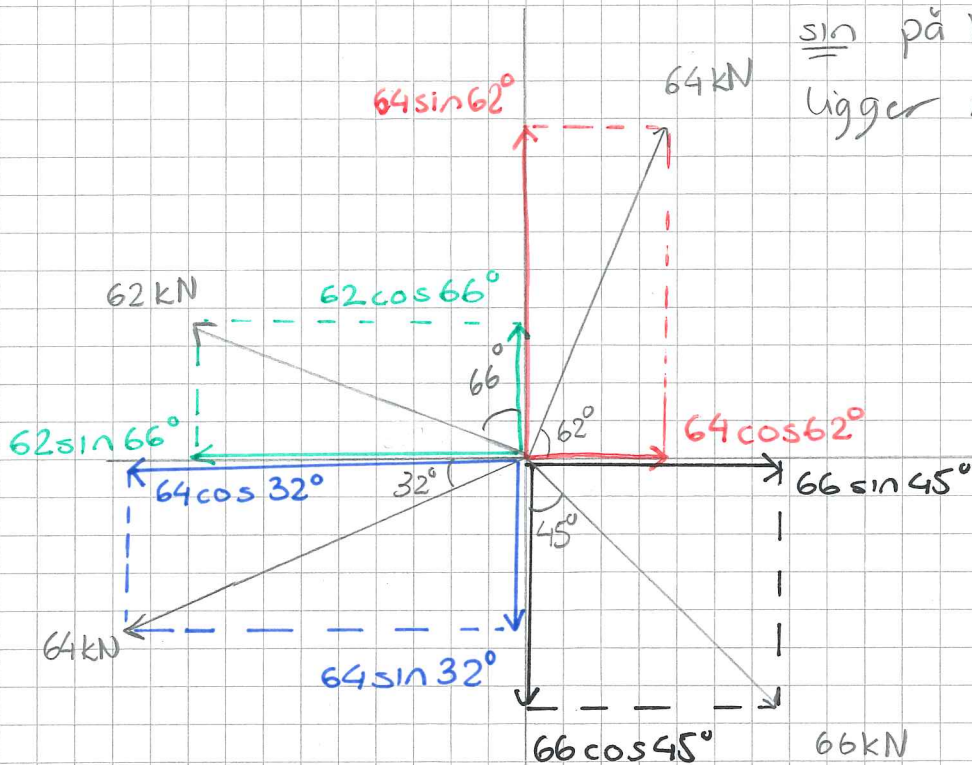
Rita med rätt riktning (gradskiva)

Fortsätt med 66 kN, 64 kN (66°+56°-90°=32°), 62 kN (66°)



cos på komponent som ligger nära vinkeln.

sin på komponent som ligger mitt emot.



Summera alla komponenter i x-led

↙ pilen anger min valda positiva riktning

$$\vec{R}_x = 64 \cos 62^\circ + 66 \sin 45^\circ - 62 \sin 66^\circ - 64 \cos 32^\circ$$

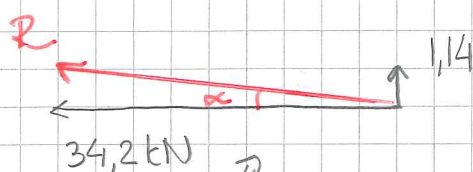
$$R_x = -34,2 \text{ kN}$$

↙ positivt uppåt

$$\vec{R}_y = 64 \sin 62^\circ + 62 \cos 66^\circ - 64 \sin 32^\circ - 66 \cos 45^\circ$$

$$R_y = 1,14 \text{ kN}$$

Rita R_x & R_y



obs ej i skala

$$R = \sqrt{34,2^2 + 1,14^2} = 34,2 \text{ kN}$$

ty liten R_y

$$\tan \alpha = \frac{1,14}{34,2}$$

$$\alpha = \arctan\left(\frac{1,14}{34,2}\right) = 1,9^\circ$$

3.8 a) Lös grafiskt med huvud/svans-metoden
(kraftpolygon)

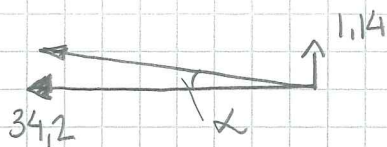
$$b) \rightarrow R_x = 64 \cos 62^\circ - 62 \sin 66^\circ - 64 \cos 32^\circ + 66 \sin 45^\circ$$

$$R_x = -34,2 \text{ kN}$$

$$\uparrow R_y = 64 \sin 62^\circ + 62 \cos 66^\circ - 64 \sin 32^\circ - 66 \cos 45^\circ$$

$$R_y = 1,14 \text{ kN}$$

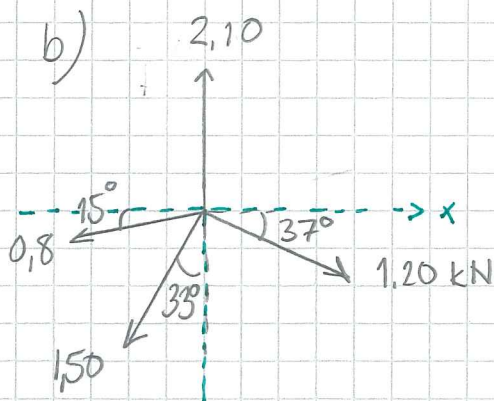
Skiss



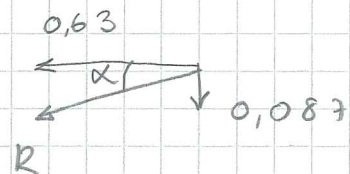
$$R = \sqrt{34,2^2 + 1,14^2} = 34,2 \text{ kN}$$

$$\alpha = \arctan\left(\frac{1,14}{34,2}\right) = 1,9^\circ$$

3.9 a) se ovan



Skiss



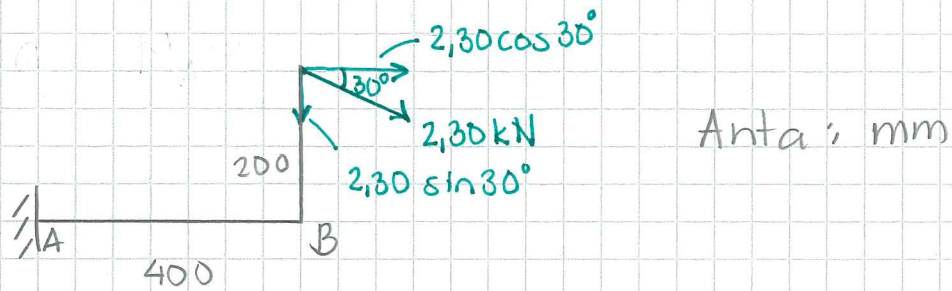
$$\alpha = \arctan\left(\frac{0,087}{0,63}\right) = \underline{\underline{7,9^\circ}}$$

$$\uparrow R_y = 2,10 - 0,8 \sin 15^\circ - 1,5 \cos 33^\circ - 1,20 \sin 37^\circ = -0,087$$

$$\rightarrow R_x = 1,20 \cos 37^\circ - 0,8 \cos 15^\circ - 1,50 \sin 33^\circ = -0,63$$

$$R = \sqrt{0,087^2 + 0,63^2} = \underline{\underline{0,63 \text{ kN}}}$$

3.10

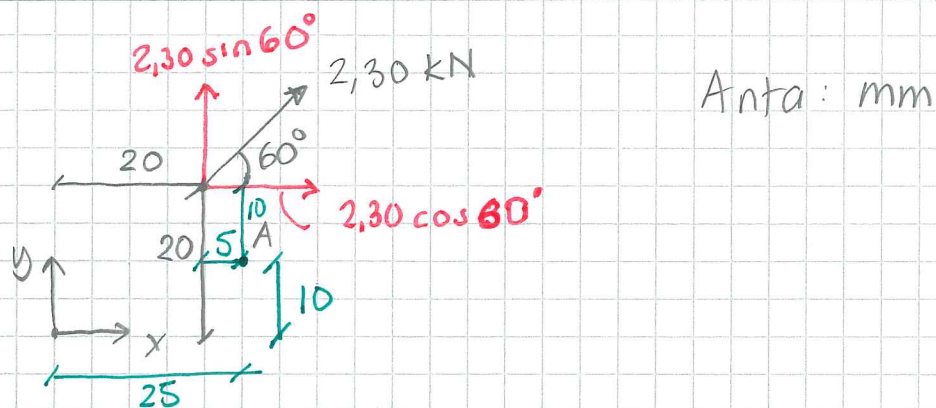


$$a) \overset{\curvearrowright}{M}_A = 2,30 \sin 30^\circ \cdot 0,4 + 2,30 \cos 30^\circ \cdot 0,2$$

$$\overset{\curvearrowright}{M}_A = 0,858 \text{ kNm} = \underline{\underline{858 \text{ Nm}}} \text{ medurs}$$

$$b) \overset{\curvearrowright}{M}_B = 0,2 \cdot 2,30 \cos 30^\circ = 0,398 \text{ kNm} = \underline{\underline{398 \text{ Nm}}} \text{ medurs}$$

3.11



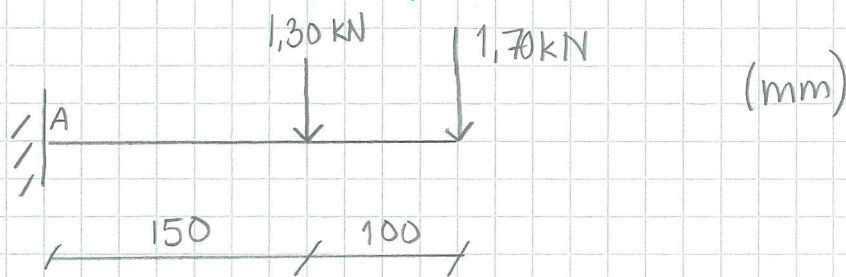
$$a) \overset{\curvearrowright}{M}_O = 0,02 \cdot 2,30 \sin 60^\circ - 0,02 \cdot 2,30 \cos 60^\circ = 16,8 \text{ Nm}$$

moturs

$$b) \overset{\curvearrowright}{M}_A = 0,01 \cdot 2,30 \cos 60^\circ + 0,005 \cdot 2,30 \sin 60^\circ = 21,5 \text{ Nm}$$

medurs

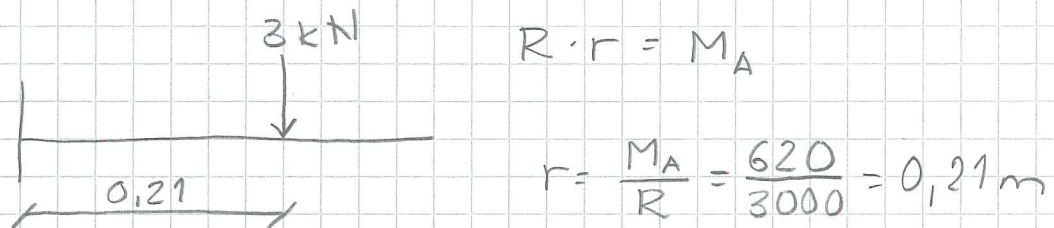
3.12



$$a) \quad \overset{\curvearrowright}{M}_A = 1,30 \cdot 0,150 + 1,70 \cdot 0,250 = 0,62 \text{ kNm}$$

$$\overset{\curvearrowright}{M}_A = 620 \text{ Nm}$$

$$b) \quad R = 1,30 + 1,70 = 3 \text{ kN}$$



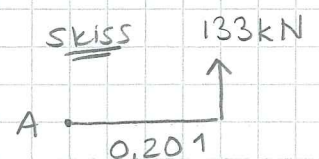
3.13

$$\uparrow R_y = 39 + 44 + 50 = 133 \text{ kN}$$

$$\overset{\curvearrowright}{M}_A = 44 \cdot 0,203 + 50(0,154 + 0,203) = 26,8 \text{ kNm}$$

(39 kN ger inget moment m.a.p A
då hävarmen är noll)

$$M_A = r \cdot R_y \quad r = \frac{26,8}{133} = 0,201$$



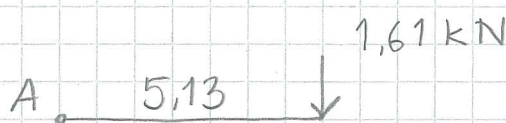
3,14

$$\downarrow R_y = R = 2,54 + 1,17 - 2,10 = 1,61 \text{ kN}$$

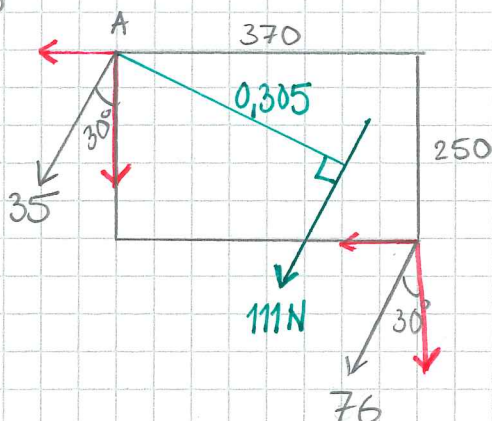
$$\overset{\curvearrowright}{M}_A = 1,17 \cdot 1,7 - 2,1 (1,7 + 2,03) + 2,54 (1,7 + 2,03 + 1,82)$$

$$\overset{\curvearrowright}{M}_A = 8,253 \text{ kNm}$$

$$M_A = r \cdot R \quad r = \frac{8,253}{1,61} = 5,13 \text{ m}$$



3,15



Båda krafterna har samma riktning \Rightarrow
 att resultanten kommer ha lika riktning (30°)

$$R = 35 + 76 = 111 \text{ N}$$

Om inte detta inses kan man räkna $R_x = R_y$

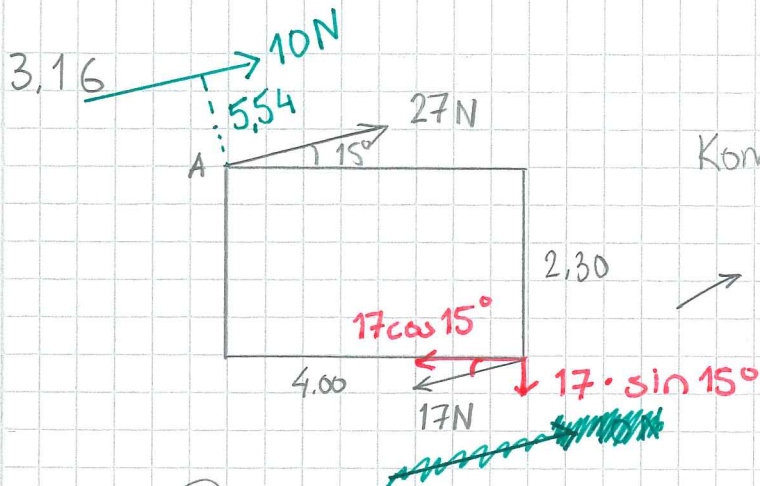
$$\leftarrow R_x = 35 \sin 30^\circ + 76 \sin 30^\circ = 55,5 \text{ N}$$

$$\downarrow R_y = 35 \cos 30^\circ + 76 \cos 30^\circ = 96,13 \text{ N}$$

$$R = \sqrt{55,5^2 + 96,13^2} = 111 \text{ N} \text{ som ovan } \ddot{u}$$

$$\overset{\curvearrowright}{M}_A = 76 \sin 30^\circ \cdot 0,250 + 76 \cos 30^\circ \cdot 0,370 = 33,85 \text{ Nm}$$

$$r = \frac{33,85}{111} = 0,305 \text{ m}$$

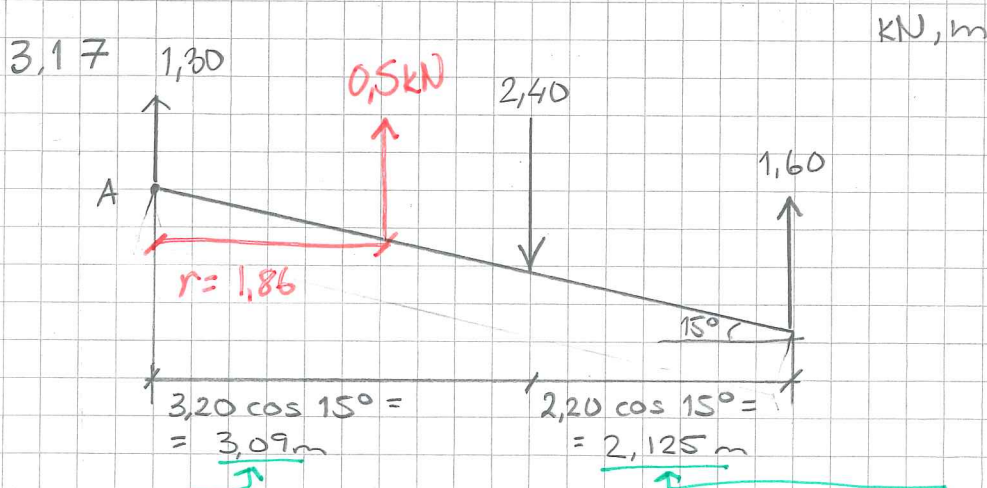


Kommentar se uppg. 3.15

$$R = 27 - 17 = 10 \text{ N}$$

$$M_A = 17 \sin 15^\circ \cdot 4,0 + 17 \cos 15^\circ \cdot 2,3 = 55,37 \text{ Nm}$$

$$r = \frac{55,37}{10} = 5,54 \text{ m}$$



$$\uparrow R_y = R = 1,3 - 2,4 + 1,6 = 0,5 \text{ kN}$$

• Bestäm hävarmar till A, vinkelräta avstånd

$$M_A = 2,40 \cdot 3,09 - 1,6(3,09 + 2,125) = -0,93 \text{ kNm}$$

$$M_A = 0,93 \text{ kNm (moturs)}$$

$$r = \frac{M_A}{R} = \frac{0,93}{0,5} = \underline{\underline{1,86 \text{ m}}}$$

3.18

$$a) \quad \overset{\curvearrowright}{M}_A^{\downarrow} = 1,30 \cdot 7,35 = 9,56 \text{ kNm}$$

$$b) \quad \overset{\curvearrowright}{M}_B^{\downarrow} = 1,30 \cdot 7,35 = 9,56 \text{ kNm}$$

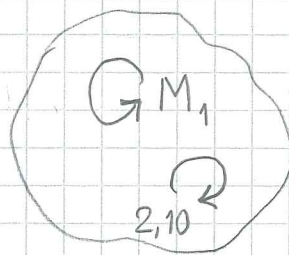
$$c) \quad \overset{\curvearrowright}{M}_C^{\downarrow} = 1,30 \cdot 4,50 + 1,3 (7,35 - 4,50)$$

$$\overset{\curvearrowright}{M}_C^{\downarrow} = \cancel{1,30 \cdot 4,50} + 1,3 \cdot 7,35 - \cancel{1,3 \cdot 4,50} = 1,3 \cdot 7,35$$

$$\overset{\curvearrowright}{M}_C^{\downarrow} = 9,56 \text{ kNm}$$

3.19

ett kraftpar är ett rent moment och är platsberoende på stela kroppar



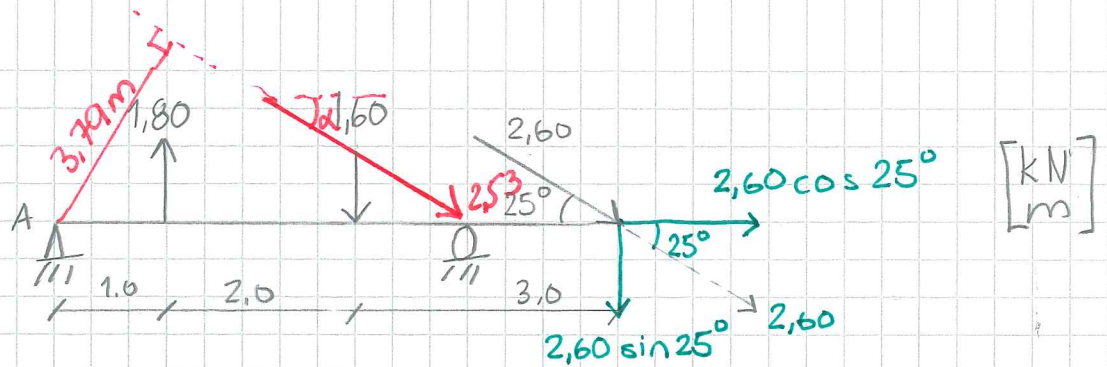
$$\overset{\curvearrowright}{M}_1 = 10 \cdot 0,5 = 5 \text{ kNm}$$

$$\overset{\curvearrowright}{M}_{\text{tot}} = 5 - 2,10 = 2,90 \text{ kNm}$$

3.20

$$\overset{\curvearrowright}{M}_A = 3,50 \cdot 2,70 - 9,30 = 0,15 \text{ kNm}$$

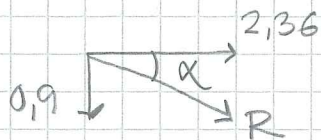
3.21



$$\rightarrow R_x = 2,60 \cos 25^\circ = 2,36 \text{ kN}$$

$$\uparrow R_y = 1,8 - 1,6 - 2,6 \sin 25^\circ = -0,90 \text{ kN}$$

Skiss



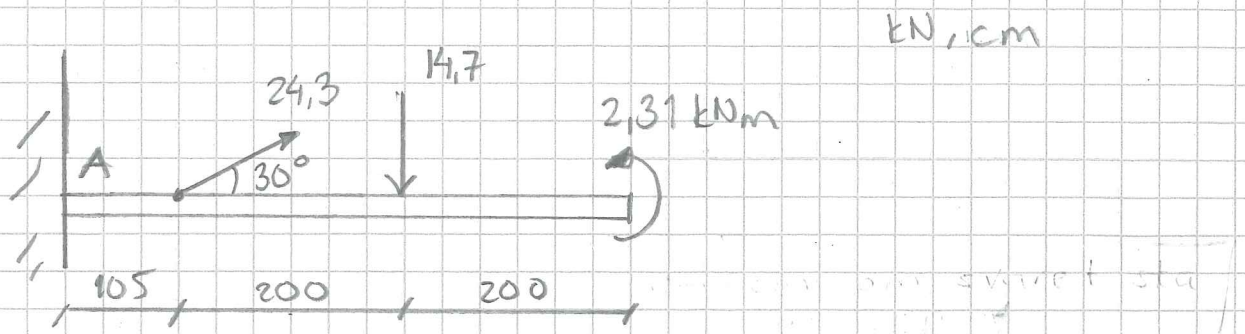
$$R = \sqrt{0,9^2 + 2,36^2} = 2,53 \text{ kN}$$

$$\alpha = \arctan\left(\frac{0,9}{2,36}\right) = 20,9^\circ$$

$$\overset{\vee}{M}_A = 1,6 \cdot 3,0 - 1,8 \cdot 1,0 + 6,0 \cdot 2,6 \sin 25^\circ = 9,59 \text{ kNm}$$

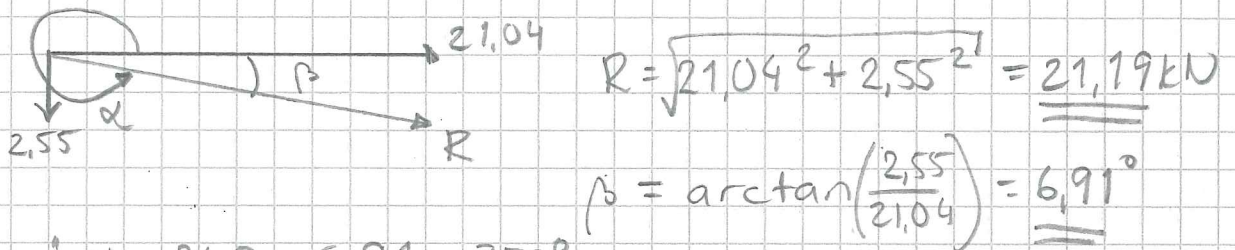
$$r = \frac{M_A}{R} = \frac{9,59}{2,53} = 3,79 \text{ m}$$

3.22 Sökt: Resultanten.



$$\rightarrow F_{Rx} = 24,3 \cos 30^\circ = 21,04 \text{ kN}$$

$$\uparrow F_{Ry} = 24,3 \sin 30^\circ - 14,7 = -2,55 \text{ kN}$$



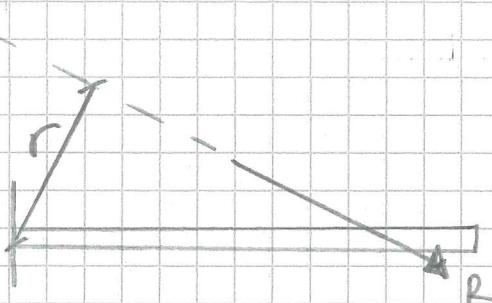
$$R = \sqrt{21,04^2 + 2,55^2} = \underline{\underline{21,19 \text{ kN}}}$$

$$\beta = \arctan\left(\frac{2,55}{21,04}\right) = \underline{\underline{6,91^\circ}}$$

"boken" $\alpha = 360 - 6,91 = 353^\circ$

R ska ge samma vridande förmåga som de två krafterna + momentet

$$\begin{aligned} \overset{\curvearrowright}{M}_A &= 2,31 - 14,7 \cdot 3,05 + 1,05 \cdot 24,3 \sin 30^\circ = \\ &= -29,8 \text{ kNm} \Rightarrow \overset{\curvearrowleft}{M}_A = 29,8 \text{ kNm} \end{aligned}$$

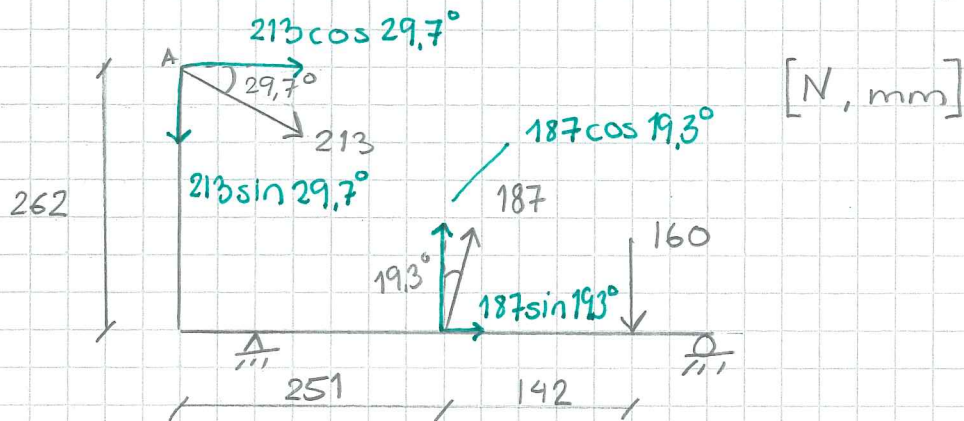


$$\overset{\curvearrowleft}{M}_A = r \cdot R$$

$$r = \frac{29,8}{21,2} = \underline{\underline{1,4 \text{ m}}}$$

$$R_y x = M_A \Rightarrow x = \frac{M_A}{R_y} = \frac{29,8}{2,55}$$

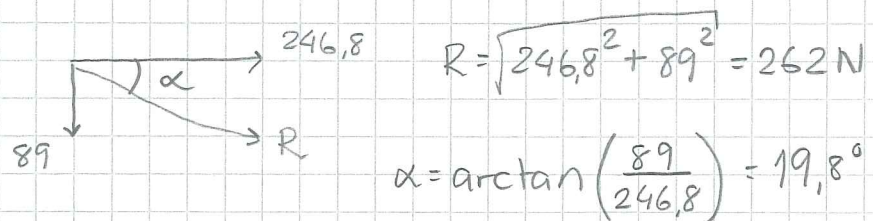
3.23



$$\downarrow R_y = 213 \sin 29,7^\circ - 187 \cos 19,3^\circ + 160 = 89,04 \text{ N}$$

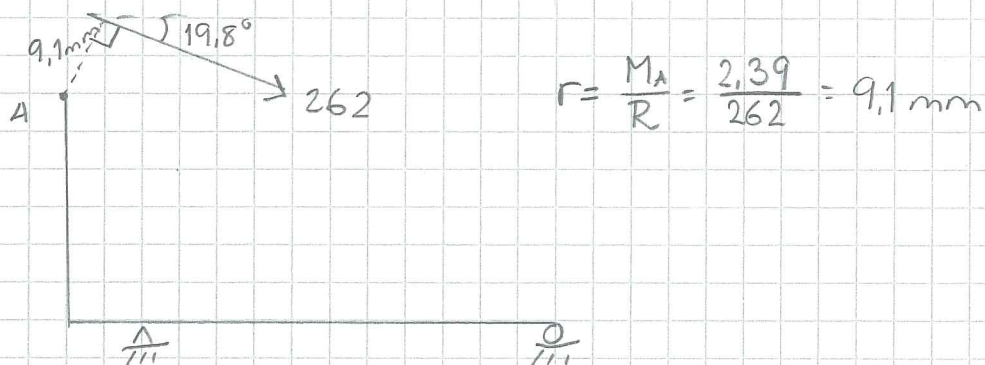
$$\rightarrow R_x = 213 \cos 29,7^\circ + 187 \sin 19,3^\circ = 246,8 \text{ N}$$

Stress

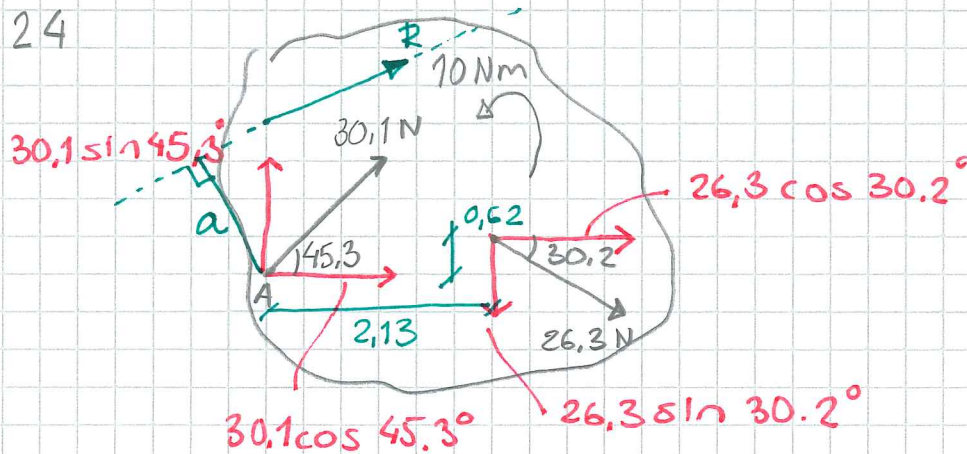


$$M_A^{\curvearrowright} = 160 (0,251 + 0,142) - 187 \cos 19,3^\circ \cdot 0,251 - \dots$$

$$\dots 187 \sin 19,3^\circ \cdot 0,262 = 2,39 \text{ Nm}$$



3.24



$$\uparrow R_y = 30,1 \sin 45,3^\circ - 26,3 \sin 30,2^\circ = 7,9 \text{ N}$$

$$\rightarrow R_x = 26,3 \cos 30,2^\circ + 30,1 \cos 45,3^\circ = 44,3 \text{ N}$$

$$R = \sqrt{7,9^2 + 44,3^2} = 45 \text{ N}$$

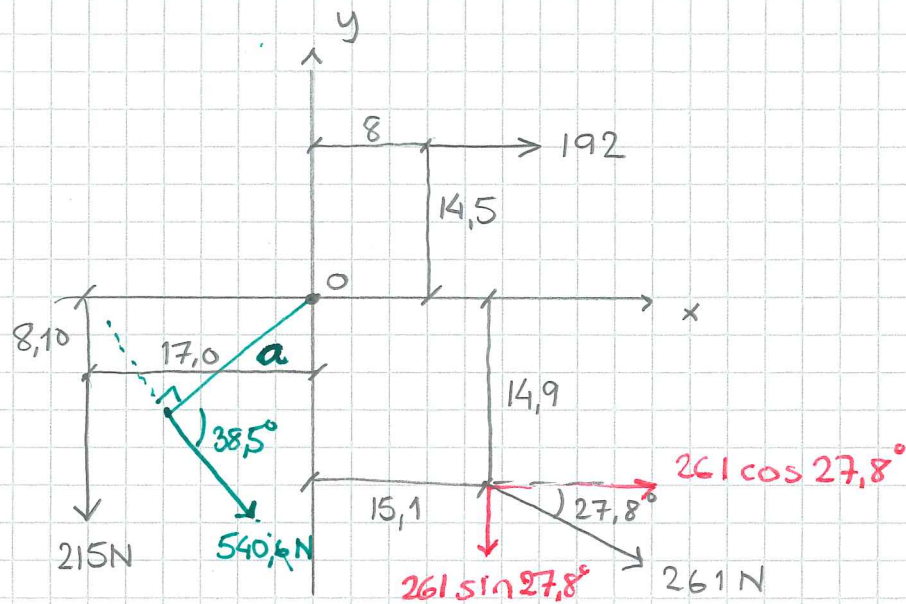
$$\alpha = \arctan\left(\frac{7,9}{44,3}\right) = 10,1^\circ$$

$$\overset{\curvearrowright}{M}_A = 0,62 \cdot 26,3 \cos 30,2^\circ + 2,13 \overset{26,3}{\sin} 30,2^\circ - 10 = 33 \text{ Nm}$$

$$R \cdot a = \overset{\curvearrowright}{M}_A \quad a = \frac{33}{45} = 0,733 \text{ m}$$

a enligt figur (vinkelrät avstånd från A till kraftens verkningslinje, placera R så a H det blir medurs i förhållande till A.

3.25



$$\rightarrow R_x = 192 + 261 \cos 27,8^\circ = 422,9 \text{ N}$$

$$\downarrow R_y = 215 + 261 \sin 27,8^\circ = 336,7 \text{ N}$$

$$R = \sqrt{422,9^2 + 336,7^2} = 540,6 \text{ N}$$

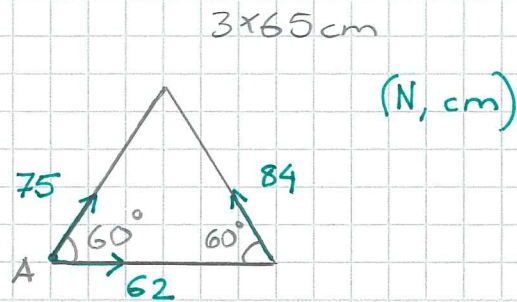
$$\alpha = \arctan\left(\frac{336,7}{422,9}\right) = 38,5^\circ$$

$$\begin{aligned} \overset{\curvearrowright}{M}_O &= 192 \cdot 14,5 - 215 \cdot 17 + 15,1 \cdot 261 \sin 27,8^\circ \dots \\ &\dots - 14,9 \cdot 261 \cos 27,8^\circ = -2470 \text{ Nmm} \end{aligned}$$

$$\overset{\curvearrowleft}{M}_O = 2470 \text{ Nmm}$$

$$a = \frac{2470}{540,6} = 4,57 \text{ mm}$$

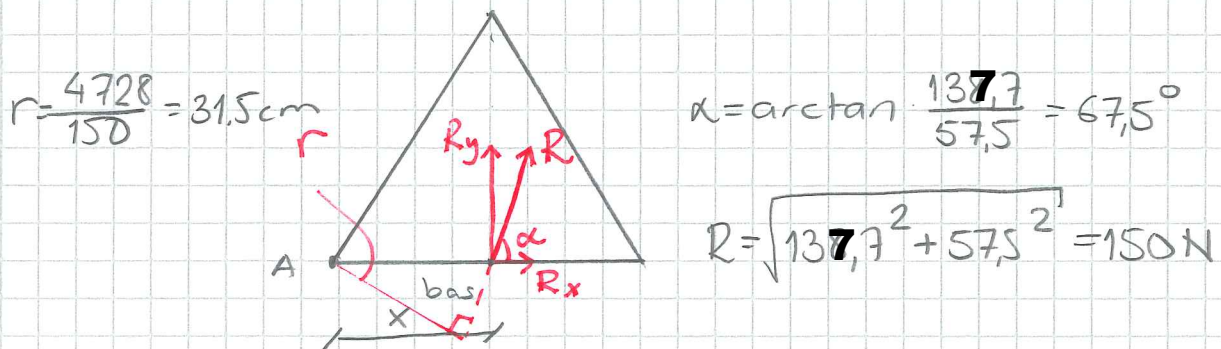
3.26



$$\uparrow R_y = 75 \sin 60^\circ + 84 \sin 60^\circ = 137,7 \text{ N}$$

$$\rightarrow R_x = 75 \cos 60^\circ - 84 \cos 60^\circ + 62 = 57,5 \text{ N}$$

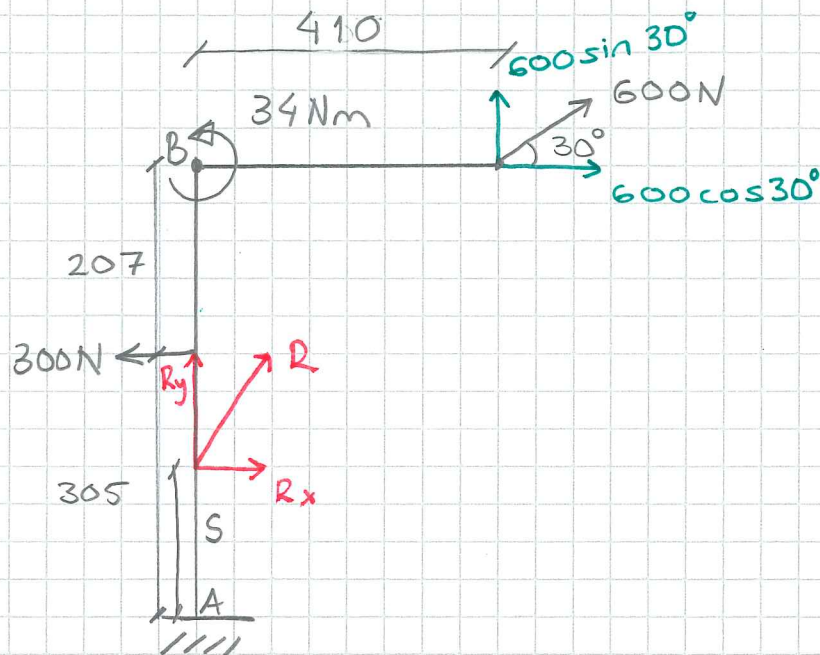
$$\curvearrowleft_A M_A = 84 \sin 60^\circ \cdot 65 = 4728 \text{ Ncm}$$



b) Då R skär basen ger endast R_y moment i A

$$R_y \cdot x = M_A \quad \Rightarrow \quad x = \frac{M_A}{R_y} = \frac{4728}{137,7} = 34 \text{ cm}$$

3,27



$$\uparrow R_y = 600 \sin 30^\circ = 300 \text{ N}$$

$$\rightarrow R_x = 600 \cos 30^\circ - 300 = 220 \text{ N}$$

$$R = \sqrt{300^2 + 220^2} = 372 \text{ N}$$

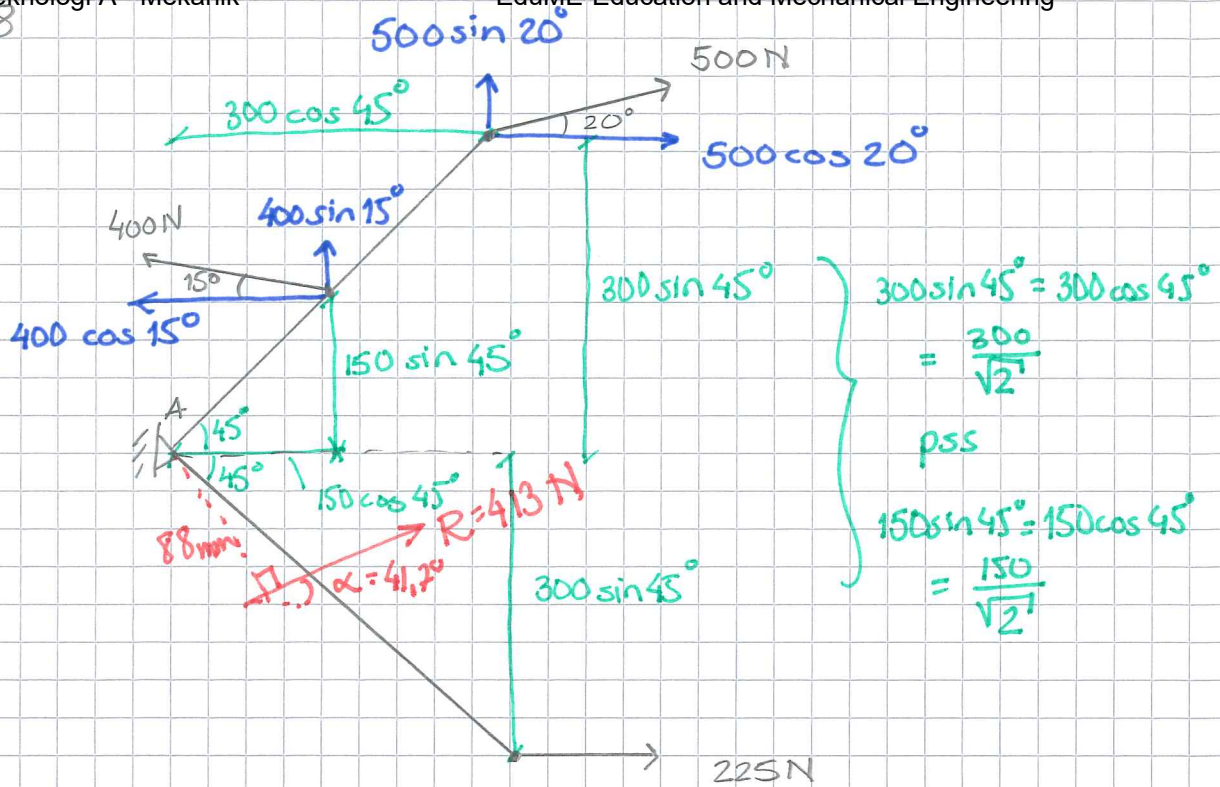
$$\alpha = \arctan\left(\frac{300}{220}\right) = 53,7^\circ$$

$$\overset{\curvearrowright}{M}_A = 0,512 \cdot 600 \cos 30^\circ - 0,41 \cdot 600 \sin 30^\circ - 34 - 300 \cdot 0,305$$

$$\overset{\curvearrowright}{M}_A = 17,5 \text{ Nm}$$

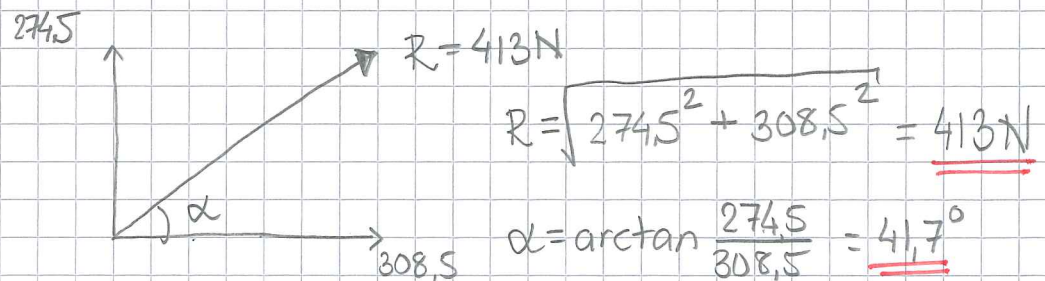
$$s = \frac{M_A}{R_x} = \frac{17,5}{220} = \underline{\underline{0,08 \text{ m}}}$$

3.2.8



$$\uparrow R_y = 500 \sin 20^\circ + 400 \sin 15^\circ = 274,5 \text{ N}$$

$$\rightarrow R_x = 225 + 500 \cos 20^\circ - 400 \cos 15^\circ = 308,5 \text{ N}$$

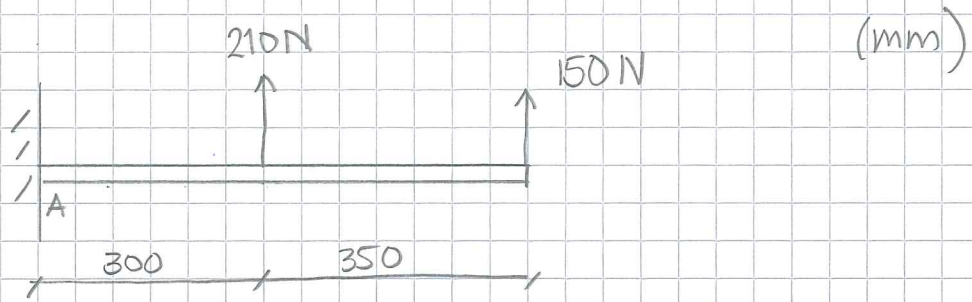


$$\curvearrowleft A \quad 225 \cdot \frac{300}{\sqrt{2}} + 400 \cos 15^\circ \cdot \frac{150}{\sqrt{2}} + 400 \sin 15^\circ \cdot \frac{150}{\sqrt{2}} + 500 \sin 20^\circ \cdot \frac{300}{\sqrt{2}} \dots$$

$$\dots - 500 \cos 20^\circ \cdot \frac{300}{\sqrt{2}} = M_A \Rightarrow \curvearrowleft M_A = 36298 \text{ Nmm}$$

$$a = \frac{36298}{413} = \underline{\underline{88 \text{ mm}}}$$

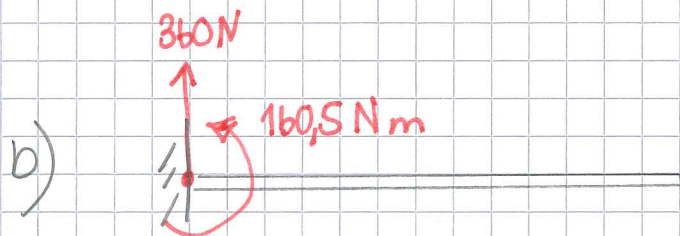
3,29



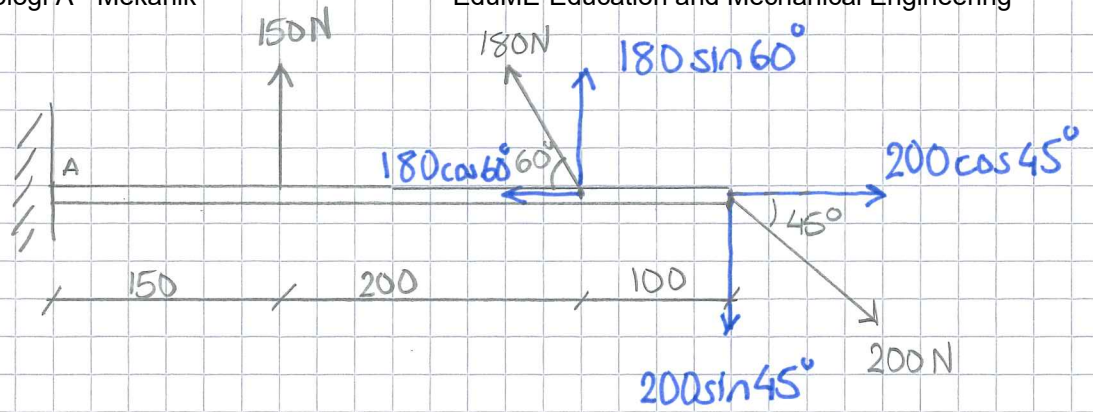
$$\uparrow R = 210 + 150 = 360 \text{ N}$$

$$\curvearrowleft M_A = 210 \cdot 300 + 150 \cdot 650 = 160500 \text{ Nmm}$$

$$a = \frac{M_A}{R} = \frac{160500}{360} = 446 \text{ mm}$$



3:30

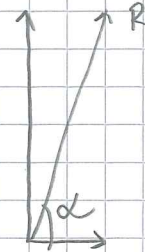


$$\uparrow R_y = 150 + 180 \sin 60^\circ - 200 \sin 45^\circ = 164,5 \text{ N}$$

$$\rightarrow R_x = 200 \cos 45^\circ - 180 \cos 60^\circ = 51,4 \text{ N}$$

$$R = \sqrt{51,4^2 + 164,5^2} = 172 \text{ N}$$

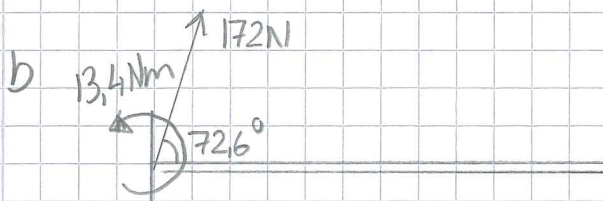
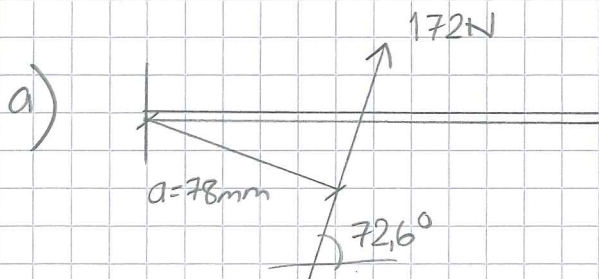
$$\alpha = \arctan\left(\frac{164,5}{51,4}\right) = 72,6^\circ$$



$$\overset{\curvearrowleft}{M}_A = 150 \cdot 150 + 180 \sin 60^\circ \cdot (150 + 200) - 200 \sin 45^\circ \cdot (150 + 200 + 100)$$

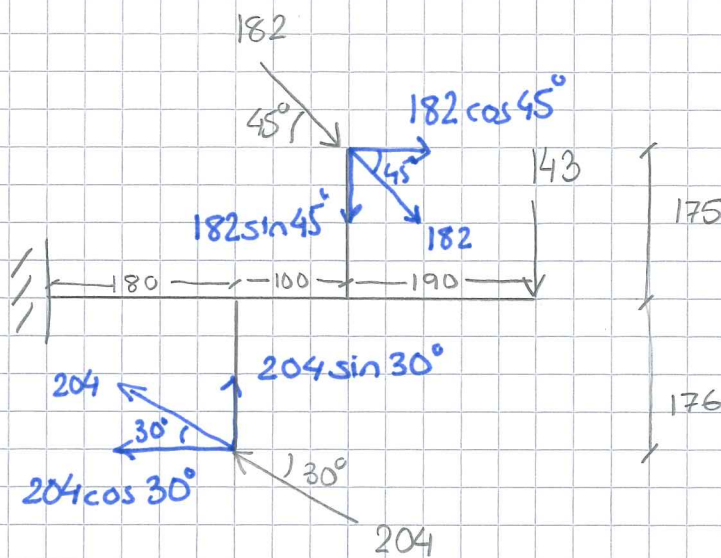
$$\overset{\curvearrowleft}{M}_A = 13420 \text{ Nmm}$$

$$a = \frac{M_A}{R} = \frac{13420}{172} = 78 \text{ mm}$$



3.31

(N, mm)

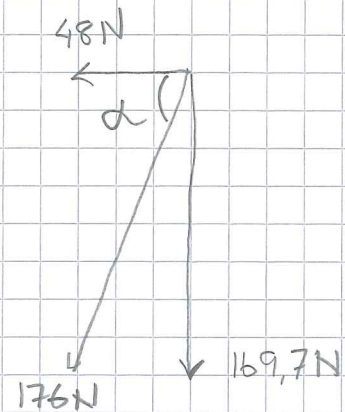


$$\uparrow R_y = 204 \sin 30^\circ - 143 - 182 \sin 45^\circ = -169,7 \text{ N}$$

$$\rightarrow R_x = 182 \cos 45^\circ - 204 \cos 30^\circ = -48,0 \text{ N}$$

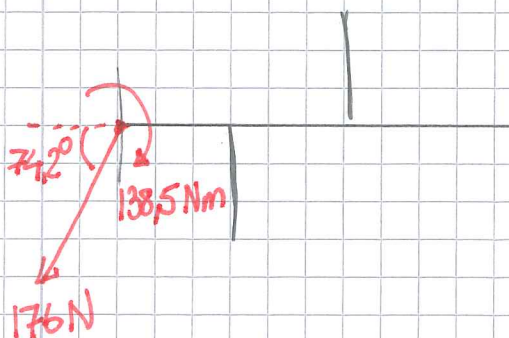
$$R = \sqrt{169,7^2 + 48^2} = 176 \text{ N}$$

$$\alpha = \arctan\left(\frac{169,7}{48}\right) = 74,2^\circ$$

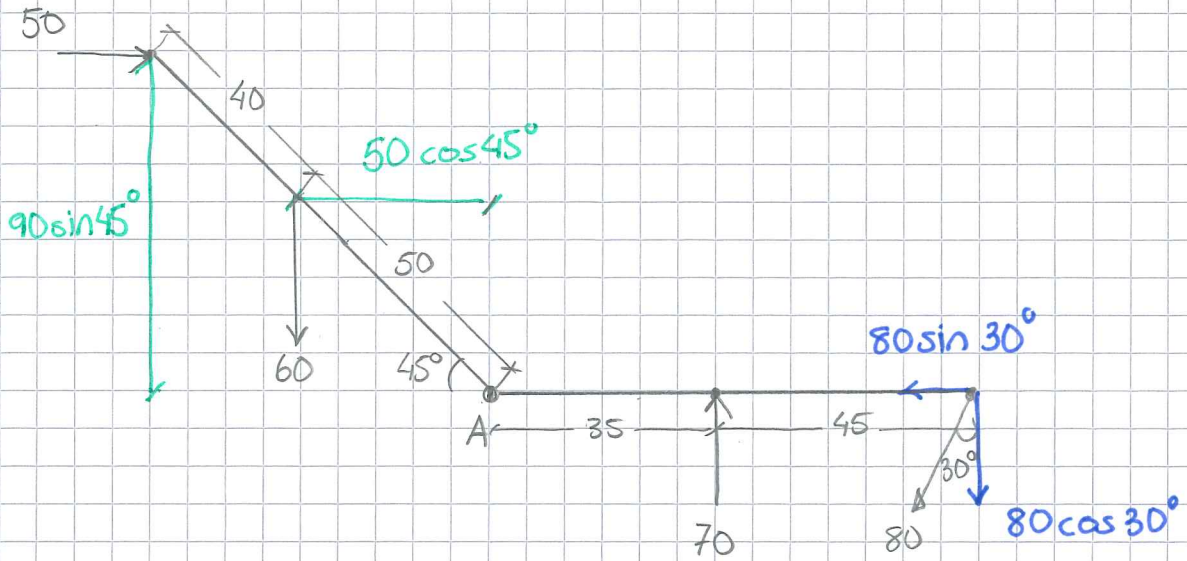


$$M_A = 143(180+100+190) + 182 \sin 45^\circ (180+100) + 182 \cos 45^\circ \cdot 175 \dots$$

$$+ 204 \cos 30^\circ \cdot 176 - 204 \sin 30^\circ \cdot 180 = 138499 \text{ Nmm}$$



3.32

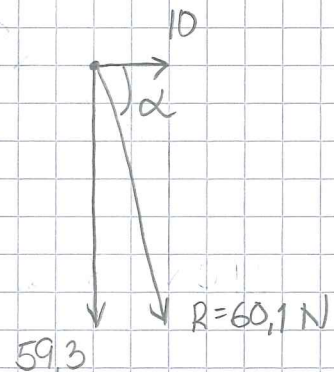


$$\uparrow R_y = 70 - 60 - 80 \cos 30^\circ = -59,3 \text{ N}$$

$$\rightarrow R_x = 50 - 80 \sin 30^\circ = 10$$

$$R = \sqrt{10^2 + 59,3^2} = 60,1 \text{ N}$$

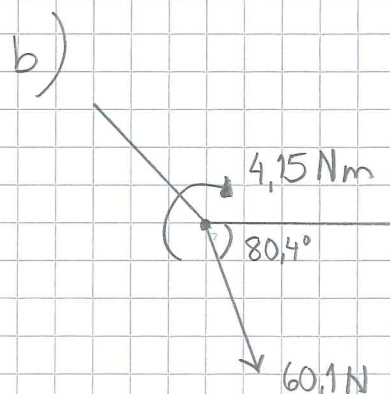
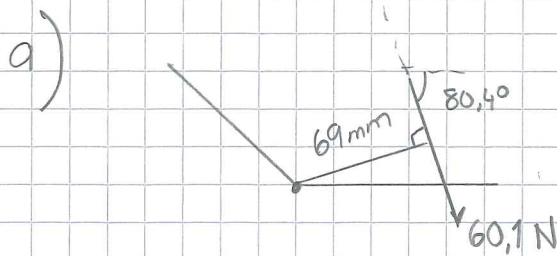
$$\alpha = \arctan\left(\frac{59,3}{10}\right) = 80,4^\circ$$



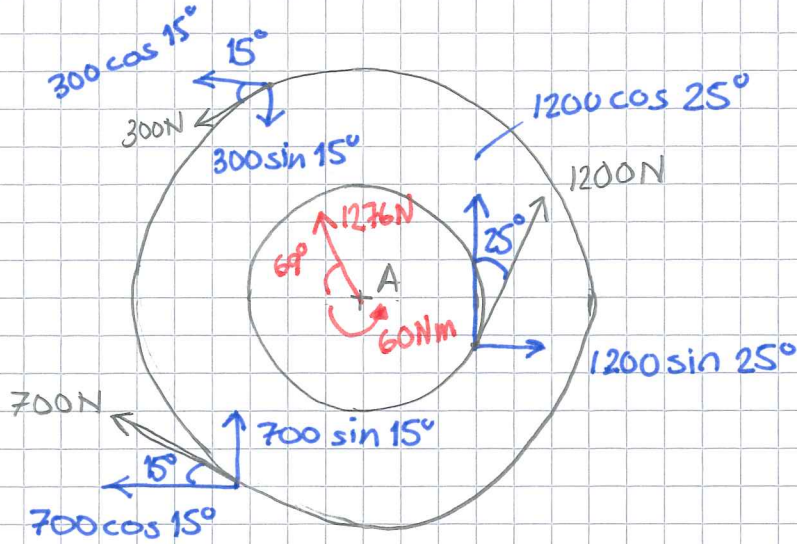
$$\overset{\curvearrowright}{M}_A = 50 \cdot 90 \sin 45^\circ - 60 \cdot 50 \cos 45^\circ - 70 \cdot 35 + 80 \cos 30^\circ \cdot 80$$

$$\overset{\curvearrowright}{M}_A = 4153 \text{ Nmm}$$

$$a = \frac{M_A}{R} = \frac{4153}{60,1} = 69 \text{ mm}$$



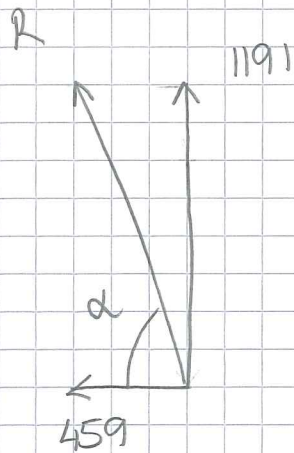
3.33



$$\rightarrow R_x = 1200 \sin 25^\circ - 700 \cos 15^\circ - 300 \cos 15^\circ = -459 \text{ N}$$

$$\uparrow R_y = 1200 \cos 25^\circ + 700 \sin 15^\circ - 300 \sin 15^\circ = 1191 \text{ N}$$

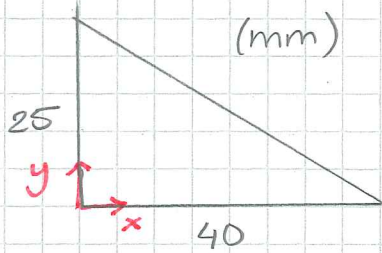
$$\curvearrowleft M_A = 300 \cdot 300 + 1200 \cdot 150 - 700 \cdot 300 = \underline{\underline{60000 \text{ Nmm}}}$$



$$R = \sqrt{1191^2 + 459^2} = \underline{\underline{1276 \text{ N}}}$$

$$\alpha = \arctan\left(\frac{1191}{459}\right) = \underline{\underline{69^\circ}}$$

4.1



Formelsamling $\Rightarrow t_p = \frac{h}{3}$

Mitt råd: Använd alltid $\frac{h}{3}$

för att beräkna t_p inte

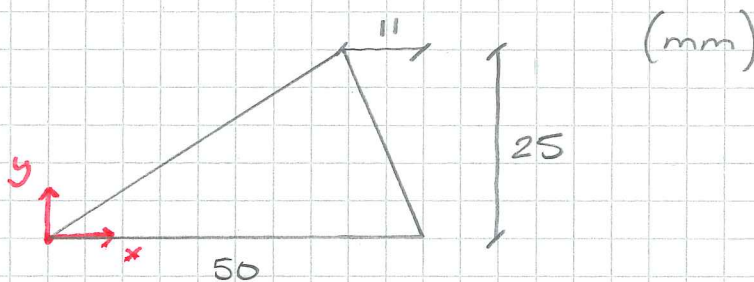
$$x_{tp} = \frac{A+C+D}{3} \text{ (Malmendahl)}$$

$\frac{h}{3}$ gäller från basen.

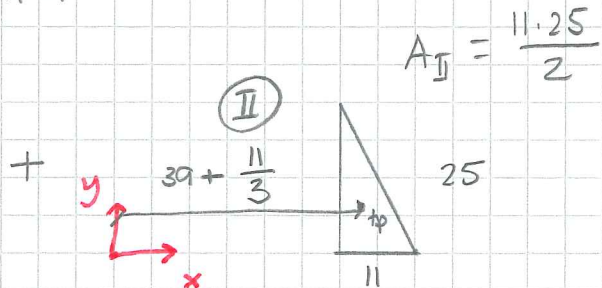
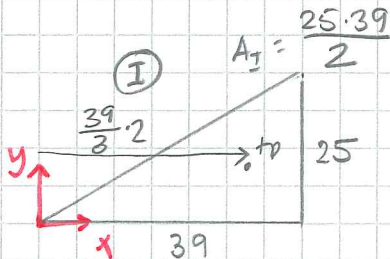
$$y_{tp} = \frac{25}{3} = 8,33 \text{ mm}$$

$$x_{tp} = \frac{40}{3} = 13,3 \text{ mm}$$

4.2



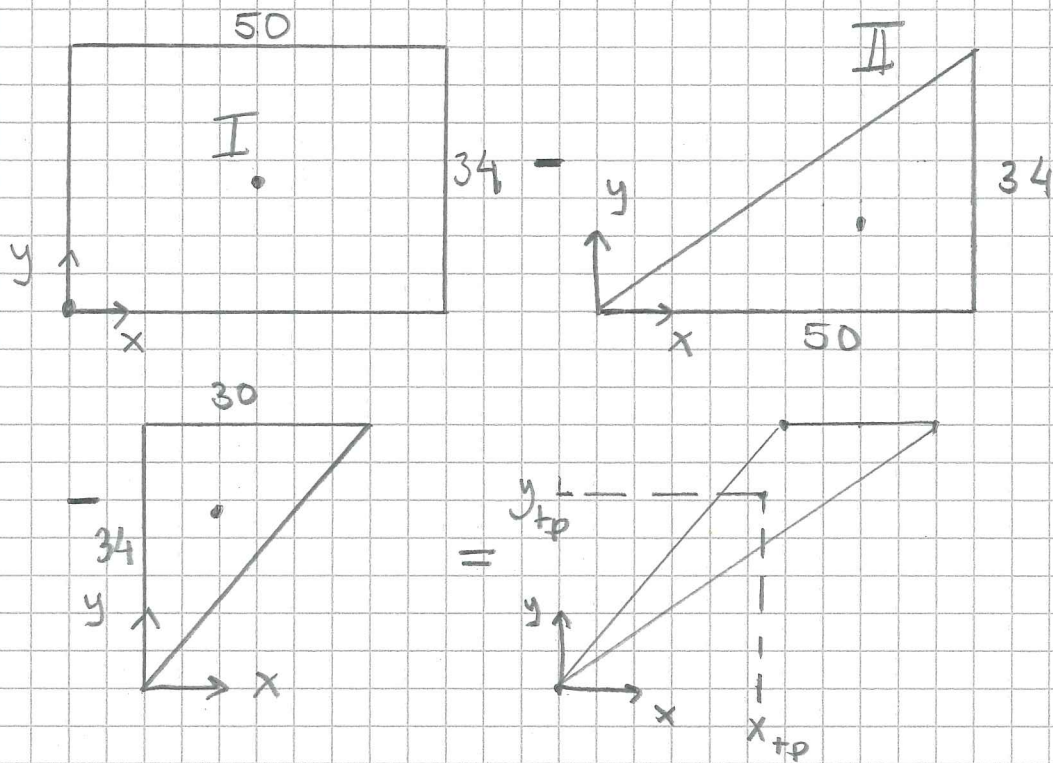
$$y_{tp} = \frac{25}{3} = 8,33 \text{ mm}$$



| | A | x_{tp} | $A x_{tp}$ |
|----|-------|----------|------------|
| I | 487,5 | 26 | 12675 |
| II | 137,5 | 42,7 | 5867 |
| | 625 | | 18542 |

$$x_{tp} = \frac{18542}{625} = \underline{\underline{29,7 \text{ mm}}}$$

4.3 Sökt: x_{tp} , y_{tp}



| n | A_n | x_{tp}^n | y_{tp}^n | $A_n x_{tp}^n$ | $A_n y_{tp}^n$ |
|----------|-------|-------------------------------|-------------------------------|----------------|----------------|
| I | 1700 | 25 | 17 | 42500 | 28900 |
| II | -850 | $\frac{2 \cdot 50}{3} = 33,3$ | $\frac{34}{3} = 11,3$ | -28333 | -9633 |
| III | -510 | $\frac{30}{3} = 10$ | $\frac{2 \cdot 34}{3} = 22,7$ | -5100 | -11560 |
| Σ | 340 | | | 9067 | 7707 |

$$x_{tp} = \frac{\Sigma A_n x_{tp}^n}{A_{tot}} = \frac{9067}{340} = \underline{\underline{26,7 \text{ mm}}}$$

$$y_{tp} = \frac{7707}{340} = \underline{\underline{22,7 \text{ mm}}}$$

4.4

$$x_{tp} = \frac{2R \sin \alpha}{3\alpha}$$

Formelbladet säger α i rad

α i nämnaren måste alltid vara i radianer.

$\sin \alpha \Rightarrow$ om α i rad måste miniräknare stå på radianer.

om α i grader kan miniräknaren vara på grader (enklast)

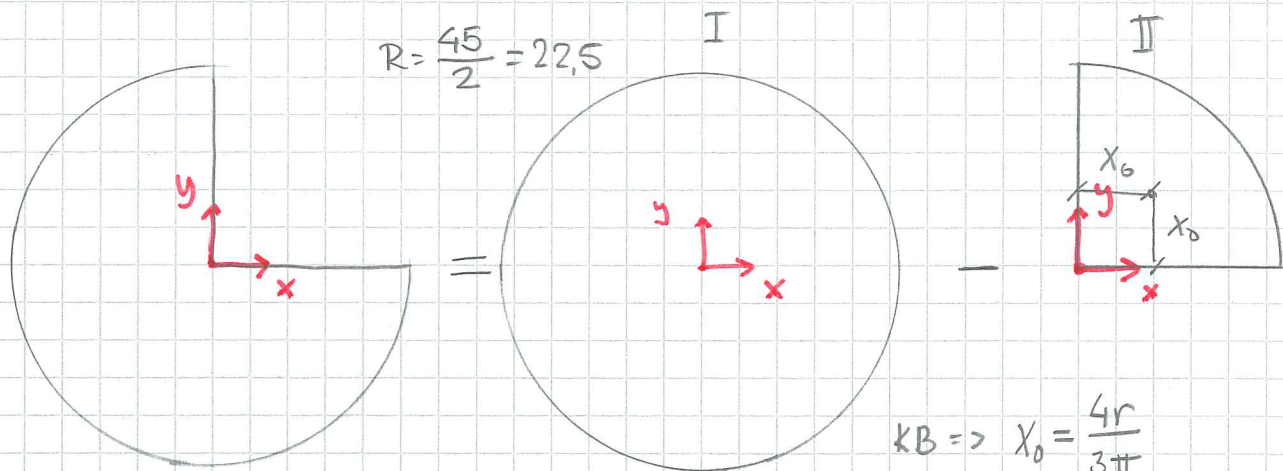
Mitt råd: radianer i nämnaren

grader för $\sin \alpha$ för miniräknare står på grader.

$$\alpha = 13^\circ \quad \alpha_{\text{rad}} = \frac{13}{360} \cdot 2\pi = 13 \cdot \frac{\pi}{180}$$

$$x_{tp} = \frac{2 \cdot 30 \cdot \sin 13^\circ}{3 \cdot 13 \cdot \frac{\pi}{180}} = 19,8 \text{ mm}$$

4.5



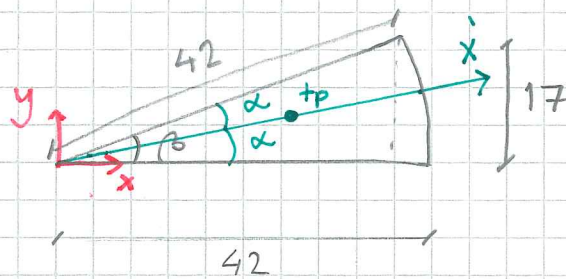
$$K_B \Rightarrow x_0 = \frac{4r}{3\pi}$$

$$x_0 = \frac{4 \cdot 22,5}{3\pi} = 9,55$$

| | A | x_{tp} | y_{tp} | Ax_{tp} | Ay_{tp} |
|----|----------------------------|----------|----------|-----------|-----------|
| I | $22,5^2 \pi = 1590$ | 0 | 0 | 0 | 0 |
| II | $-\frac{1590}{4} = -397,5$ | 9,55 | 9,55 | -3796 | -3796 |
| | 1193 | | | -3796 | -3796 |

$$x_{tp} = \frac{-3796}{1193} = \underline{\underline{-3,18}} = y_{tp}$$

4,6



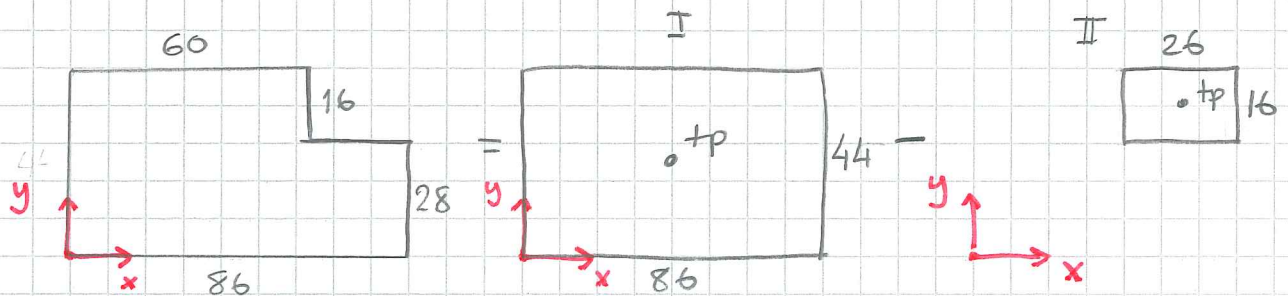
$$\beta = \arcsin\left(\frac{17}{42}\right) = 23,88^\circ$$

$$x'_{tp} = \frac{2 \cdot 42 \cdot \sin 11,9^\circ}{3 \cdot 11,9 \cdot \frac{\pi}{180}} = 27,8 \text{ mm}$$

$$x_{tp} = 27,8 \cdot \cos 11,9^\circ = \underline{\underline{27,2 \text{ mm}}}$$

$$y_{tp} = 27,8 \cdot \sin 11,9^\circ = \underline{\underline{5,73 \text{ mm}}}$$

4,7

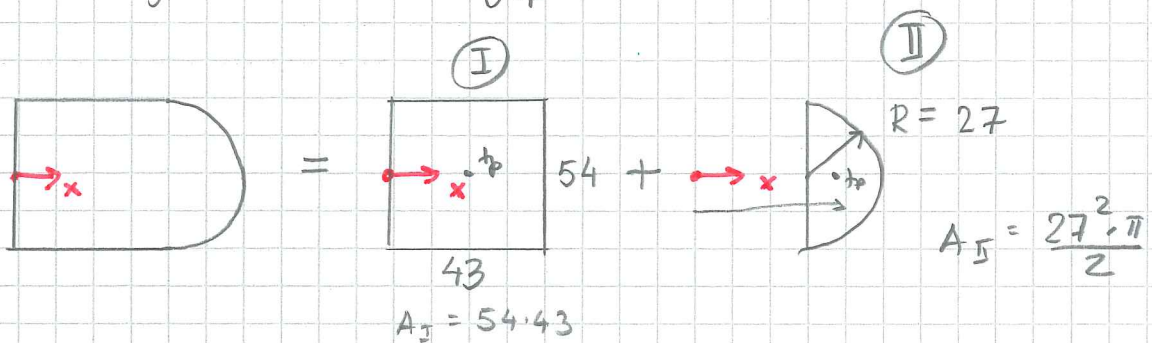


| | A | x_{tp} | y_{tp} | Ax_{tp} | Ay_{tp} |
|----|-----------------------|--------------------------|--------------------------|-----------|-----------|
| I | $44 \cdot 86 = 3784$ | $\frac{86}{2} = 43$ | $\frac{44}{2} = 22$ | 162712 | 83248 |
| II | $-26 \cdot 16 = -416$ | $60 + \frac{26}{2} = 73$ | $28 + \frac{16}{2} = 36$ | -30368 | -14976 |
| | 3368 | | | 132344 | 68272 |

$$x_{tp} = \frac{132344}{3368} = \underline{\underline{39,3}}$$

$$y_{tp} = \frac{68272}{3368} = \underline{\underline{20,3}}$$

4.8 Symmetri $\Rightarrow y_{tp} = 0$

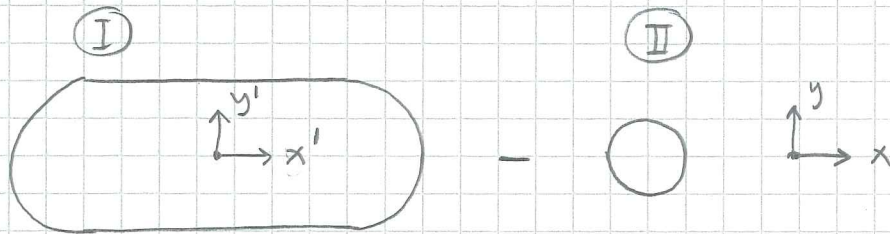


| | A | x_{tp} | $A x_{tp}$ |
|----------|------|---------------------------------------|------------|
| I | 2322 | 21,5 | 49923 |
| II | 1145 | $43 + \frac{4 \cdot 27}{3\pi} = 54,5$ | 62402 |
| Σ | 3467 | — | 112325 |

$$x_{tp} = \frac{112325}{3467} = \underline{\underline{32.4 \text{ mm}}}$$

4,9 Tänk till INNAN! Leta symmetrier & kända tyngdpunkter

Här: flytta koordinatsystemet



$$A_I = 19,26 + 95^2 \pi = 777$$

$$x'_{tp} = 0$$

$$A_{II} = 5^2 \pi = 78,5$$

$$x'_{tp} = -\left(\frac{45}{2} - 95\right) = -13$$

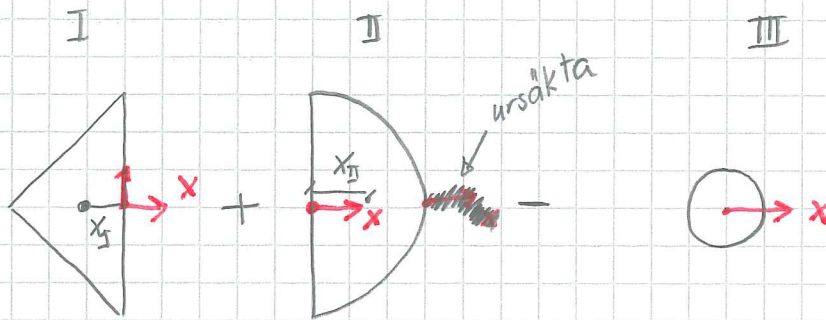
| | A | x'_{tp} | Ax'_{tp} |
|----|-------|-----------|------------|
| I | 777 | 0 | 0 |
| II | -78,5 | -13 | 1020,5 |
| | 698,5 | | 1020,5 |

$$x'_{tp} = \frac{1020,5}{698,5} = 1,46$$

Koordinatsystem enl. uppg. $\Delta 13$ mm

$$x_{tp} = 13 + 1,46 = \underline{\underline{14,5 \text{ mm}}}$$

4.10 Symmetri $\Rightarrow y_{tp} = 0$



$$A_I = \frac{17 \cdot 218}{2} = 306$$

$$A_{II} = \frac{18^2 \cdot \pi}{2} = 509$$

$$A_{III} = \frac{11^2 \cdot \pi}{4} = 95$$

$$x_I = -\frac{17}{3} = -5,67$$

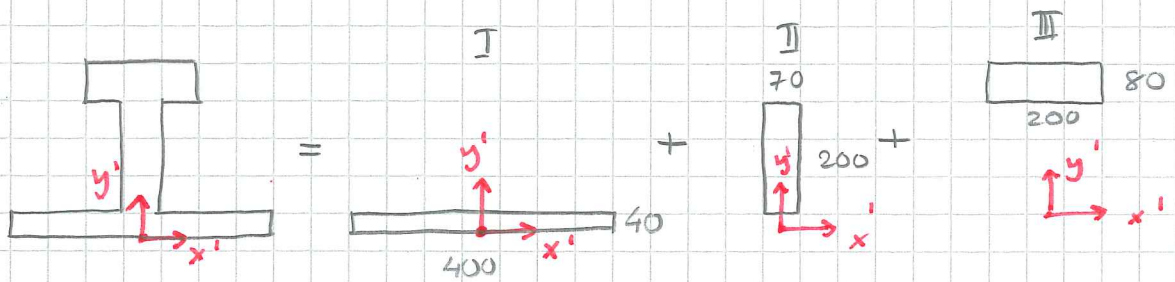
$$x_{II} = \frac{4 \cdot 18}{3\pi} = 7,64$$

$$x_{III} = 0$$

| | A | x_{tp} | Ax_{tp} |
|----------|-----|----------|-----------|
| I | 306 | -5,67 | -1735 |
| II | 509 | 7,64 | 3888 |
| III | -95 | 0 | 0 |
| Σ | 720 | | 2153 |

$$x_{tp} = \frac{2153}{720} = \underline{\underline{2,99}}$$

4.11 Flytta koordinatsystemet



Symmetri $\Rightarrow X_{tp} = 0$ eller $X_{tp} = 200 \text{ mm}$

$$A_I = 400 \cdot 40$$

$$A_{II} = 200 \cdot 70$$

$$A_{III} = 200 \cdot 80$$

$$y_I = 20$$

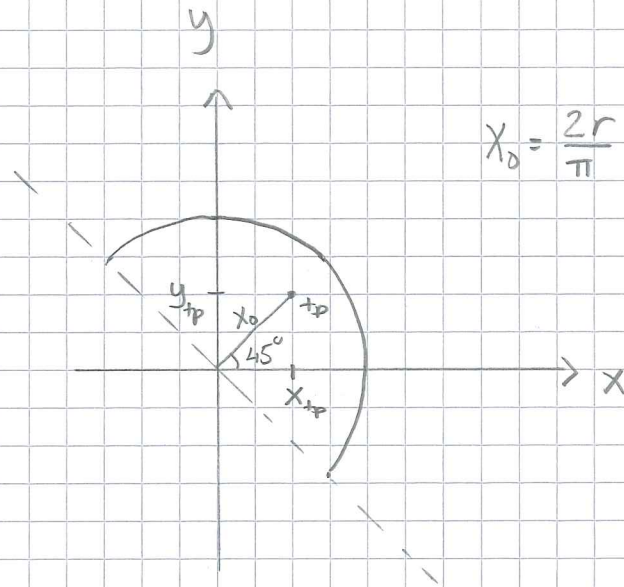
$$y_{II} = 100 + 40$$

$$y_{III} = 240 + 40$$

| | A | y_{tp} | $A y_{tp}$ |
|-----|-------|----------|------------|
| I | 16000 | 20 | 320 000 |
| II | 14000 | 140 | 1960 000 |
| III | 16000 | 280 | 4480 000 |
| | 46000 | / | 6760 000 |

$$y_{tp} = \frac{6760 000}{46000} = \underline{\underline{147 \text{ mm}}}$$

4.12

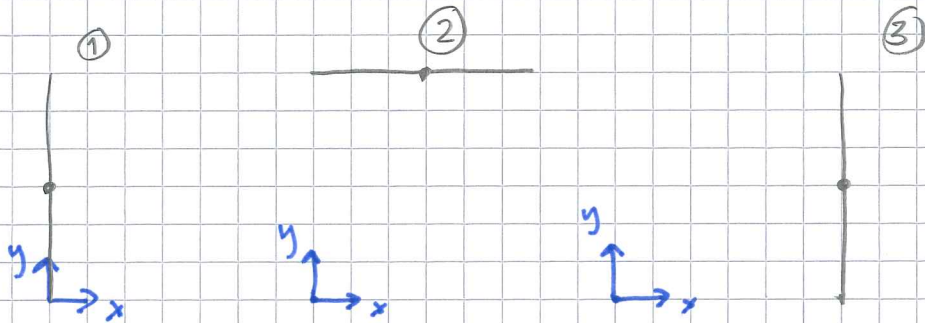


$$x_0 = \frac{2r}{\pi} = \frac{2 \cdot 15}{\pi} = 9,55$$

$$x_{tp} = 9,55 \cos 45^\circ = \underline{\underline{6,75}}$$

$$y_{tp} = x_{tp} = \underline{\underline{6,75}}$$

4.13 Area ersätts med längd då tvärsnitt är konstant.



$$L = 30$$

$$x_{tp}^1 = 0$$

$$y_{tp}^1 = 15$$

$$L = 30$$

$$x_{tp}^2 = 15$$

$$y_{tp}^2 = 30$$

$$L = 30$$

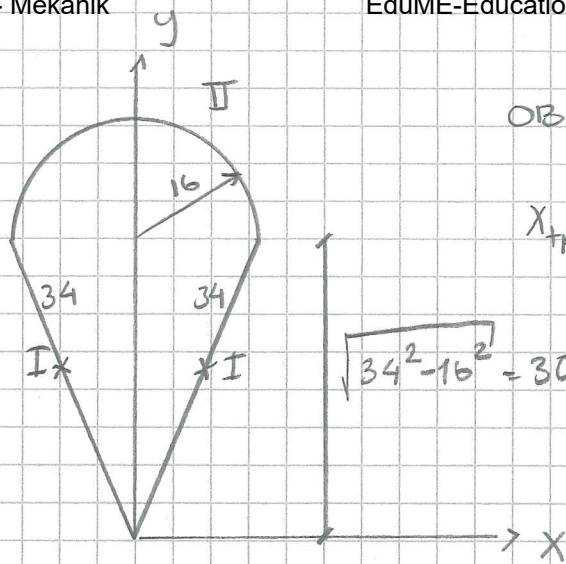
$$x_{tp}^3 = 30$$

$$y_{tp}^3 = 15$$

$$x_{tp} = \frac{\sum L \cdot x_{tp}}{L_{tot}} = \frac{30 \cdot 0 + 30 \cdot 15 + 30 \cdot 30}{3 \cdot 30} = \underline{\underline{15}} \quad \text{"inses"}$$

$$y_{tp} = \frac{30 \cdot 15 + 30 \cdot 30 + 30 \cdot 15}{3 \cdot 30} = \underline{\underline{20}}$$

4,14



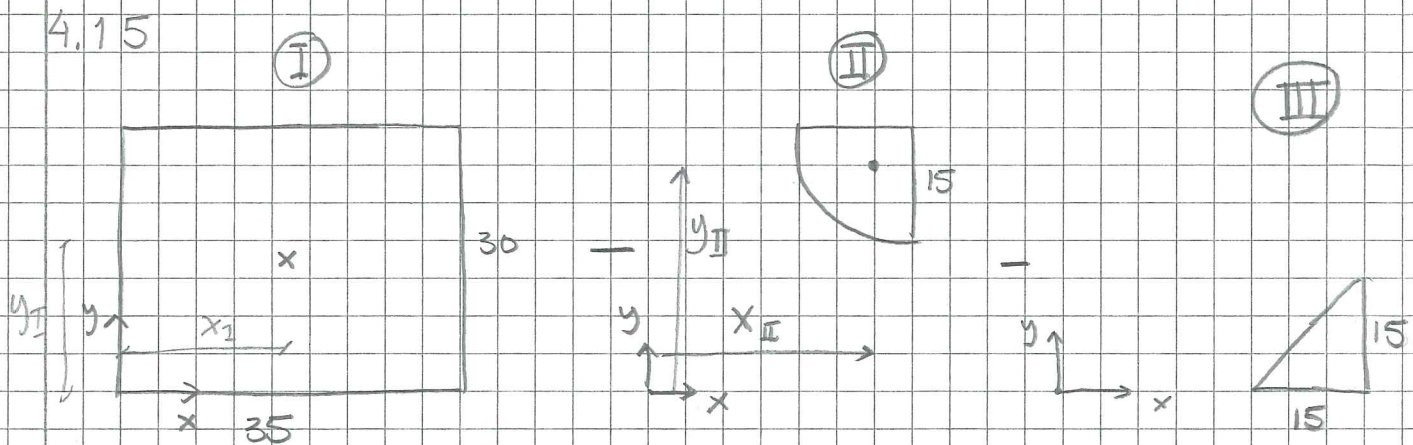
OBS! tråd

$x_{tp} = 0$ ty symmetri

| | L | y_{tp} | $L \cdot y_{tp}$ |
|----------|----------------------------------|---------------------------------------|------------------|
| I | $2 \cdot 34$ | $\frac{\sqrt{34^2 - 16^2}}{2} = 15$ | 1020 |
| II | $\frac{32 \cdot \pi}{2} = 16\pi$ | $30 + \frac{2 \cdot 16}{\pi} = 40,18$ | 2020 |
| Σ | 118,27 | | 3039,7 |

$$y_{tp} = \frac{3039,7}{118,27} = \underline{\underline{25,7}}$$

$$\underline{\underline{x_{tp} = 0}}$$



$$\textcircled{\text{I}} \quad A_I = 35 \cdot 30 = 1050$$

$$x_I = \frac{35}{2} = 17,5$$

$$y_I = \frac{30}{2} = 15$$

$$\textcircled{\text{II}} \quad A_{II} = \frac{15^2 \cdot \pi}{4} = 176,7$$

$$x_{II} = 35 - x_0 = 35 - \frac{4 \cdot 15}{3\pi} = 28,6$$

$$y_{II} = 30 - x_0 = 30 - 6,37 = 23,6$$

$$\textcircled{\text{III}} \quad A_{III} = \frac{15 \cdot 15}{2} = 112,5$$

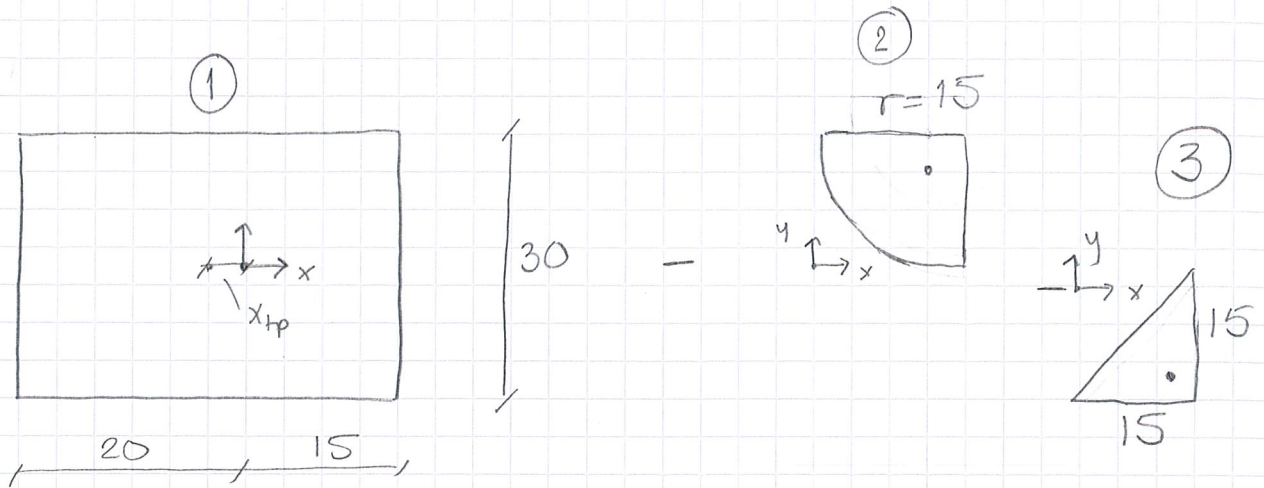
$$x_{III} = 35 - \frac{h}{3} = 35 - 5 = 30$$

$$y_{III} = \frac{15}{3} = 5$$

$$x_{tp} = \frac{1050 \cdot 17,5 - 177 \cdot 28,6 - 112,5 \cdot 30}{1050 - 177 - 112,5} = 13,07 \text{ mm}$$

$$y_{tp} = \frac{1050 \cdot 15 - 177 \cdot 23,6 - 112,5 \cdot 5}{1050 - 177 - 112,5} = 14,48 \text{ mm}$$

4.15 Bokens koordinatsystem

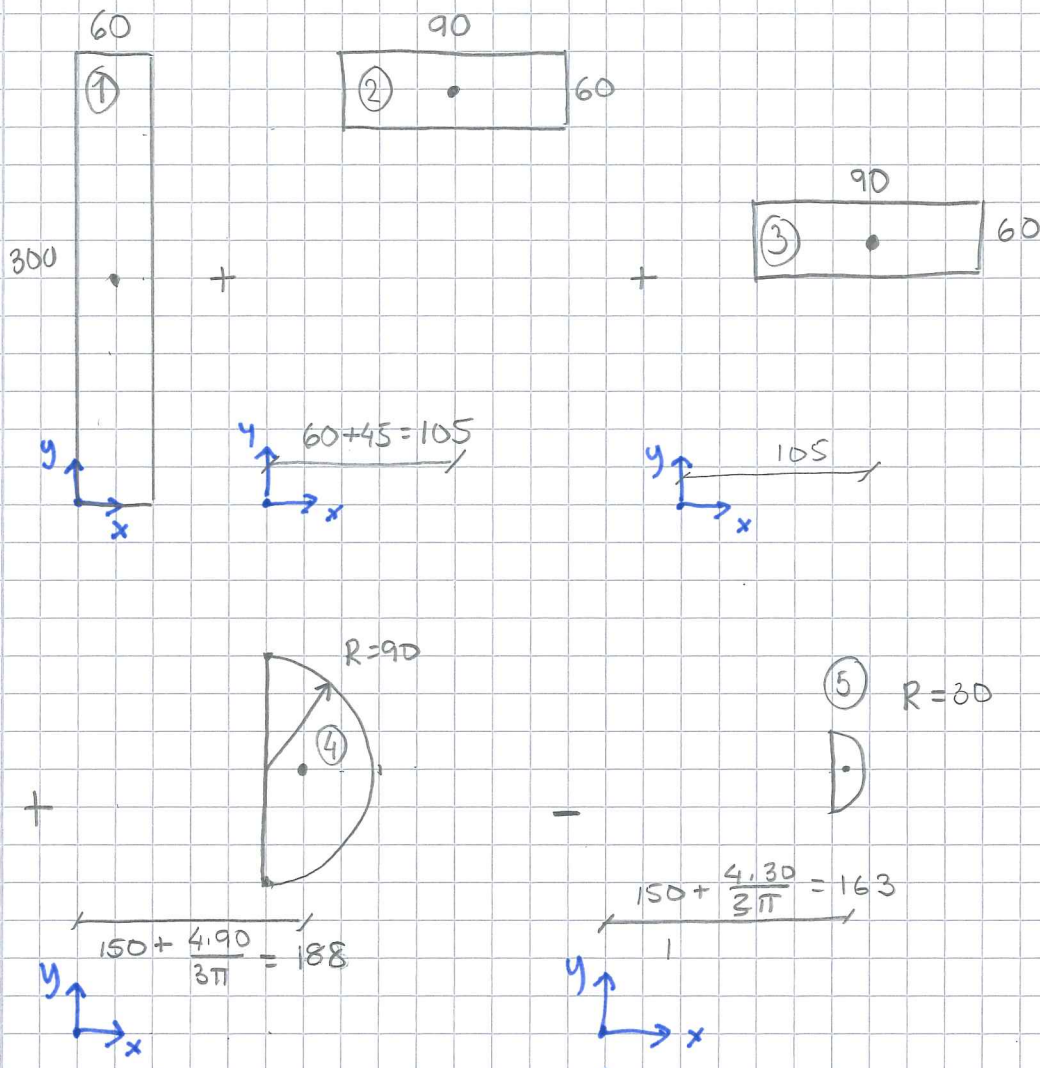


| | A | x_{tp} | y_{tp} | $A x_{tp}$ | $A y_{tp}$ |
|---|---------------------------------|--|----------|------------|------------|
| ① | $30 \cdot 35 =$ 1050 | -2,5 | 0 | -2625 | 0 |
| ② | $-\frac{15^2 \pi}{4} =$ -177 | $15 - \frac{4 \cdot 15}{3\pi} =$ 8,63 | 8,63 | -1527 | -1527 |
| ③ | $-\frac{15^2}{2} =$ -112,5 | 10 | -10 | -1125 | 1125 |
| | 760,5 | | | -5277 | -402 |

$$x_{tp} = \frac{-5277}{760,5} = -6,9$$

$$y_{tp} = \frac{-402}{760,5} = 0,53$$

4.16 Många lösningsstrategier är möjliga.
Försök att minska antal delar



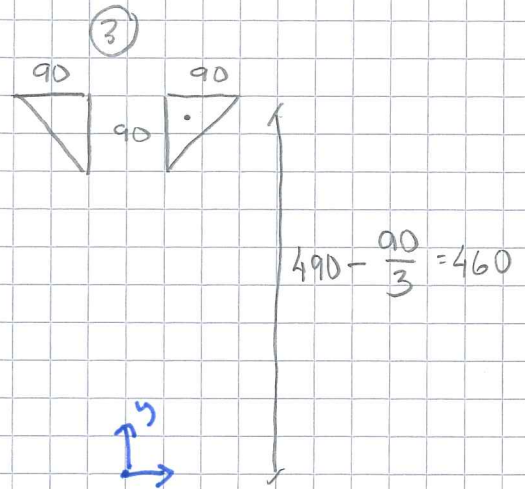
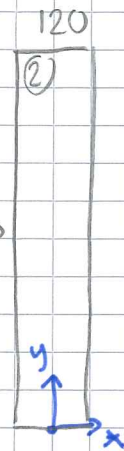
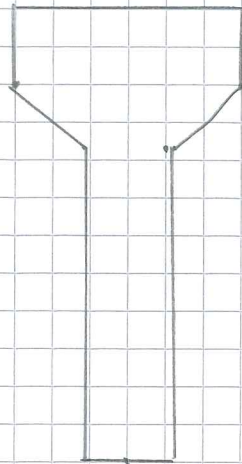
| | A_n | x_{tp} | y_{tp} | $A_n x_{tp}$ | $A_n y_{tp}$ |
|----------|-------------------------------------|----------|----------|--------------|--------------|
| 1 | $60 \cdot 300 = 18000$ | 30 | 150 | 540000 | 2700000 |
| 2 | $90 \cdot 60 = 5400$ | 105 | 270 | 567000 | 1458000 |
| 3 | $90 \cdot 60 = 5400$ | 105 | 150 | 567000 | 810000 |
| 4 | $\frac{90^2 \cdot \pi}{2} = 12723$ | 188 | 210 | 2391924 | 2671830 |
| 5 | $\frac{-30^2 \cdot \pi}{2} = -1414$ | 163 | 210 | -230482 | -296940 |
| Σ | 40109 | \ | \ | 3835442 | 7342890 |

$$x_{tp} = \frac{3835442}{40109} = \underline{\underline{95,6}}$$

$$y_{tp} = \frac{7342890}{40109} = \underline{\underline{183,42}}$$

4,17

$X_{tp} = 0$ (symmetri)



| | A_n | y_{tp} | $A_n y_{tp}$ |
|---|---------------------------------|----------|--------------|
| ① | $300 \cdot 110 = 33000$ | 545 | 17985 000 |
| ② | $490 \cdot 120 = 58800$ | 245 | 14406 000 |
| ③ | $\frac{90^2}{2} \cdot 2 = 8100$ | 460 | 3726 000 |
| | 99900 | | 36117 000 |

$$y_{tp} = \frac{36117000}{99900} = \underline{\underline{362 \text{ mm}}}$$